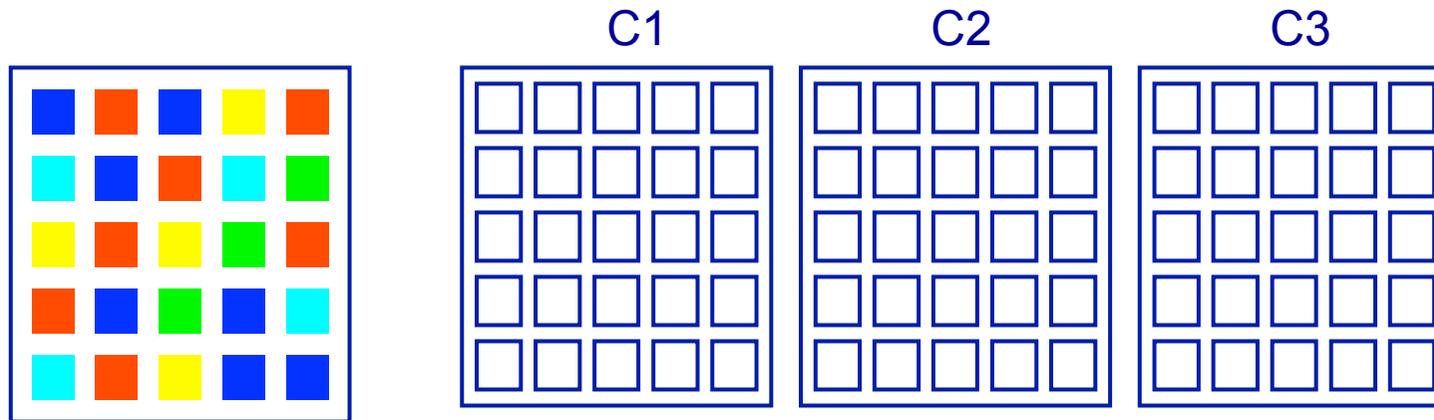


Color imaging

Color images



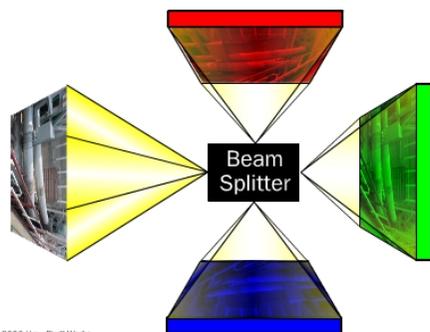
- Each colored pixel corresponds to a *vector* of three values $\{C1, C2, C3\}$
- The characteristics of the components depend on the chosen *colorspace* (RGB, YUV, CIE Lab,..)

Digital Color Images

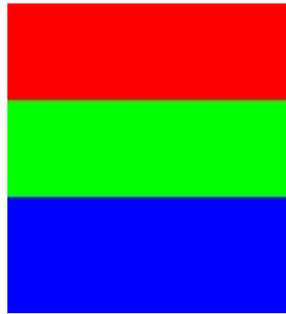
- $x_R[n_1, n_2]$

$$x_G[n_1, n_2]$$

$$x_B[n_1, n_2]$$



Color channels



Red



Green



Blue

Color channels



Red



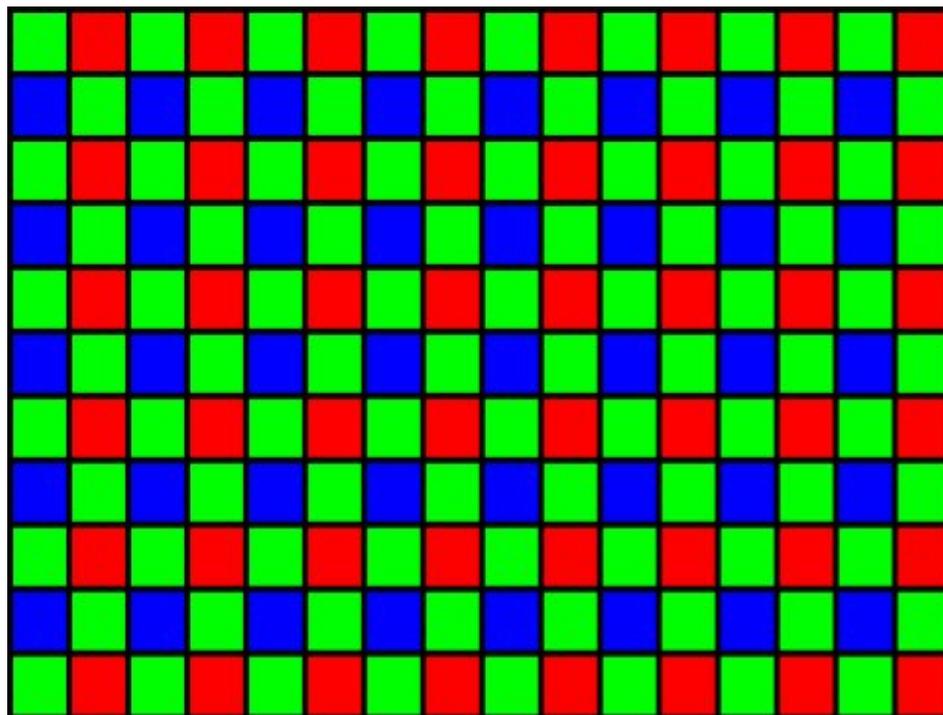
Green



Blue



Bayer matrix



Typical sensor topology in CCD devices.
The green is twice as numerous as red and blue.

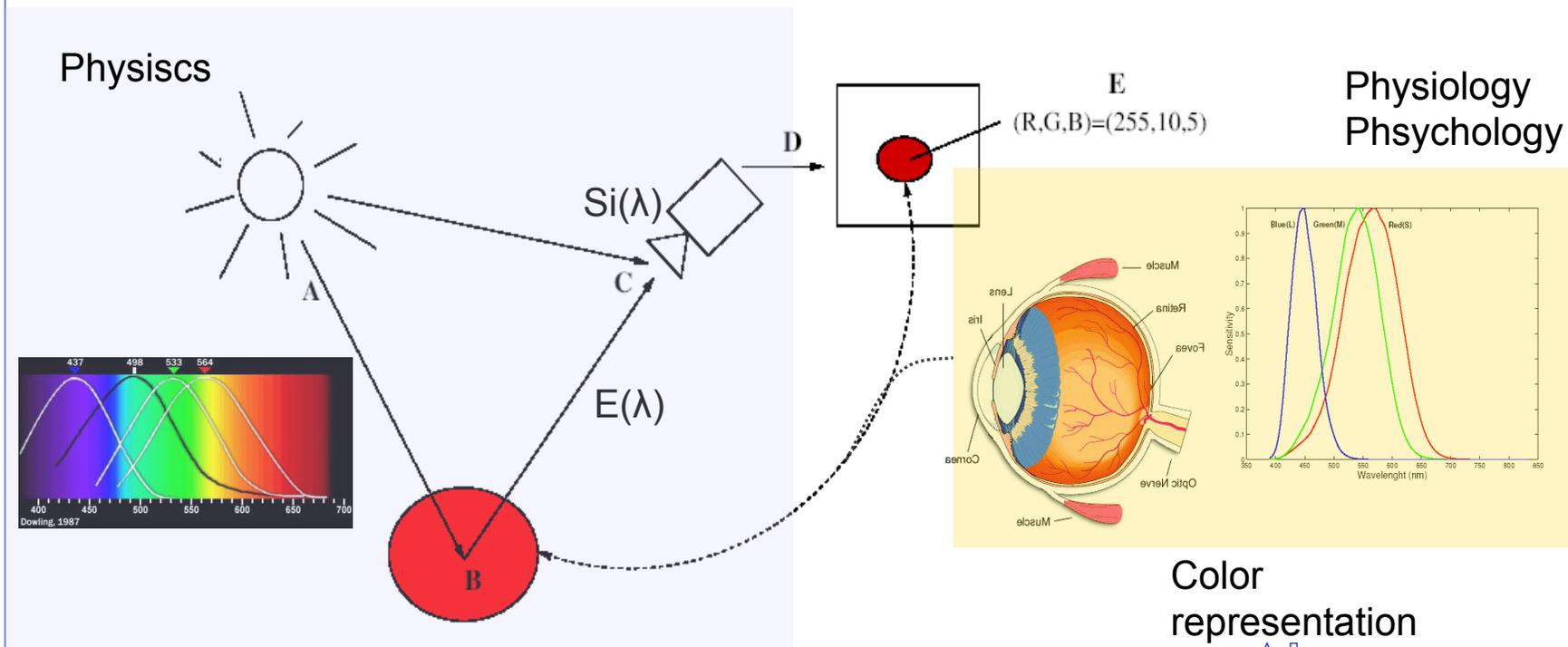
Color imaging

- Color reproduction
 - Printing, rendering
- Digital photography
 - High dynamic range images
 - Mosaicking
 - Compensation for differences in illuminant (CAT: chromatic adaptation transforms)
- Post-processing
 - Image enhancement
- Coding
 - Quantization based on color CFSs (contrast sensitivity function)
 - Downsampling of chromatic channels with respect to luminance

Color science

- Color vision
 - *Seeing* colors
 - Foundations of color vision
 - Trichromatic model
- Color naming
 - *Attaching labels* to colors
- Colorimetry & Photometry
 - *Measuring* colors: radiometric & photometric units
- Applications
 - Image rendering, cross-media color reproduction, image analysis, feature extraction, image classification, data mining...

What is color?



$$C_i = \int_{\lambda} E(\lambda) S_i(\lambda) d\lambda$$

$S_i(\lambda)$: sensitivity of the i^{th} sensor

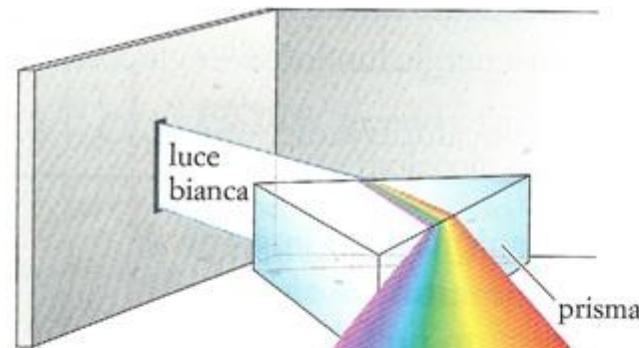
$E(\lambda)$: Spectral Power Distribution (SPD) of the diffused light

Color representation

Modeling

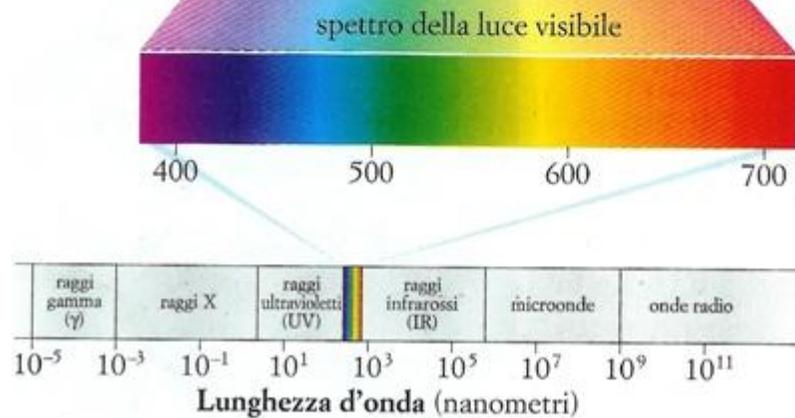
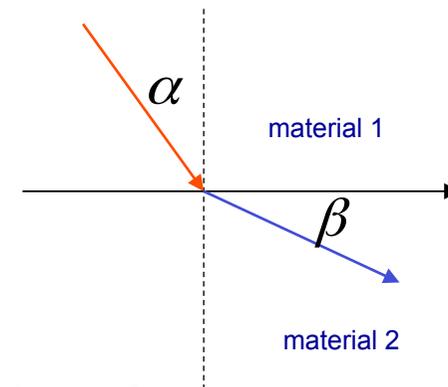
Color perception

