Moto smorzato esponenzialmente:

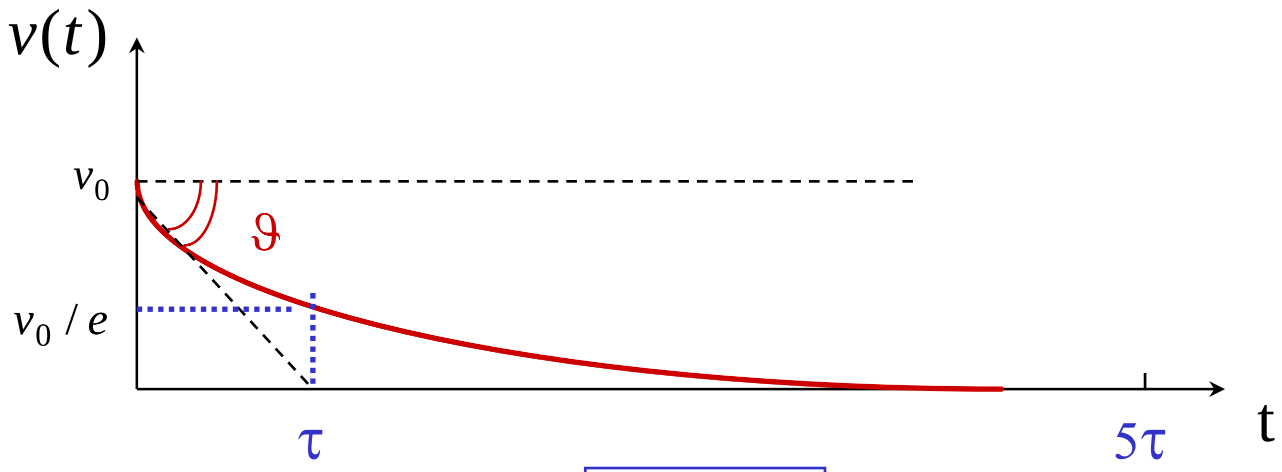
si verifica in presenza di una decelerazione di tipo “viscoso”,

ossia proporzionale alla velocità :

$$a(t) = \frac{dv(t)}{dt} = -kv(t)$$

$$\Rightarrow \quad \frac{dv(t)}{v} = -kdt \quad \Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -k\int_0^t dt$$

$$\Rightarrow \quad \ln\left(\frac{v}{v_0}\right) = -kt \quad \Rightarrow \quad v(t) = v_0 e^{-kt}$$



$$v(t = 1/k) = v_0 e^{-1} \quad \Rightarrow \quad \tau \equiv 1/k$$

“costante di tempo” dello smorzamento

Per t ≈5 τ : $v(t = 5\tau) = v_0 e^{-5} \approx 0.006 v_0 \approx 0$

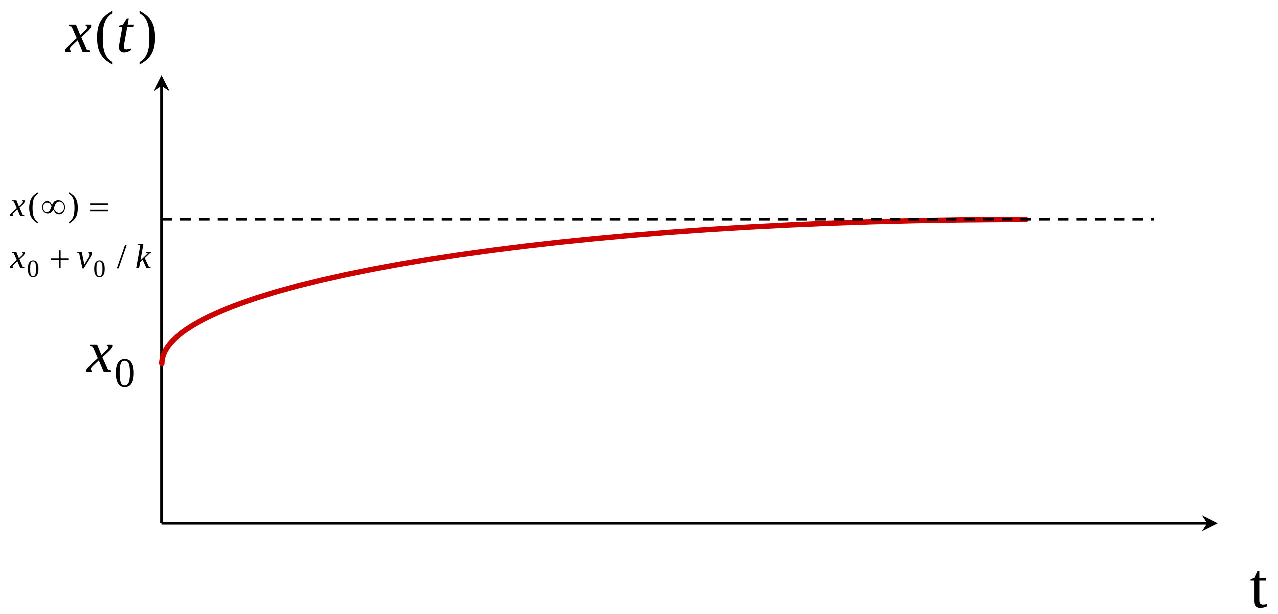
τ è l’intersezione con l’asse dei tempi della retta tangente alla curva $v(t)$ al tempo t = 0 :

