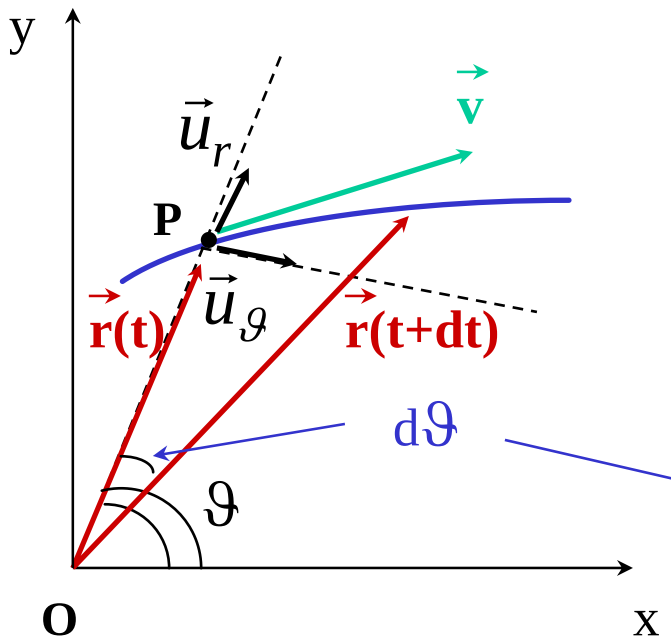
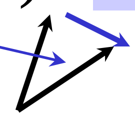
Componenti polari della velocità:



$$\vec{u}_r^{\,2} \equiv 1 = \text{ costante}$$

$$\Rightarrow \quad d\vec{u}_r^{\,2} = 2\vec{u}_r \cdot d\vec{u}_r = 0$$

$$\Rightarrow \quad \vec{u}_r \perp d\vec{u}_r$$



$$d\vec{u}_r = d\vartheta \vec{u}_\vartheta$$

$$\vec{r}(t) = r(t)\vec{u}_r(t)$$

$$= \frac{d\vartheta}{dt}\vec{u}_\vartheta$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dr(t)}{dt}\vec{u}_r + r(t)\frac{d\vec{u}_r(t)}{dt}$$

$$\Rightarrow \quad \boxed{\vec{v}(t) = \frac{dr(t)}{dt}\vec{u}_r + r(t)\frac{d\vartheta(t)}{dt}\vec{u}_\vartheta}$$

$\vec{v}_r$ "velocità radiale"

$\vec{v}_\vartheta$ "velocità trasversa"

$$\vec{v}(t) = \left(\frac{dr(t)}{dt}, r(t)\frac{d\vartheta(t)}{dt}\right) = \left(\frac{dx(t)}{dt}, \frac{dy(t)}{dt}\right)$$

componenti polari

componenti cartesiane

Componenti polari dell'accelerazione

$$\vec{a} \equiv \frac{d\vec{v}(t)}{dt} = \frac{d}{dt}\left(\frac{dr}{dt}\vec{u}_r + r\frac{d\vartheta}{dt}\vec{u}_\vartheta\right) =$$

$$= \frac{d^2r}{dt^2}\vec{u}_r + \frac{dr}{dt}\frac{d\vec{u}_r}{dt} + \frac{dr}{dt}\frac{d\vartheta}{dt}\vec{u}_\vartheta + r\frac{d^2\vartheta}{dt^2}\vec{u}_\vartheta + r\frac{d\vartheta}{dt}\frac{d\vec{u}_\vartheta}{dt}$$

$$\frac{d\vec{u}_r}{dt} = \frac{d\vartheta}{dt}\vec{u}_\vartheta \qquad\qquad \frac{d\vec{u}_\vartheta}{dt} = -\frac{d\vartheta}{dt}\vec{u}_r$$

$$\frac{dr}{dt}\frac{d\vec{u}_r}{dt} + \frac{dr}{dt}\frac{d\vartheta}{dt}\vec{u}_\vartheta = 2\frac{dr}{dt}\frac{d\vartheta}{dt}\vec{u}_\vartheta$$