Newton's prism

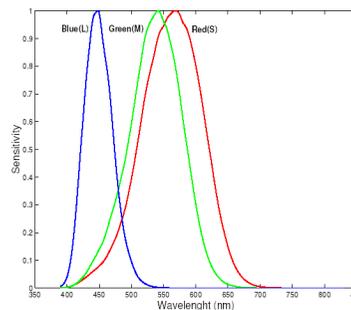
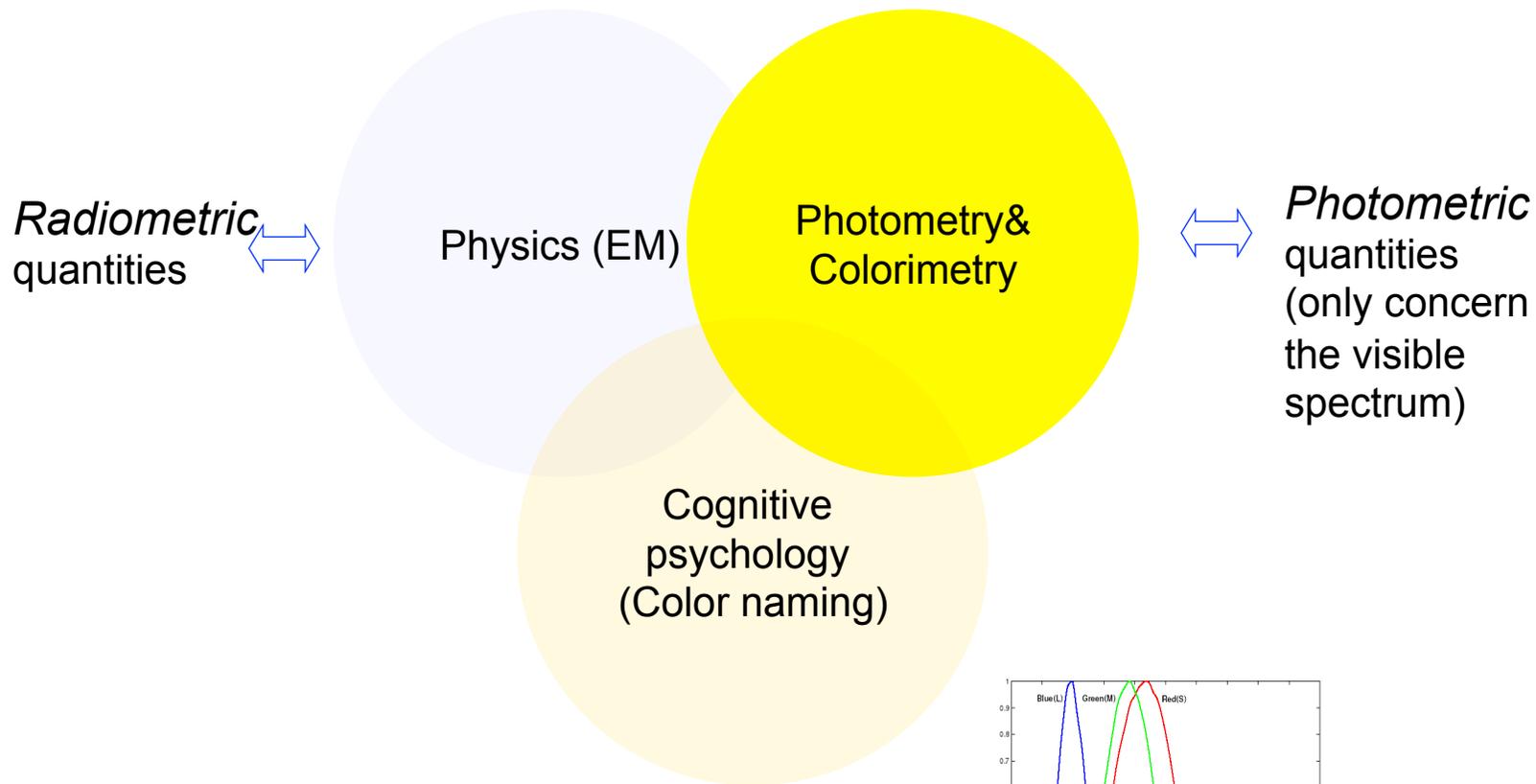


9.2 La luce bianca si separa nelle sue componenti quando passa attraverso un prisma.

$$\sin \alpha = n(\lambda) \sin \{ \beta(\lambda) \}$$

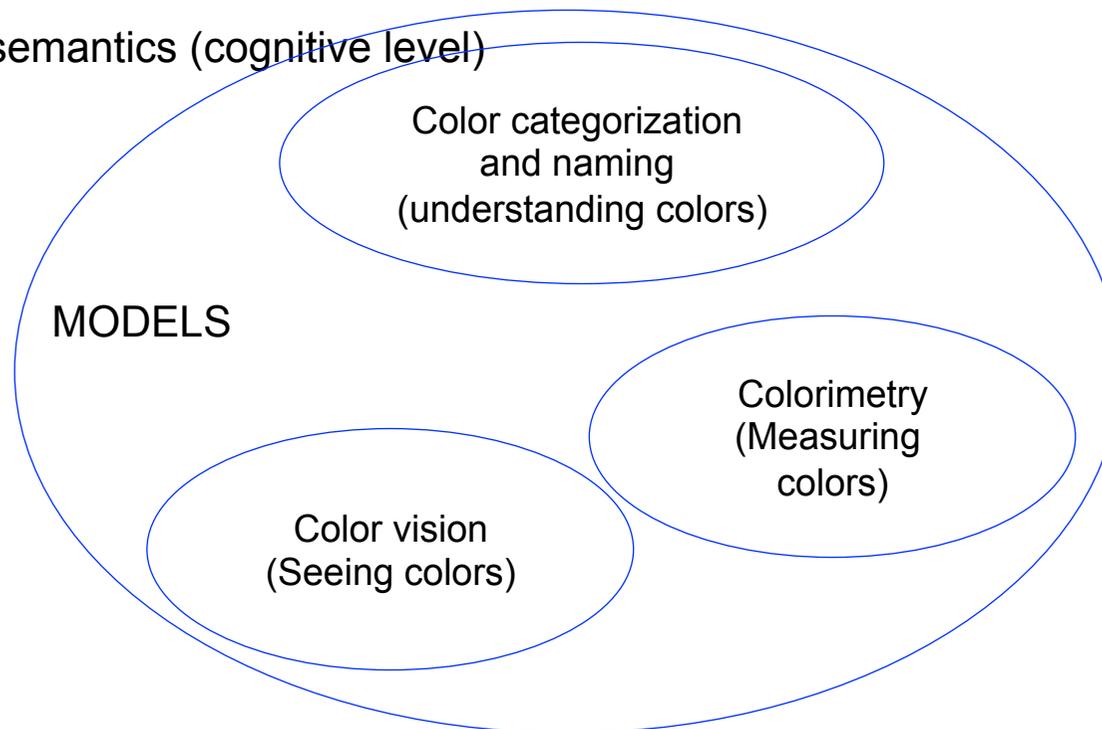


What is color?

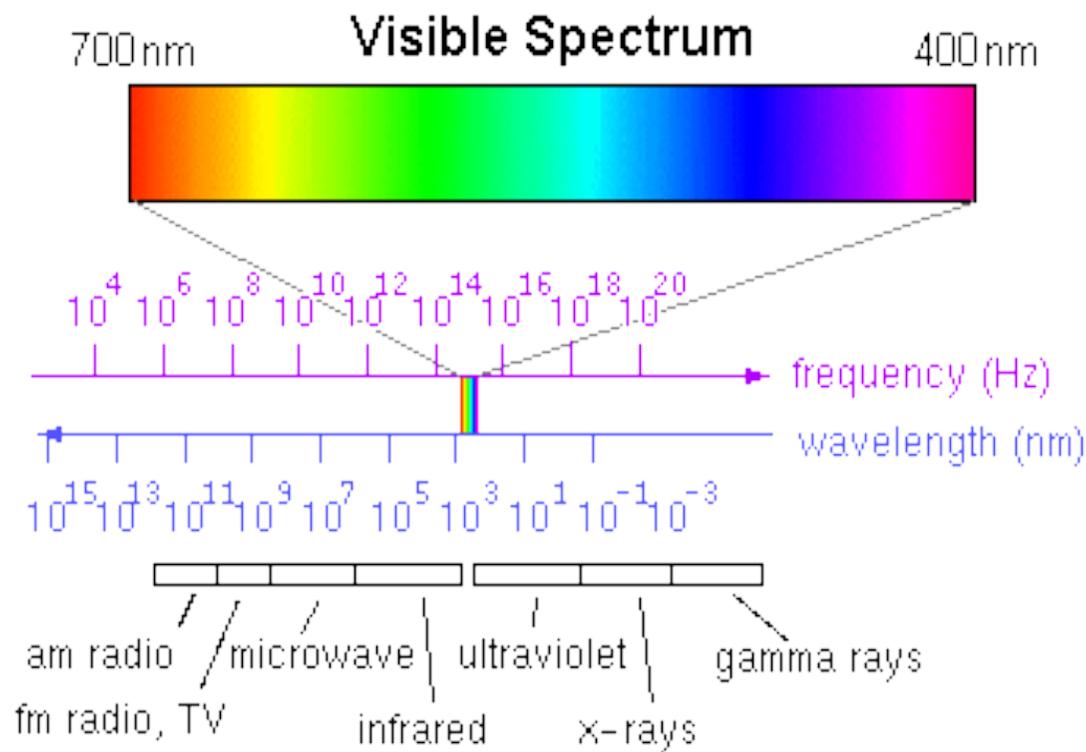


Color

- Human vision
 - Color encoding (receptor level)
 - Color perception (post-receptor level)
 - Color semantics (cognitive level)
- Colorimetry
 - Spectral properties of radiation
 - Physical properties of materials

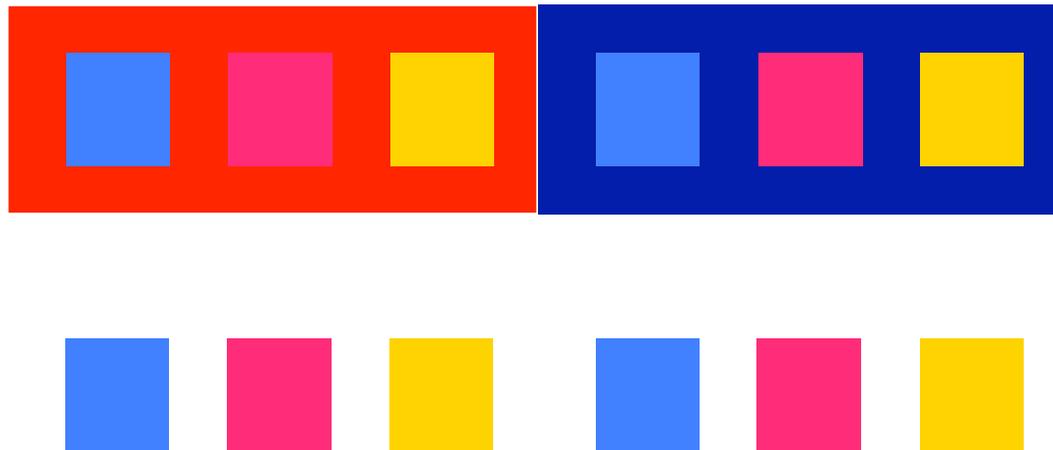


The physical perspective



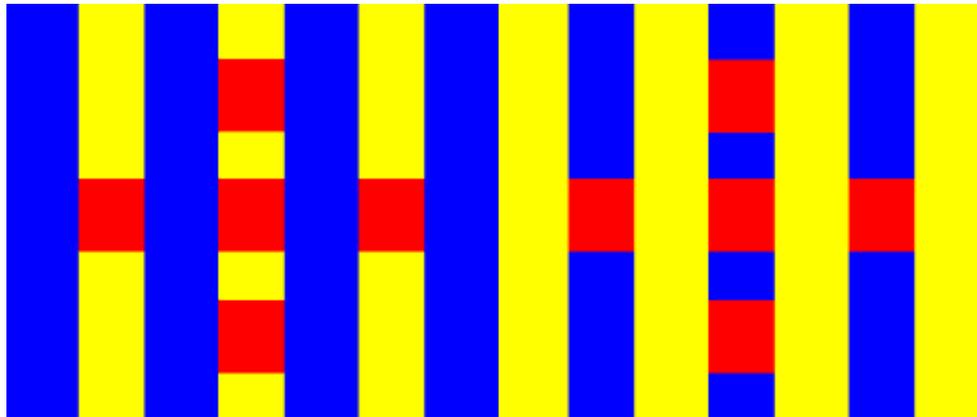
The perceptual perspective

Simultaneous contrast



Color

- Chromatic induction



Basic quantities

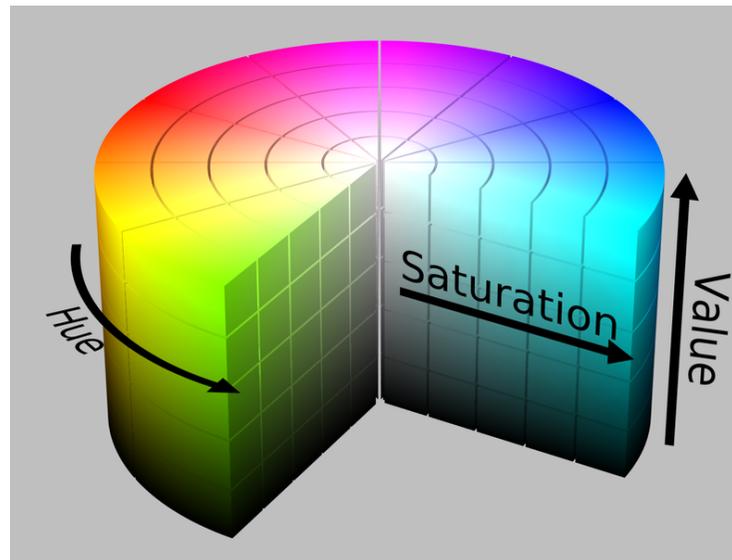
- **Radiance:** total amount of energy that flows from the light source
 - Physical quantity
 - measured in Watts [W] by a radiometer
- **Luminance:** measure of the amount of light emitted by the source that a person *perceives*
 - *Perceptual* quantity
 - measured in lumens [lm]
 - it is assessed by “weighting” the light emitted by the source by the absorption curves of the “standard subject”
- **Brightness:** *psychological* quantity that is it impossible to measure “objectively”. It embodies the achromatic notion of “intensity”
 - *Psychological* quantity