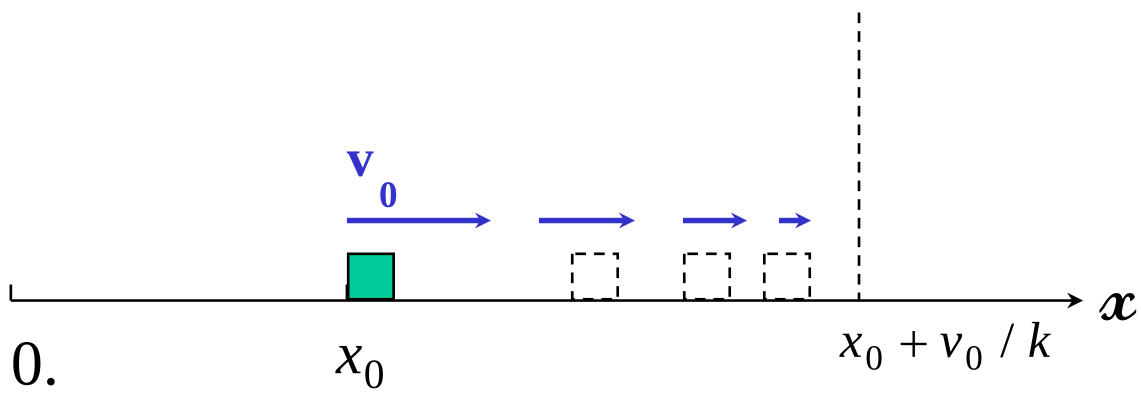
$$\tan\vartheta \equiv \left.\frac{dv}{dt}\right|_{t=0} = -\left.kv_0 e^{-kt}\right|_{t=0} = -kv_0 = -\frac{v_0}{\tau}$$

Spazio percorso in un moto smorzato esponenzialmente :

$$x(t) = x_0 + \int_0^t v(t)dt = x_0 + \int_0^t v_0 e^{-kt} dt = x_0 - \frac{v_0}{k} e^{-kt}\Big|_0^t$$

$$\boxed{x(t) = x_0 + \frac{v_0}{k}\left(1 - e^{-kt}\right)}$$





Moto accelerato in presenza di un attrito viscoso:

$$\boxed{\frac{dv(t)}{dt} = a - kv(t)}$$

termine costante (es: g)

$$\Rightarrow \quad \frac{dv(t)}{a - kv(t)} = dt$$

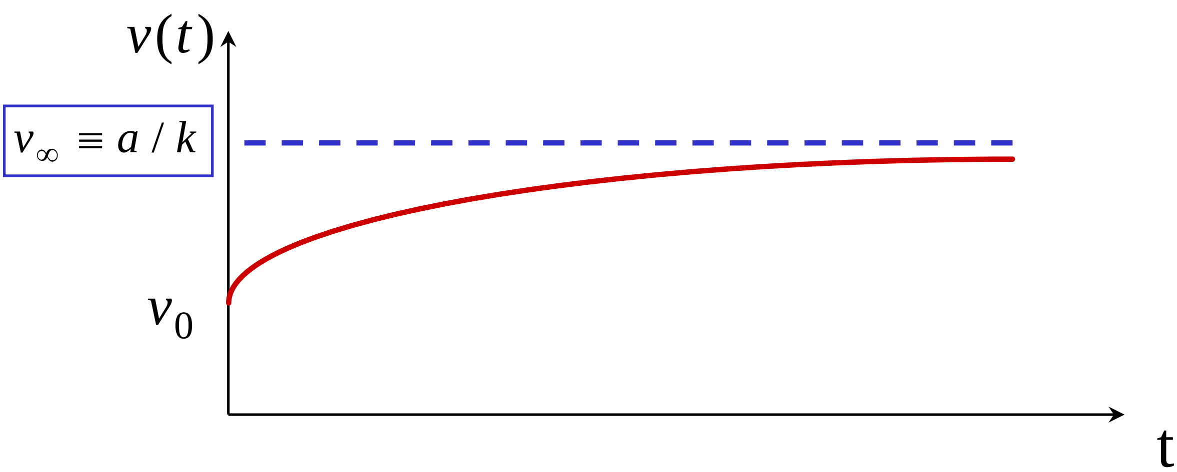
Posto: $w(t) \equiv a - kv(t)$

$\rightarrow dw \equiv -k\,dv$

$$\Rightarrow \quad \frac{1}{k}\frac{dw(t)}{w} = -dt \quad \Rightarrow \quad \ln\left(\frac{w}{w_0}\right) = -kt \quad \Rightarrow \quad w(t) = w_0 e^{-kt}$$

$$\Rightarrow \quad a - kv(t) = (a - kv_0)e^{-kt} \quad \Rightarrow \quad v(t) = \frac{a}{k} - \left(\frac{a}{k} - v_0\right)e^{-kt}$$

$$\Rightarrow \quad \boxed{v(t) = \frac{a}{k} + \left(v_0 - \frac{a}{k}\right)e^{-kt}}$$



"velocità limite" : $v_\infty \equiv \lim_{t \to \infty} v(t) = a / k$ (indipendente da v_0)