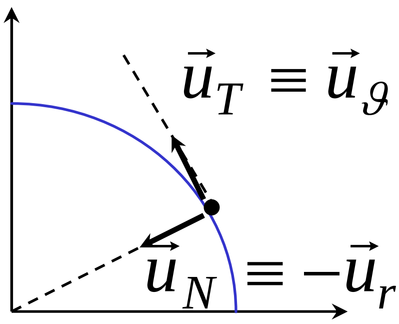
$$\Rightarrow \quad \boxed{\vec{a} = \left(\frac{d^2r}{dt^2} - r\left(\frac{d\vartheta}{dt}\right)^2\right)\vec{u}_r + \left(2\frac{dr}{dt}\frac{d\vartheta}{dt} + r\frac{d^2\vartheta}{dt^2}\right)\vec{u}_\vartheta}$$

$\vec{a}_r$ "accelerazione radiale" $\qquad$ $\vec{a}_\vartheta$ "accelerazione trasversa"

In un moto circolare (r = costante) :

$$a_r = -r\left(\frac{d\vartheta}{dt}\right)^2 = -r\omega^2 \equiv -a_N$$

$$a_\vartheta = r\frac{d^2\vartheta}{dt^2} = r\frac{d\omega}{dt} = r\alpha \equiv a_T$$



Moto circolare uniforme:

velocità con modulo costante:

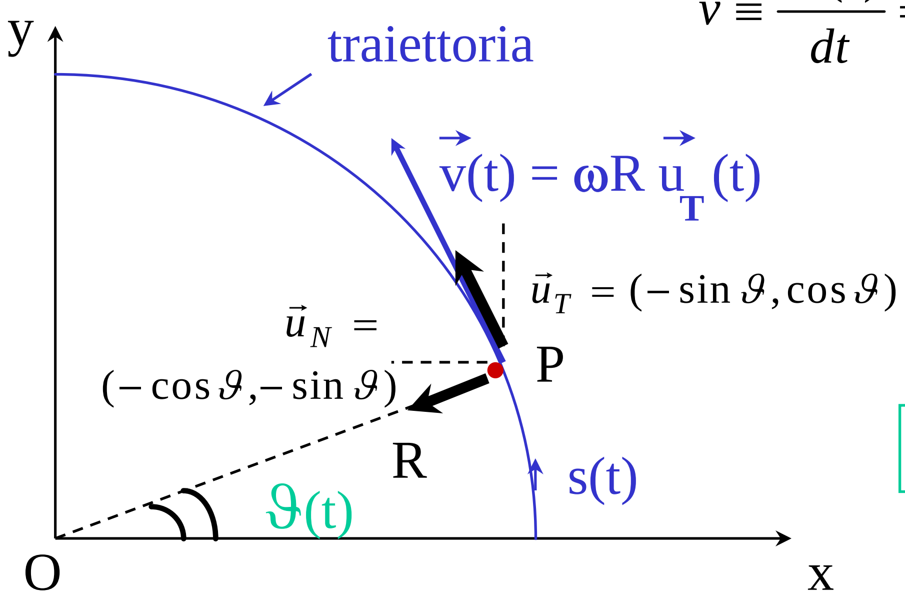
coordinata curvilinea
$s(t)=R\vartheta(t)$

$$v \equiv \frac{ds(t)}{dt} = R\frac{d\vartheta(t)}{dt} = \omega R$$

"velocità angolare"

$$\omega \equiv \frac{d\vartheta(t)}{dt}$$

$$\vartheta(t) = \vartheta_0 + \omega t$$



$$x(t) = R\cos\vartheta(t)$$
$$y(t) = R\sin\vartheta(t)$$

$$\Rightarrow \quad v_x(t) = \frac{dx(t)}{dt} = -R\sin\vartheta(t)\frac{d\vartheta}{dt} \equiv -R\omega\sin\vartheta(t)$$
$$v_y(t) = \frac{dy(t)}{dt} = R\cos\vartheta(t)\frac{d\vartheta}{dt} \equiv R\omega\cos\vartheta(t)$$

$$\Rightarrow \quad \vec{v}(t) = (v_x(t), v_y(t)) = R\omega(-\sin\vartheta(t), \cos\vartheta(t))$$

$$\Rightarrow \quad \vec{v}(t) = R\omega\vec{u}_T(t) = v\vec{u}_T(t) \qquad \vec{u}_T$$