Color models

- A color model is a 3D *unique* representation of a color
- There are different color models and the use of one over the other is problem oriented. For instance
 - RGB color model is used in hardware applications like PC monitors, cameras and scanners
 - CMY color model is used in color printers
 - YIQ model in television broadcast
 - In color image manipulation the two models widely used are HSI and HSV
 - *Uniform* color models (CIELAB, CIELUV) are used in color imaging
- [Gonzalez Chapter 6]

Color models

- User-oriented color models
 - Emphasize the intuitive color notions of brightness, hue and saturation
 - HSV (Hue, saturation, Value)
 - HSI (Hue, Saturation, Intensity)
 - HSL (Hue, Saturation, Lightness)



Color models

- Device-oriented color models
 - The color representation depends on the device.
- Concerns both acquisition and display devices
 - Acquisition
 - The value of the color numerical descriptors depend on the spectral sensitivity of the camera sensors
 - Display
 - A color with given numerical descriptors appears different if displayed on another device or if the set-up changes
 - In RGB for instance, the R,G and B components depend on the chosen red, green and blue primaries as well as on the reference white
 - Amounts of ink expressed in CMYK or digitized video voltages expressed in RGB
 - RGB, Y' CbCr, Y' UV, CMY, CMYK
 - Towards device independence: sRGB

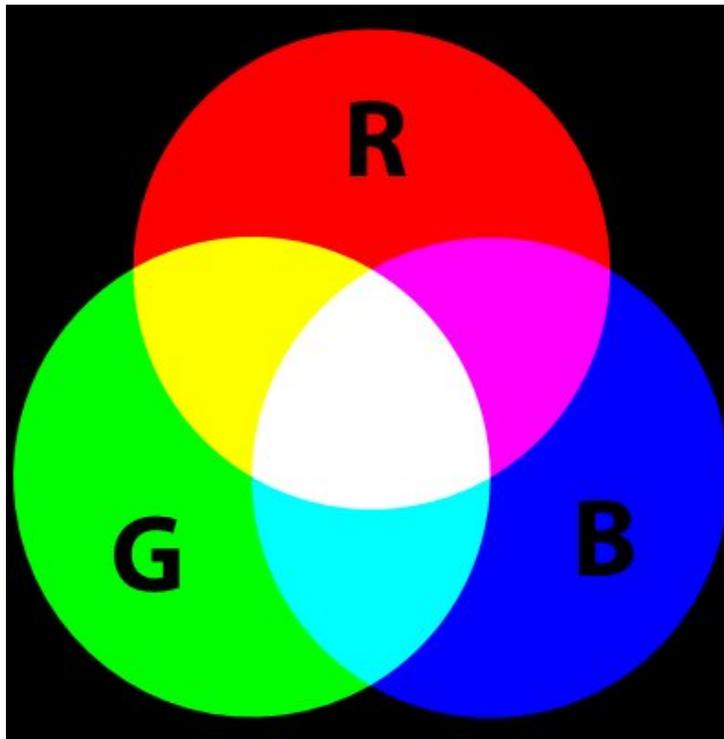
Color models

- Colorimetric color models
 - Based on the principles of *trichromacy*
 - Allow to predict if two colors *match in appearance* in given observation conditions
 - CIE XYZ
 - Perceptually uniform color models (CIELAB, CIELUV)

Device-oriented color models

RGB color model

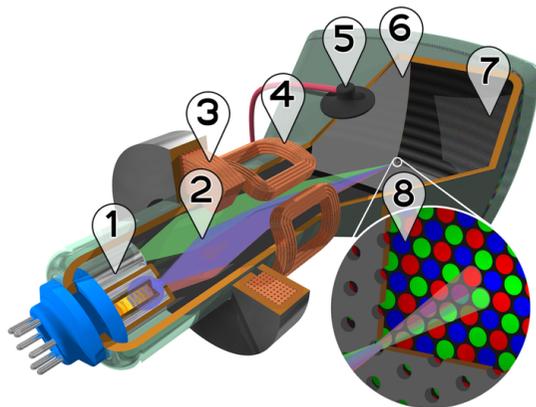
- Additive color model
 - The additive reproduction process usually uses red, green and blue light to produce the other colors



RGB displays

- Each pixel on the screen is built by driving three small and very close but still separated RGB light sources.
- At common viewing distance, the separate sources are indistinguishable, which tricks the eye to see a given solid color.
- All the pixels together arranged in the rectangular screen surface conforms the color image.

CRT



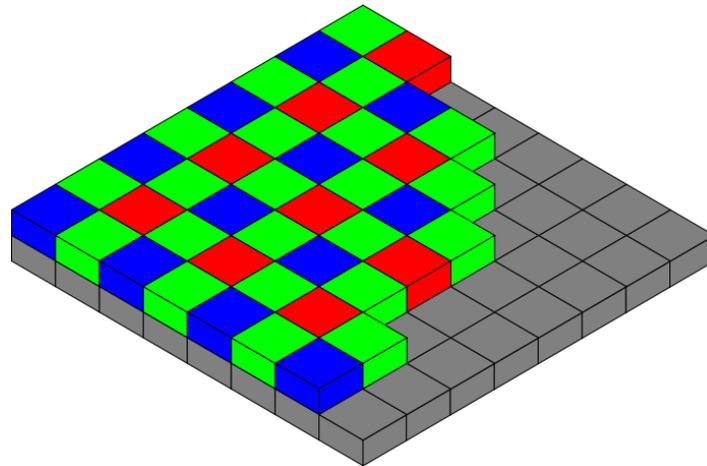
LCD



Close-up of red, green, and blue LEDs that form a single pixel in a large scale LED screen

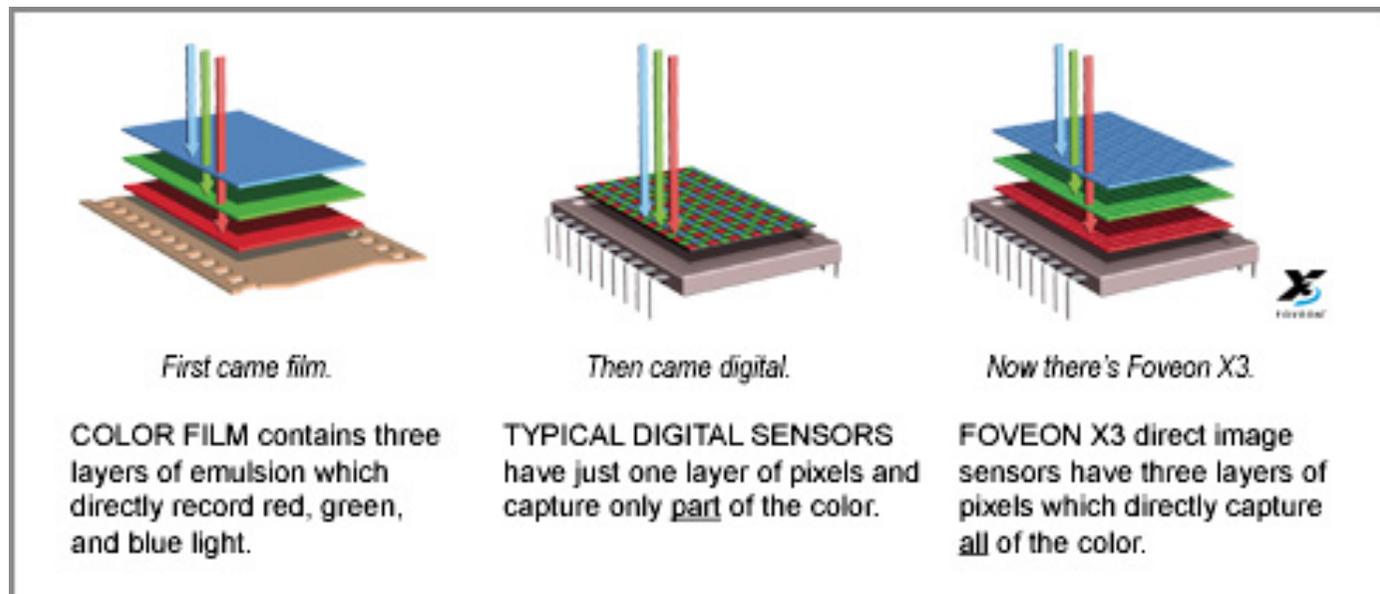
RGB digital cameras

- CCD camera sensor with Bayer array
 - Only one color channel is recorded in each physical location (pixel)
 - Twice as many green sensors than red and blue
 - Demosaicing is needed to recover full size images for the three color channels



RGB digital cameras

- CCD cameras with full color sensors
 - The three color channels are recorded in each physical location (pixel)

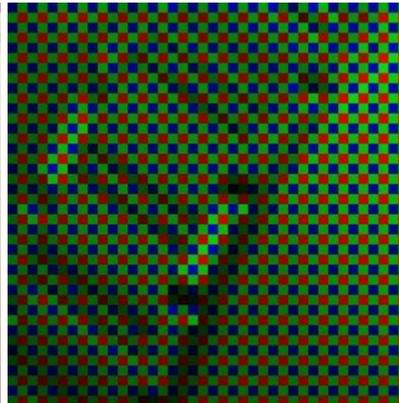


RGB digital cameras

Full color sensors



Image as seen through a Bayer sensor

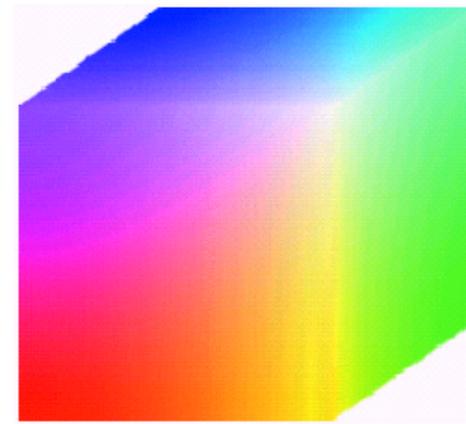
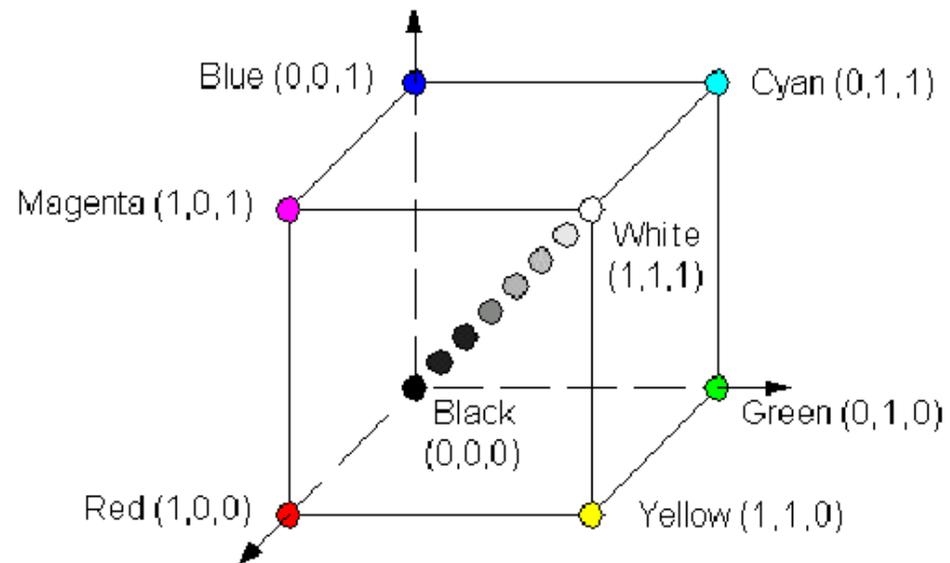


