Research Day 2017

Gruppo di matematica discreta e computazionale

11 aprile 2017

Representation of algebras

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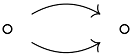
A simple example is the quiver



corresponding to the algebra

$$\begin{bmatrix} k & k^2 \\ 0 & k \end{bmatrix}$$

Representation of algebras

Modules over the Kronecker algebra can be seen as quadruples

$$V \underset{g}{\overset{f}{\rightrightarrows}} W$$

the algebra itself corresponding to $V = W = k$ and $f = g$ the identity

Such algebras are called *path algebras*

Representation of algebras

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It is so interesting to classify indecomposable modules

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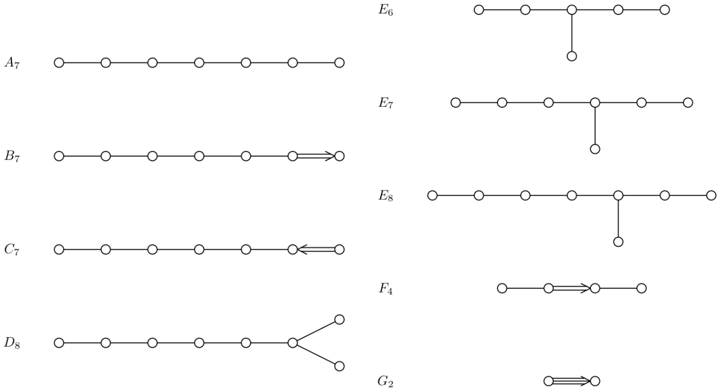
A classical theorem by Gabriel determines all path algebras of tame representation type as the ones whose quiver is built from a Dynkin diagram

Dynkin diagrams



Representation of algebras

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It has several connections with geometry and other branches of mathematics, mainly category theory, spectral sequences and derived categories

Listing of faces of polytopes

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Some recent developments suggest the possibility of approaching this problem from a novel point of view

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Suppose you have to place durations or plan temporal events, subject to certain constraints and where some durations depend on external variables (not under our control): can we still control such a situation? Are there algorithms for determining such dynamic choices?

Circular flux

Given a graph, we look for an orientation of the edges and an attribution of numbers so that the sum of the in-going values equals the sum of the out-going values

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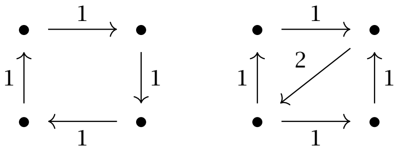

Circular flux

More precisely, given a graph (V, E), we want an orientation of the edges and a map

$$f : E \to [1, v-1]$$

such that, for each $v \in V$,

$$\sum_{\overset{\longrightarrow}{e} v} f(e) = \sum_{v \overset{\longrightarrow}{e}} f(e)$$

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Such a pair of orientation and map is called a v-CF

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Problems

- For what graphs does equality hold?
- Find minimal counterexamples

Constructive proofs

A well known proof of the infinitude of primes goes along like

Suppose, by contradiction, that $p_1, p_2, \ldots, p_n$ are all the primes
Then $p_1 p_2 \cdots p_n + 1$ is composite, but it is not divisible by any of the listed primes: contradiction

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Consider the minimum number q *such that* $q > 1$ *and* $q \mid m$
Then q *is a prime number not appearing in the given list*

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- $2, 5, 11$: $m = 110 + 1 = 111$, $q = 3$

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- $2, 2, 3$: $m = 12 + 1 = 13$, $q = 13$
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There are many other proof in elementary and higher mathematics that can be transformed in a constructive fashion