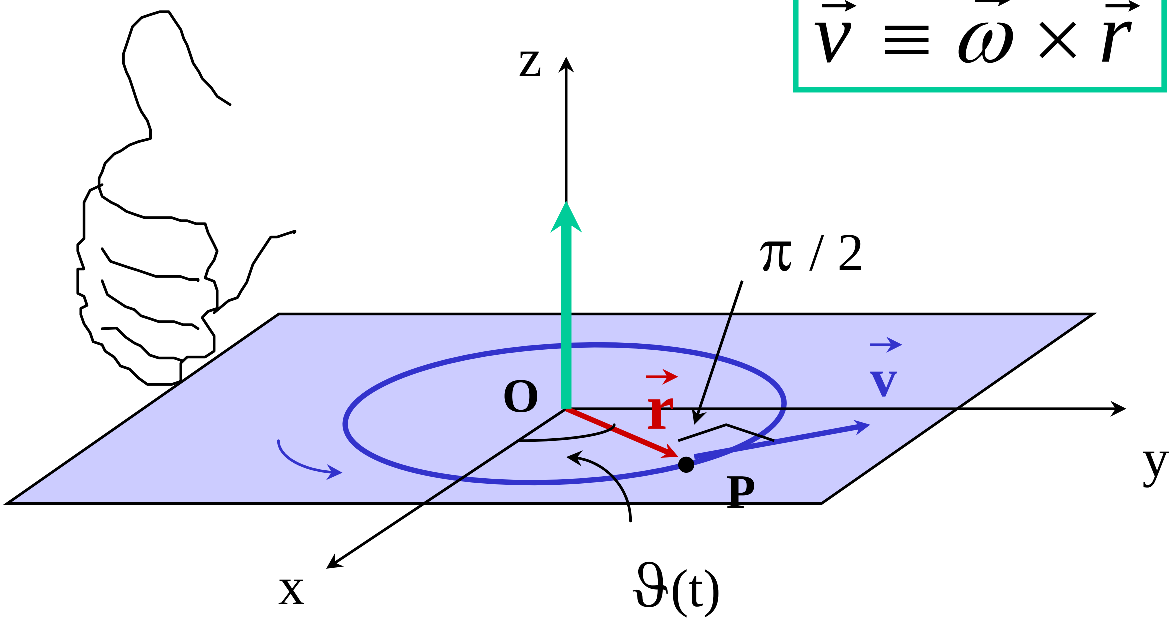
$$a_x(t) = \frac{dv_x(t)}{dt} = -R\omega\cos\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\cos\vartheta(t)$$
$$a_y(t) = \frac{dv_y(t)}{dt} = -R\omega\sin\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\sin\vartheta(t)$$

$$\Rightarrow \quad \vec{a}(t) = (a_x(t), a_y(t)) = R\omega^2(-\cos\vartheta(t), -\sin\vartheta(t))$$

$$\Rightarrow \quad \vec{a}(t) = R\omega^2\vec{u}_N(t) = \frac{v^2}{R}\vec{u}_N(t) \qquad \vec{u}_N$$

Vettore velocità angolare $\vec{\omega}$:

$$\vec{v} \equiv \vec{\omega} \times \vec{r}$$



$$\omega \equiv \frac{d\vartheta(t)}{dt}$$

$\Rightarrow$

$\vec{\omega}$ è $\perp$ al piano del moto, con verso definito dalla "regola della mano destra"

Infatti:

$$|\vec{\omega} \times \vec{r}| \equiv \omega r \sin\frac{\pi}{2} = \omega r \equiv v = \frac{ds(t)}{dt} = r\frac{d\vartheta(t)}{dt} \quad \Rightarrow \quad \omega \equiv \frac{d\vartheta(t)}{dt}$$

Vettore accelerazione angolare $\vec{\alpha}$

$$\vec{\alpha} \equiv \frac{d\vec{\omega}(t)}{dt}$$

Accelerazione:

$$\vec{a} \equiv \frac{d\vec{v}(t)}{dt} = \frac{d(\vec{\omega}\times\vec{r})}{dt} = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$\frac{d\vec{\omega}}{dt} \equiv \vec{\alpha}$ $\qquad$ $\frac{d\vec{r}}{dt} = \vec{v} \equiv \vec{\omega}\times\vec{r}$

$$\Rightarrow \quad \vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$\vec{\alpha}\times\vec{r} \rightarrow \vec{a}_T$ $\qquad$ $\vec{\omega}\times(\vec{\omega}\times\vec{r}) \rightarrow \vec{a}_N$