Reconstructed image after demosaicing



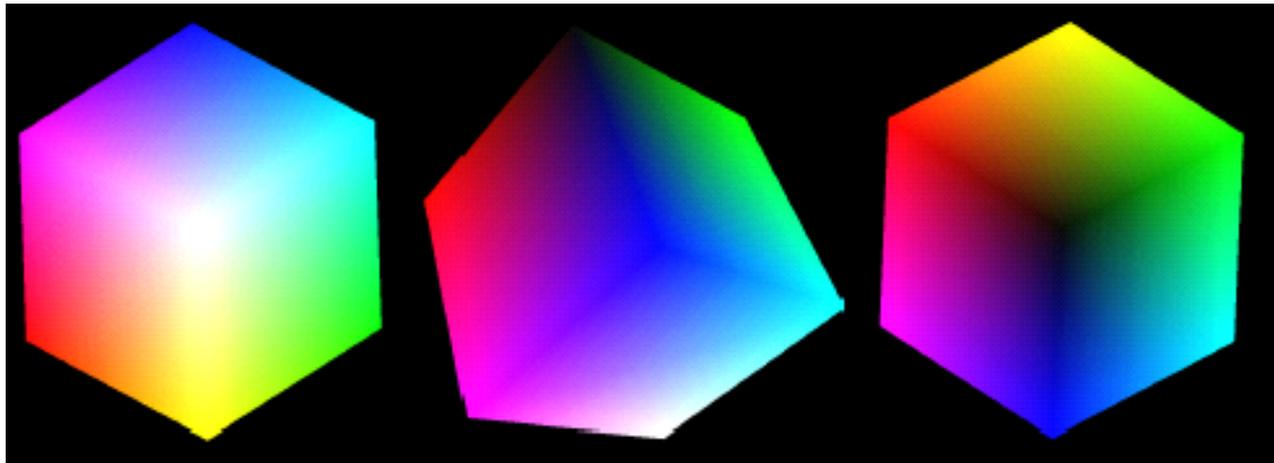
JPEG compression was added to the images

RGB model



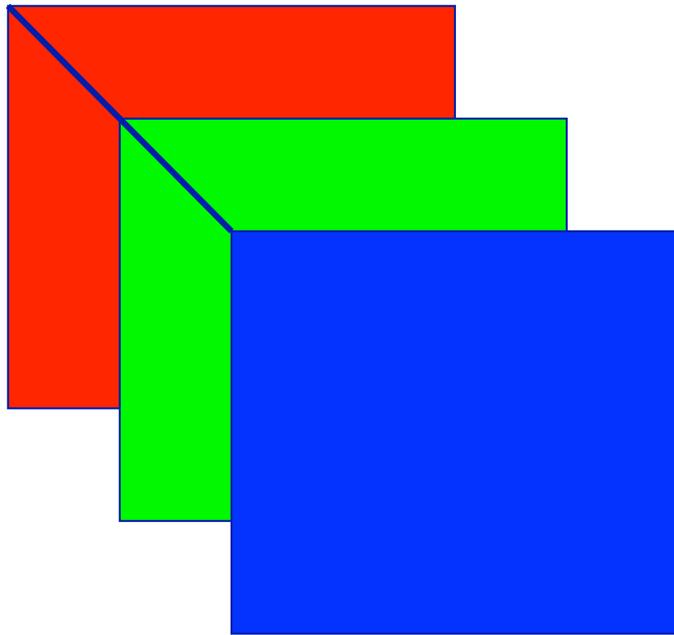
RGB model

- Normalized values in $[0,1]$ (chromaticity coordinates) may be convenient for some applications
- For a given device, the set of manageable colors lies inside the RGB cube



RGB model

(0,0)



A single pixel consists of three components.

128	251	60
-----	-----	----

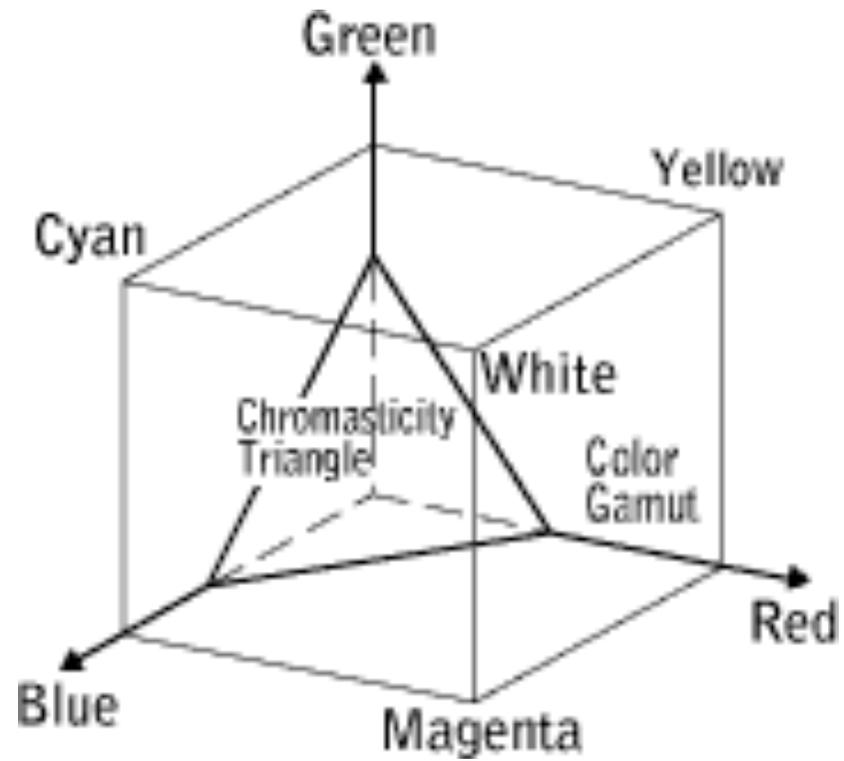
=



Final pixel in the image

If R,G, and B are represented with 8 bits (24-bit RGB image), the total number of colors is $256^3=16,777,216$

RGB Color Space



Exemple RGB

Original Image



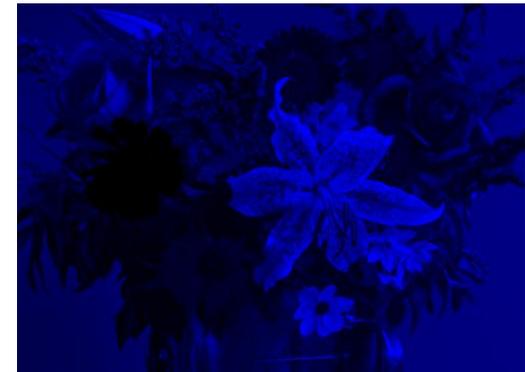
G-Component



R-Component



B-Component



False colors are used to represent the color channels, which all consists of gray values in the range [0,255]

Color channels



Red



Green

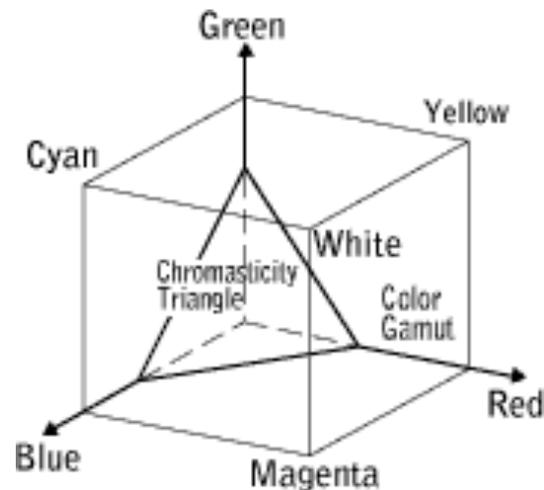


Blue



Device-oriented color models: CYM(K)

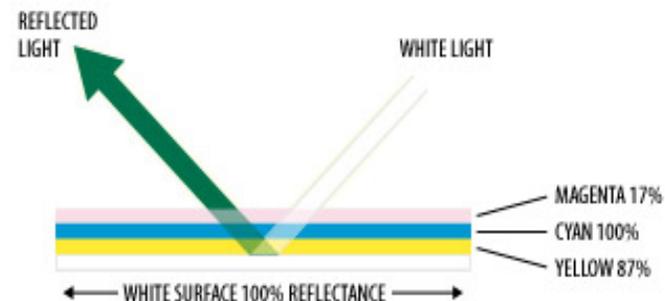
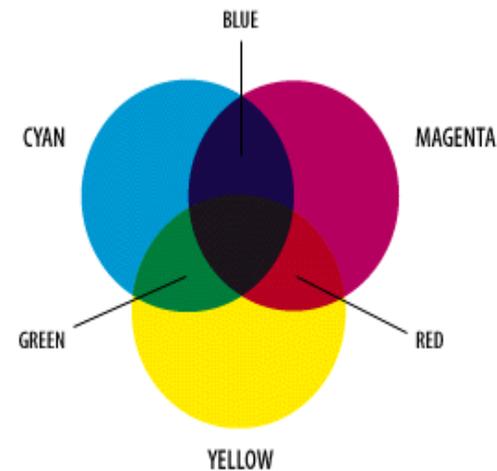
- Cyan, Yellow and Magenta are the “secondary” colors of light or the “primary” colors of pigments
- Model of “color subtraction”
- Used in printing devices



CMY(K)

- *Color subtraction*
 - Cyan, Magenta, Yellow filters
 - The Y filter removes B and transmits the R and G
 - The M filter removes G and transmits R and B
 - The C filter removes R and transmits G and B
 - Adjusting the transparency of these filters the amounts of R, G and B can be controlled

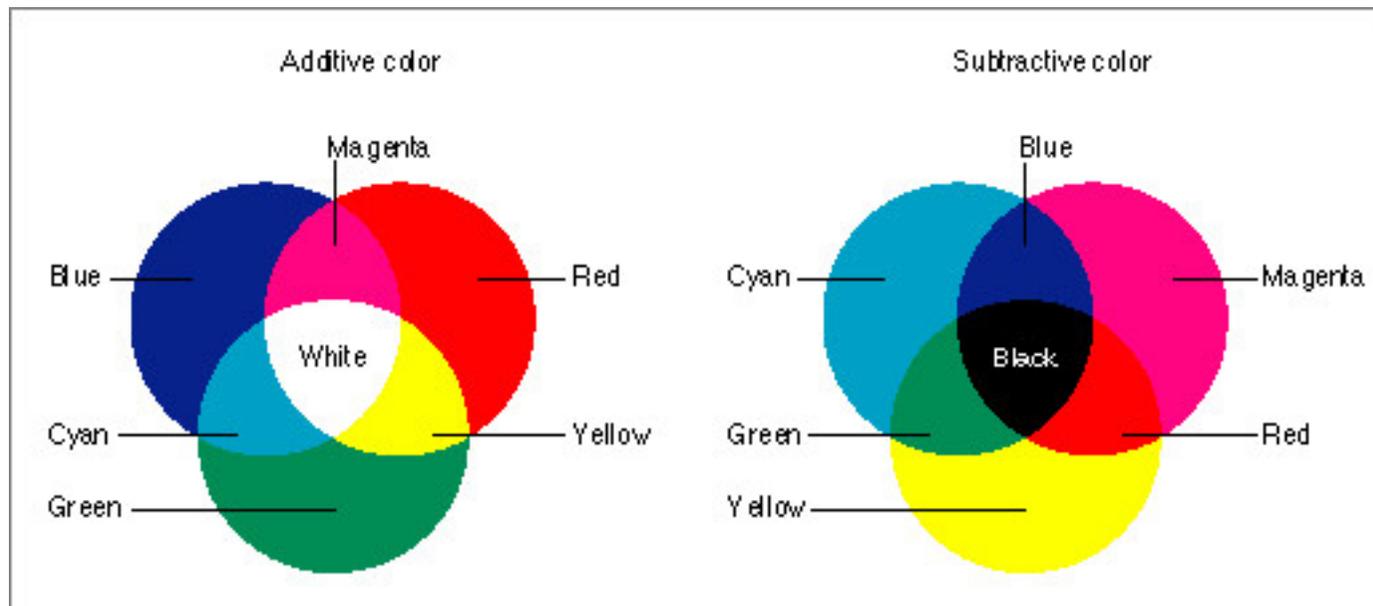
cyan=white-red
magenta=white-green
yellow=white-blue



CMY model

- CMY (Cyan, Magenta, Yellow)
- Used in printing devices
- Subtractive color synthesis
- CMYK: adding the black ink
 - Equal amounts of C,M and Y should produce black, but in practice a dark brown results. A real black ink is then added to the printer

CYM(K)



- cyan (C) absorbs red
- magenta (M) absorbs green
- yellow (Y) absorbs blue

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

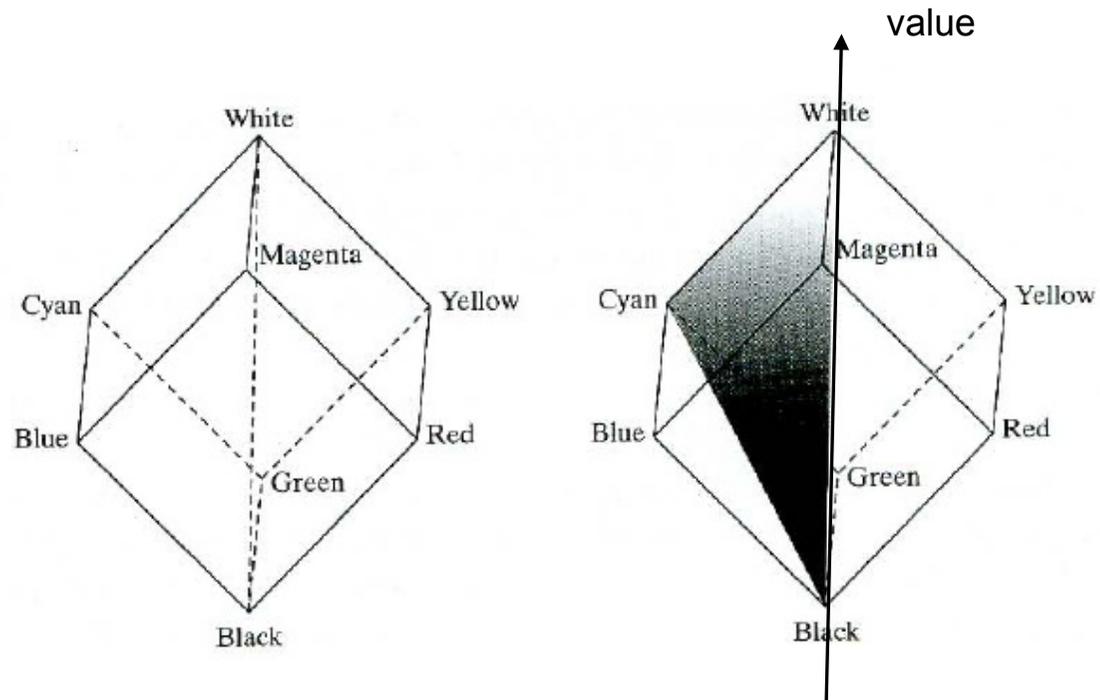
CMY(K) model

- Red, Green, Blue are the primary colors of light
- Cyan, Magenta, Yellow are the
 - Secondary colors of light
 - Primary colors of pigments
- When a cyan-colored object is illuminated with white light, no red light will be reflected from its surface! *Cyan subtracts red!*
- *The pigment when illuminated with white light absorbs its complementary color and reflects the others*

User-oriented CM

- Color is encoded in a way that is most “natural” to humans for describing colors
- Based on the decoupling of chromatic and achromatic information
 - One of the three axis represents the “value” or “intensity on the black-white axis” of the color
 - “dark-” or “bright-” ness of the color
 - The other two independent variables represent
 - Hue, which “qualifies” the color as belonging to a category (ex: red, green)
 - Saturation, or colorfulness, expressing how far the color is from neutral gray
 - Can be thought of as a deformation of the RGB cube

User-oriented CM



They all are effectively the RGB space twisted so that the neutral diagonal becomes the lightness axis, the saturation the distance from the central lightness axis and the hue the position around the center.

The only difference between these models is the measurement of saturation, or the strength of the colour

User-oriented CM

- HSV (Hue, Saturation, and Value). Sometimes variations include HSB (Brightness), HSL (Lightness/Luminosity), HSI (Intensity)
 - The *hue* of a color places it on the color wheel where the color spectrum (rainbow) is evenly spaced
 - The *saturation or chroma* of a hue defines its intensity
 - Decreasing the saturation via a contrast control adds gray.
 - The value of a hue defines how bright or dark a color is
 - They all are effectively the RGB space twisted so that the *neutral diagonal becomes the lightness axis*, the *saturation the distance from the central lightness axis* and the *hue the position around the center*.
 - The only difference between these models is the measurement of saturation, or the strength of the colour

HSI (HSV, HSL) Color Space

- Recall:
 - **Hue** is color attribute that describes a pure color
 - **Saturation** gives the measure to which degree the pure color is diluted by white light.
- 1. Intensity (Value or Lightness) component I (V,L), is decoupled from the chromaticity information!
- 2. Hue and saturation can be accessed independently from illumination

HSI

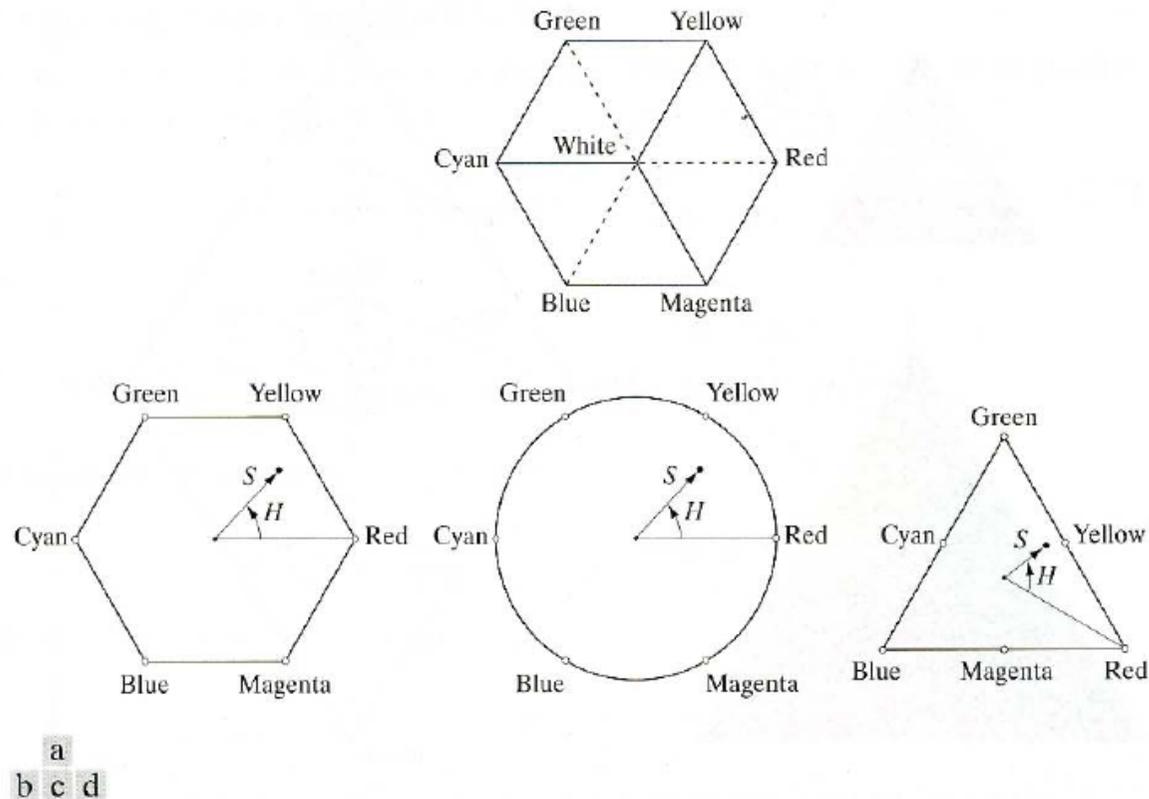
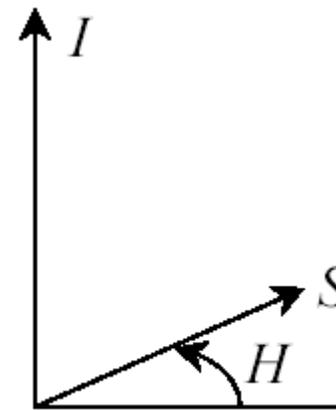


FIGURE 6.13 Hue and saturation in the HSI color model. The dot is an arbitrary color point. The angle from the red axis gives the hue, and the length of the vector is the saturation. The intensity of all colors in any of these planes is given by the position of the plane on the vertical intensity axis.

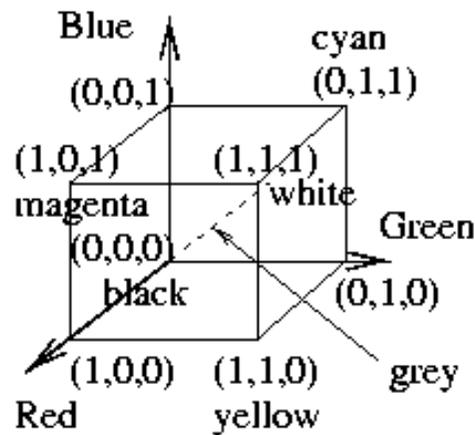
HSI model

- Two values (H & S) encode *chromaticity*
- Convenient for *designing* colors
- Hue H is defined by an angle between 0 and 2π :
 - “red” at angle of 0;
 - “green” at $2\pi/3$;
 - “blue” at $4\pi/3$
- Saturation S models the *purity* of the color
 - $S=1$ for a completely pure or saturated color
 - $S=0$ for a shade of “gray”

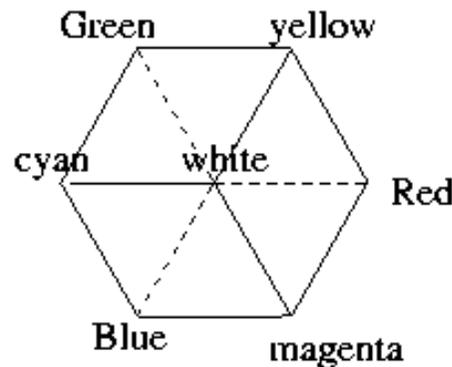


Color hexagon for HSI (HSV)

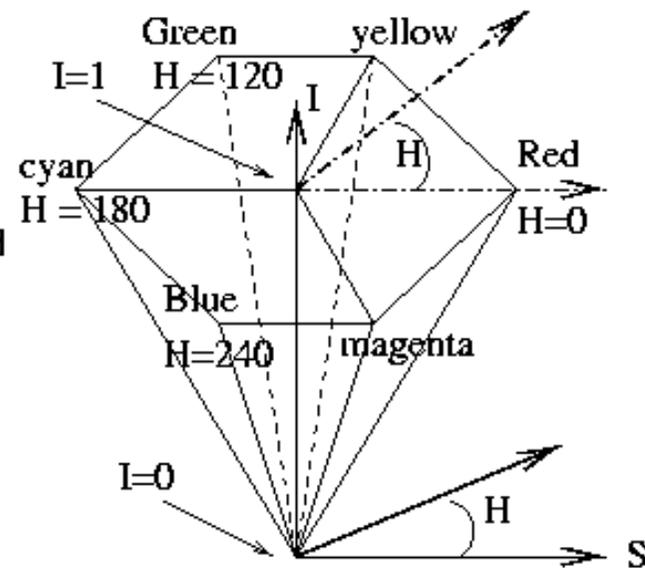
- Color is coded relative to the diagonal of the color cube. Hue is encoded as an angle, saturation is the relative distance from the diagonal, and intensity is height.



(a) RGB color cube

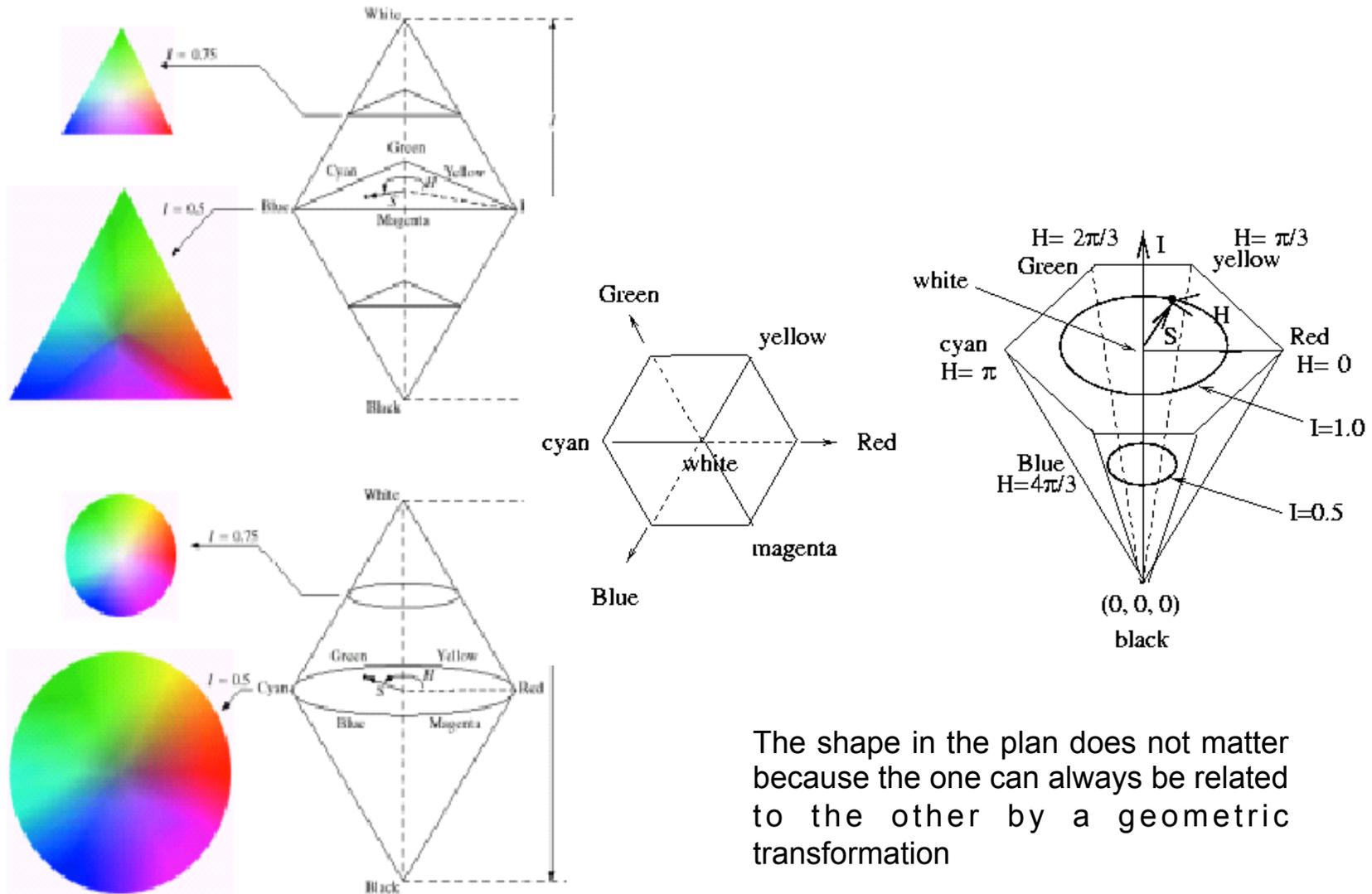


(b) view on diagonal from white to black



(c) single hexacone HSI model

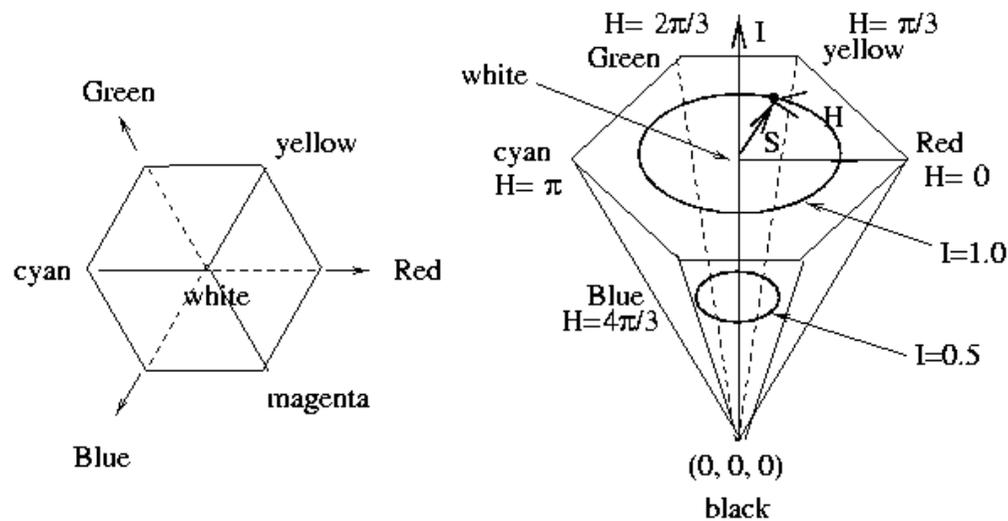
Variations on the theme



The shape in the plan does not matter because the one can always be related to the other by a geometric transformation

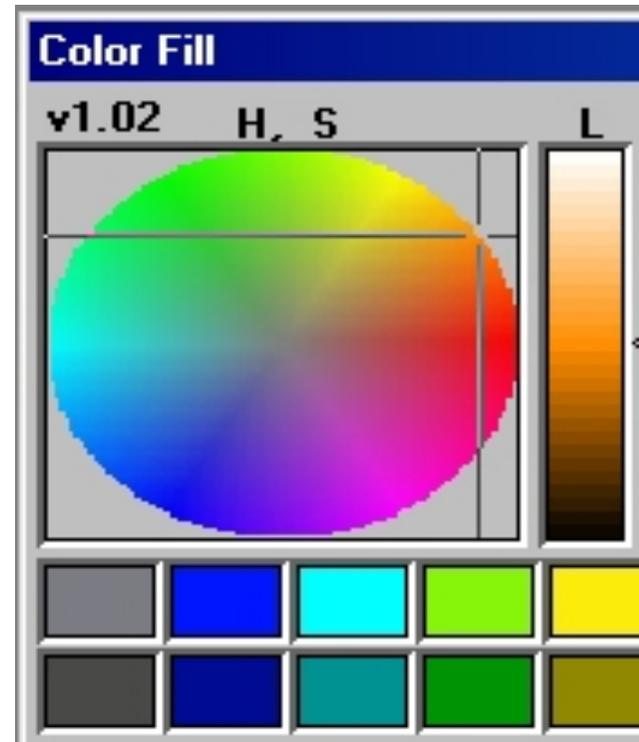
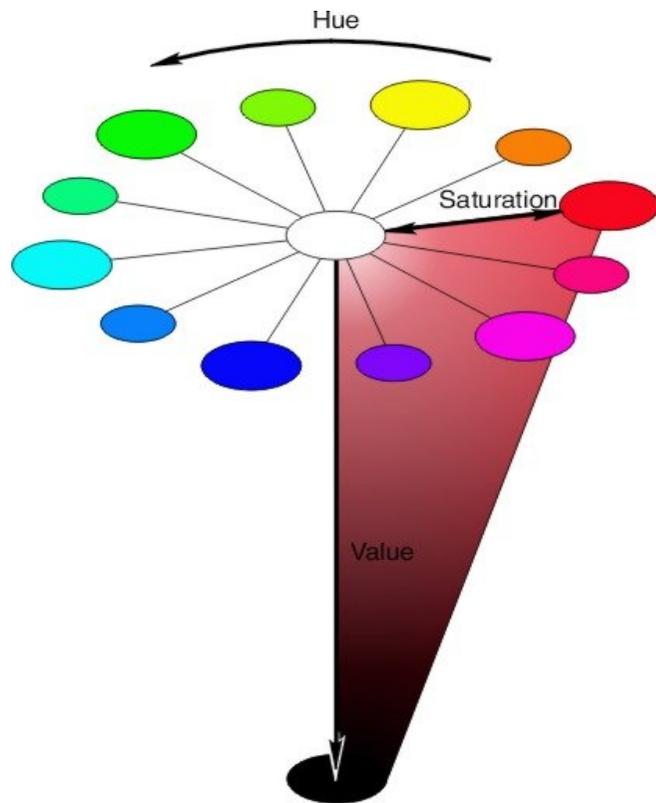
Color hexacone for HSI (HSV)

- (Left) Projection of RGB cube perpendicular to the diagonal $(0,0,0) - (1,1,1)$.
- Color names now at vertices of a hexagon.
- Colors in HIS :
 - intensity I is vertical axis
 - hue H is angle with R at 0
 - saturation is 1 at periphery and 0 on I axis



HSI-like model

- Hue, Saturation, Value (HSV) model

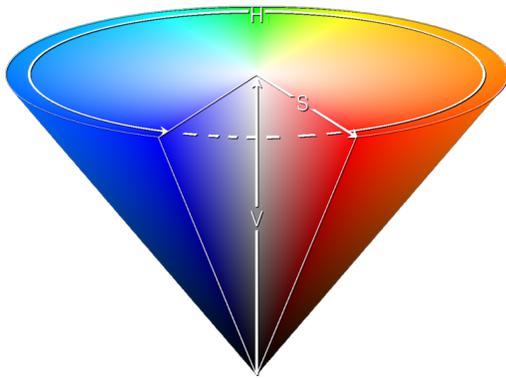


from [http://www2.ncsu.edu/scivis/lessons/colormodels/color_models2.html#saturation.](http://www2.ncsu.edu/scivis/lessons/colormodels/color_models2.html#saturation)

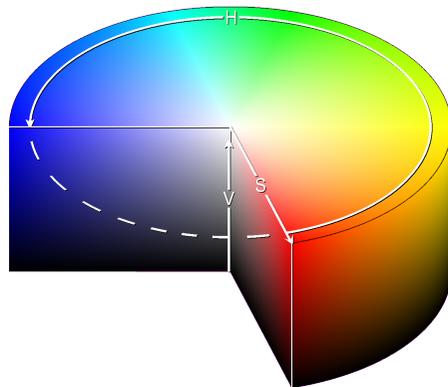
HSV, HSL

Hue, Saturation, Value (Brightness)

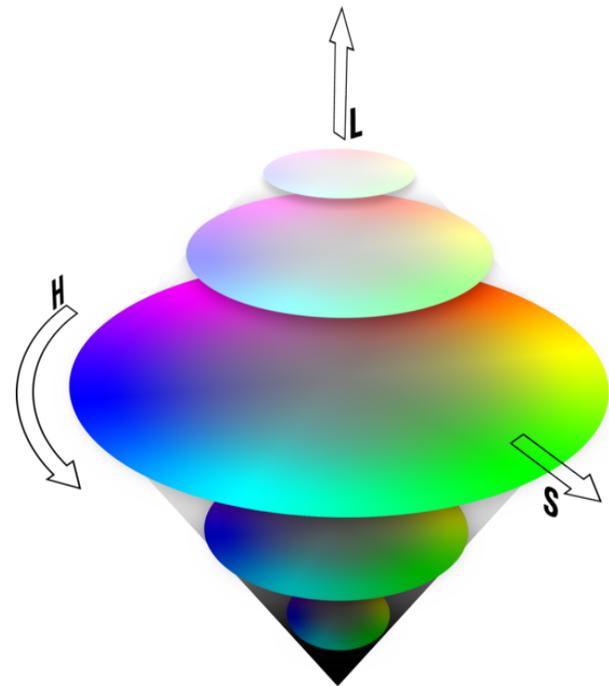
HSV cone



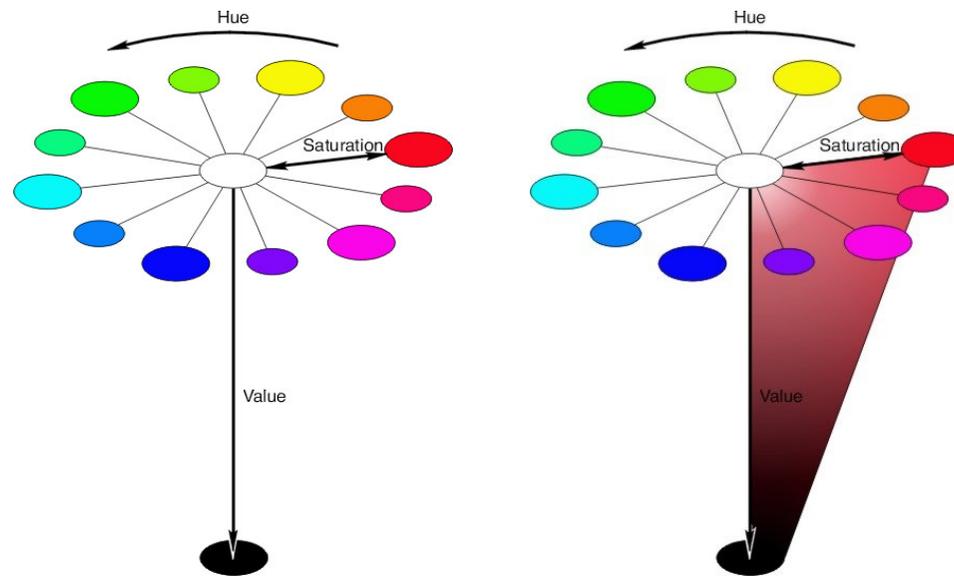
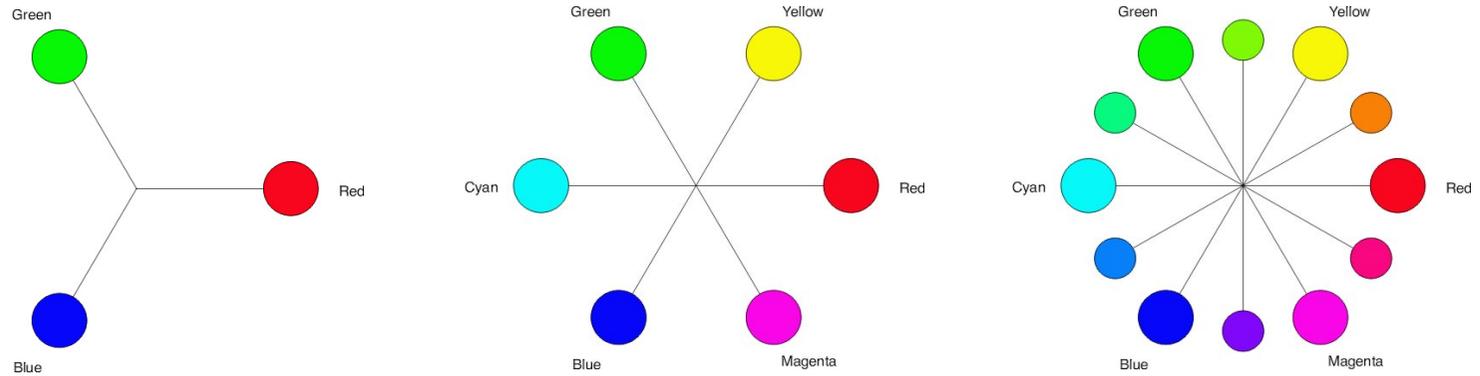
HSV cylinder



Hue, Saturation, Lightness



User-oriented CM: HSV



RGB to HSI

Given an image in RGB color format, the H component of each RGB pixel is obtained using the equation

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases} \quad (6.2-2)$$

with

$$\theta = \cos^{-1} \left\{ \frac{\frac{1}{2}[(R - G) + (R - B)]}{[(R - G)^2 + (R - B)(G - B)]^{1/2}} \right\}$$

The saturation component is given by

$$S = 1 - \frac{3}{(R + G + B)} [\min(R, G, B)]. \quad (6.2-3)$$

Finally, the intensity component is given by

$$I = \frac{1}{3} (R + G + B). \quad (6.2-4)$$

RGB 2 HSI

$R, G, B \in \{0, 1\}$

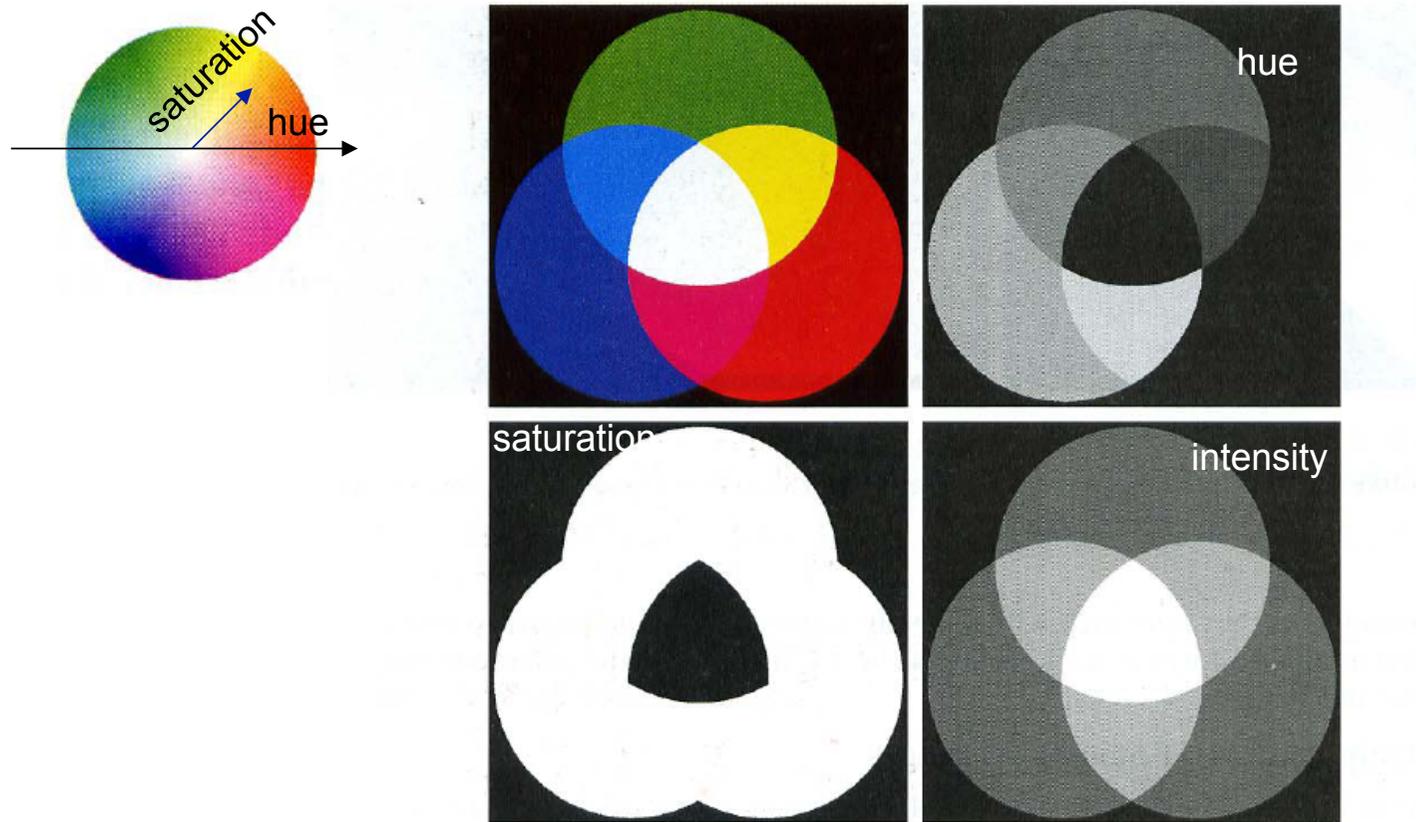
θ is measured counterclockwise from the red axis

H can be normalized to be in $\{0, 1\}$ by dividing by 360

The other values (for chroma and saturation) are in $\{0, 1\}$

The inverse formulas are also defined.

RGB vs HSI



a b
c d

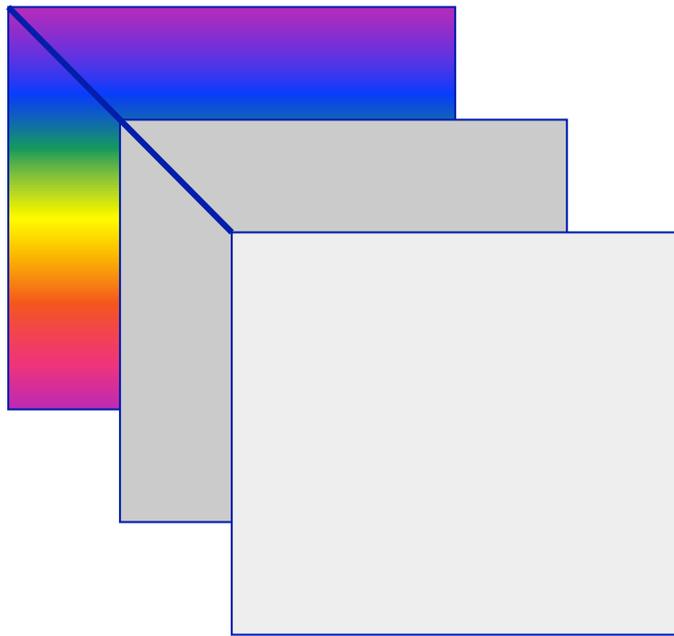
FIGURE 6.16 (a) RGB image and the components of its corresponding HSI image: (b) hue, (c) saturation, and (d) intensity.

User-oriented CM

- Drawbacks
 - Singularities in the transform (such as undefined hue for achromatic points)
 - Sensitivity to small deviations of RGB values near the singularities
 - Numerical instability when operating on hue due to its angular nature

HSI Representation

(0,0)



A single pixel consists of three components.

Each pixel is a **Vector / Array**.

128	251	60
-----	-----	----

=



Pixel-Vector in
the computer
memory

Final pixel in
the image

HSI Examples

Original Image



Hue



Saturation



Intensity



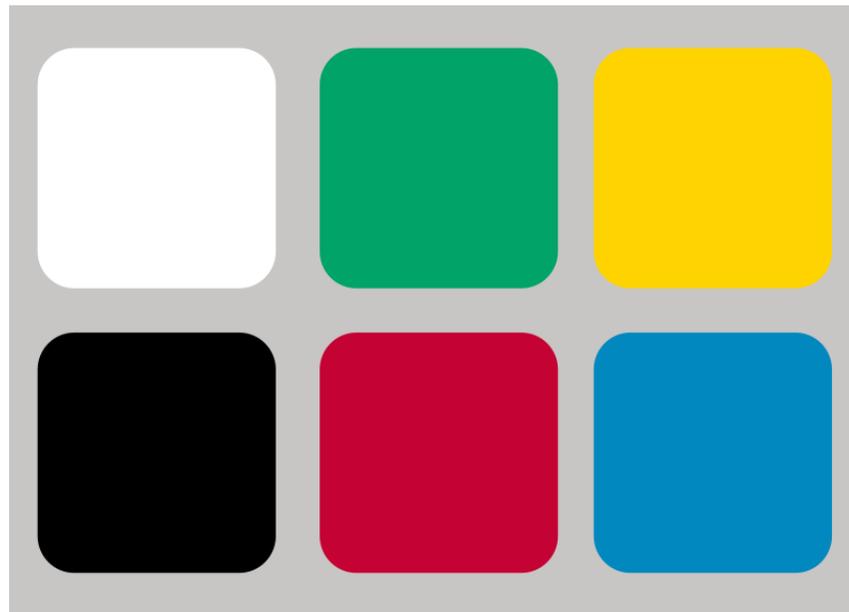
Editing saturation of colors



(Left) Image of food originating from a digital camera;
(center) saturation value of each pixel decreased 20%;
(right) saturation value of each pixel increased 40%.

Opposite channels model

- Encode color images taking human perception into account
- RGB \rightarrow luminance + 2 chrominances
- Going from Y' (physical entity) to Y implies a non linear operation



YUV Color model

- YUV color model “imitates” human vision.
 - Implementation of the opposed channel model, also called luminance / chrominance color spaces
- Historically, YUV color space was developed to provide compatibility between color and black /white analog television systems.
 - YUV color image information transmitted in the TV signal allowed proper reproducing an image contents at the both types of TV receivers, at the color TV sets as well as at the black / white TV sets.
- PAL TV standard
 - YCbCr similar, used in JPEG and MPEG
 - YCbCr color space is defined in the ITU-R BT.601-5 [1] and ITU-R BT.709-5 [2] standards of ITU (International Telecommunication Union).
 - YIQ (similar) used in NTSC

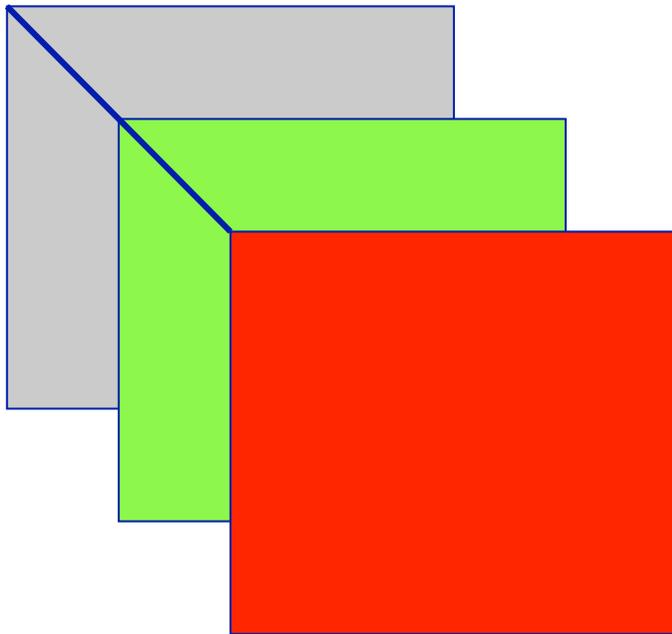
[1] RECOMMENDATION ITU-R BT.601-5, 1982-1995; [2] RECOMMENDATION ITU-R BT.709-5, 1990-2002.

YUV color model

- Color channels
 - Y: luminance
 - UV (Cb, Cr): chrominance. These are often downsampled exploiting the lower cutting frequency and sensitivity of the human visual system with respect to the luminance component
- Conversion formulas from/to RGB are available in the literature and implemented in Matlab

YUV representation

(0,0)



A single pixel consists of three components.
Each pixel is a Vector / Array.

128	251	60
-----	-----	----



Pixel-Vector in
the computer
memory

Final pixel in
the image

Same Caution as before applies here!

YUV example

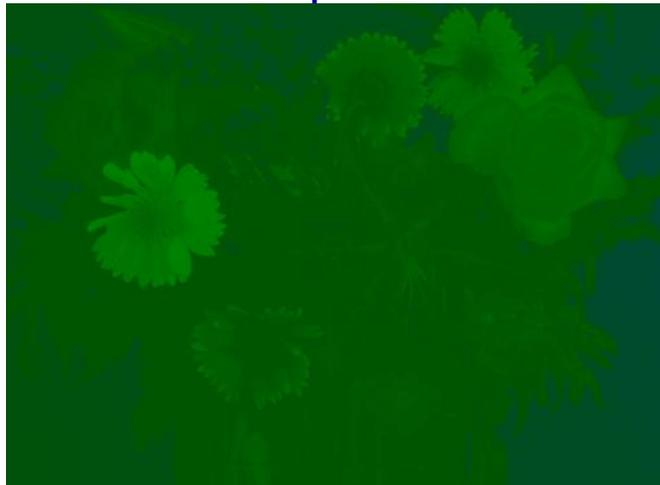
Original Image



Y-Component



U-Component



V-Component



YUV possible subsampling patterns

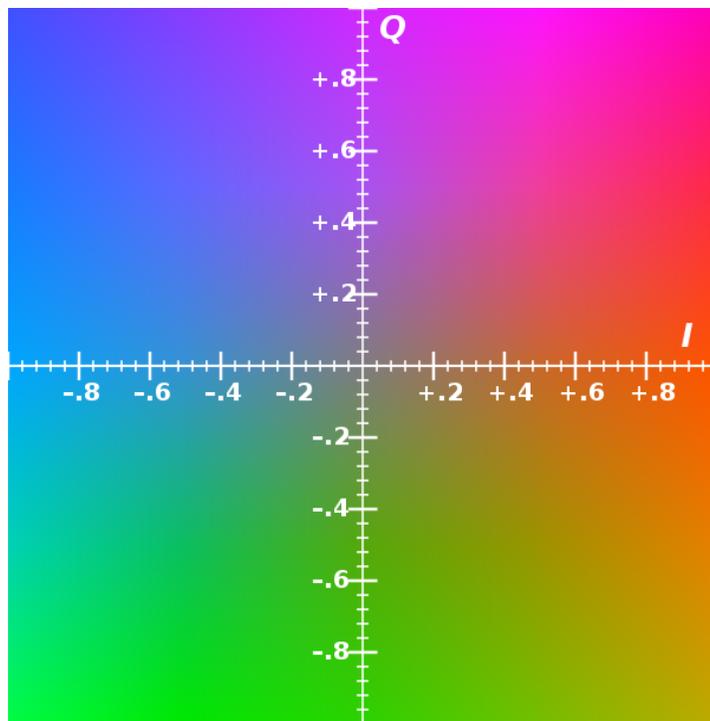
Sub sampling ratio	Sub sampling pattern				Color component size		
	Uniform	Co site Even	Co site Odd	Centered	Luma Y	Chroma Cb	Chroma Cr
4:4:4					1	1	1
4:2:2					1	1/2	1/2
4:2:0					1	1/4	1/4

Designation of used symbols are the following:

- position of luma sample only
- position of 2 chroma samples only
- positions of luma and 2 chroma samples are co sited.

YIQ model

- NTSC (National Television Color System)
- Y is the luminance, meaning that light intensity is nonlinearly encoded based on gamma corrected RGB primaries



The YIQ color space at $Y=0.5$. Note that the I and Q chroma coordinates are scaled up to 1.0

YIQ

- Chromaticity is represented by I and Q
 - in phase and in quadrature components
- RGB2YIQ

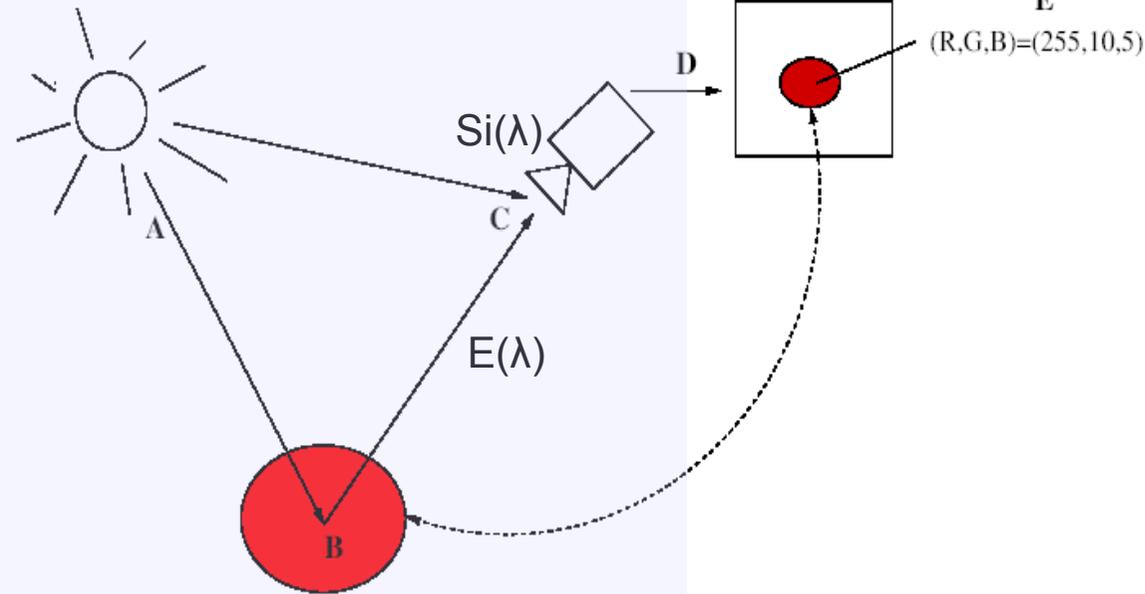
$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.528 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Colorimetric color models

- CIE-RGB
- CIE-XYZ
- CIELAB
- CIELUV

RGB color model

Image formation



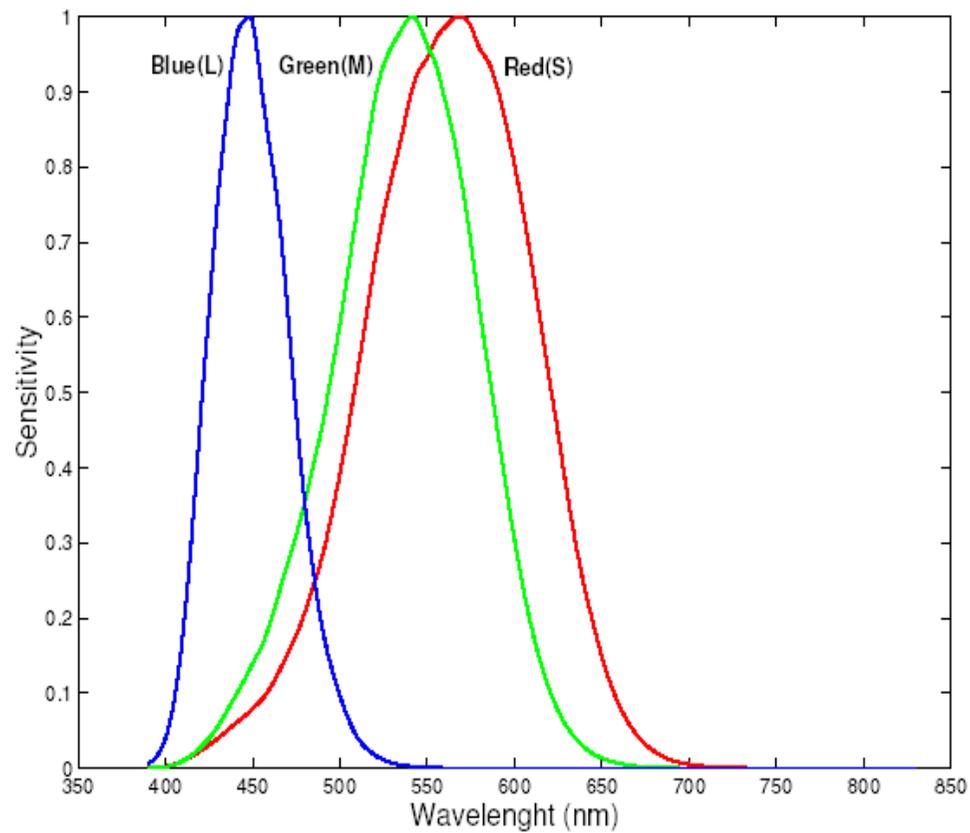
$$C_i = \int_{\lambda} E(\lambda) S_i(\lambda) d\lambda$$

$S_i(\lambda)$: sensitivity of the i^{th} sensor

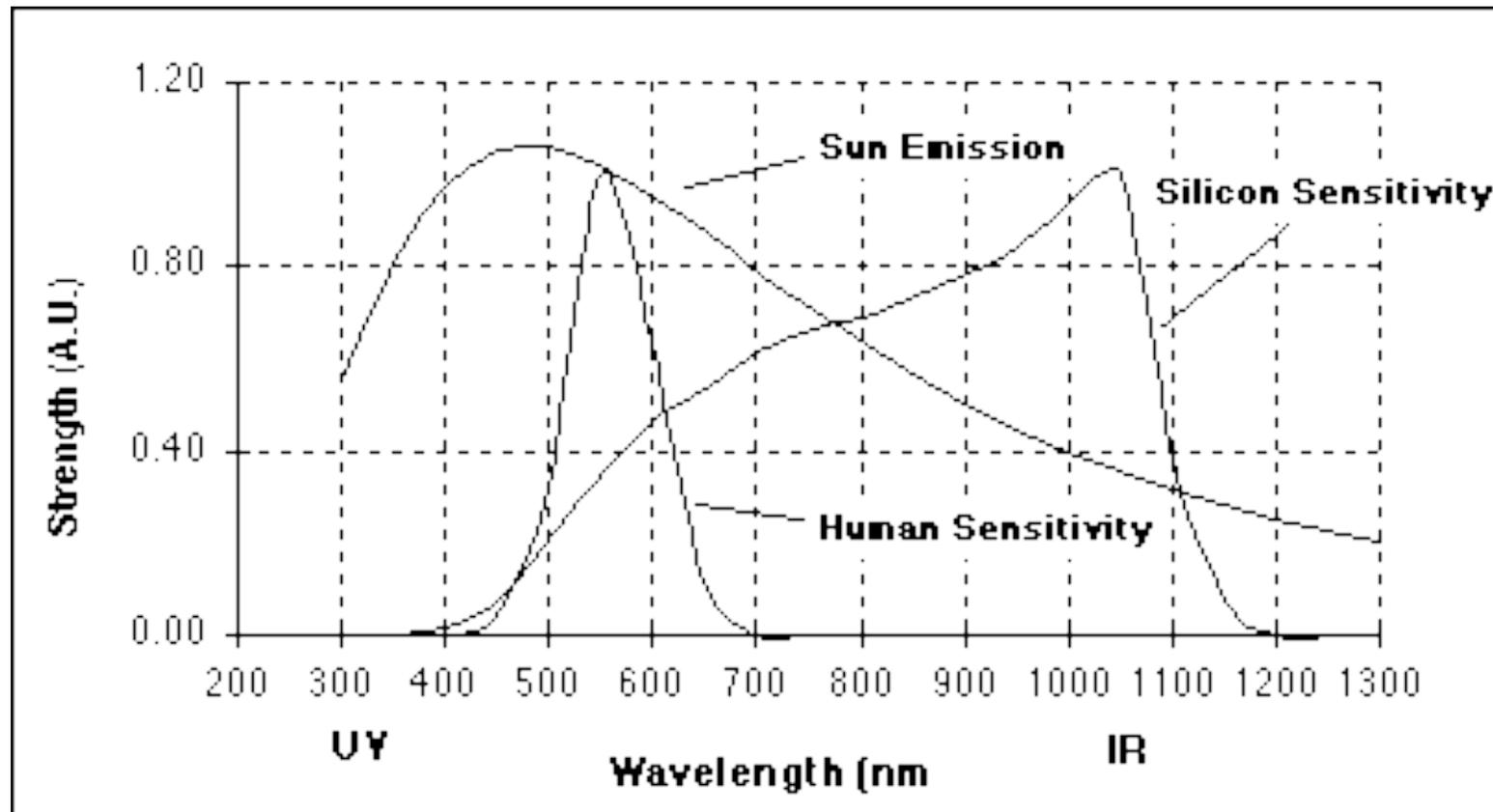
$E(\lambda)$: Spectral Power Distribution (SPD) of the diffused light

Spectral sensitivities

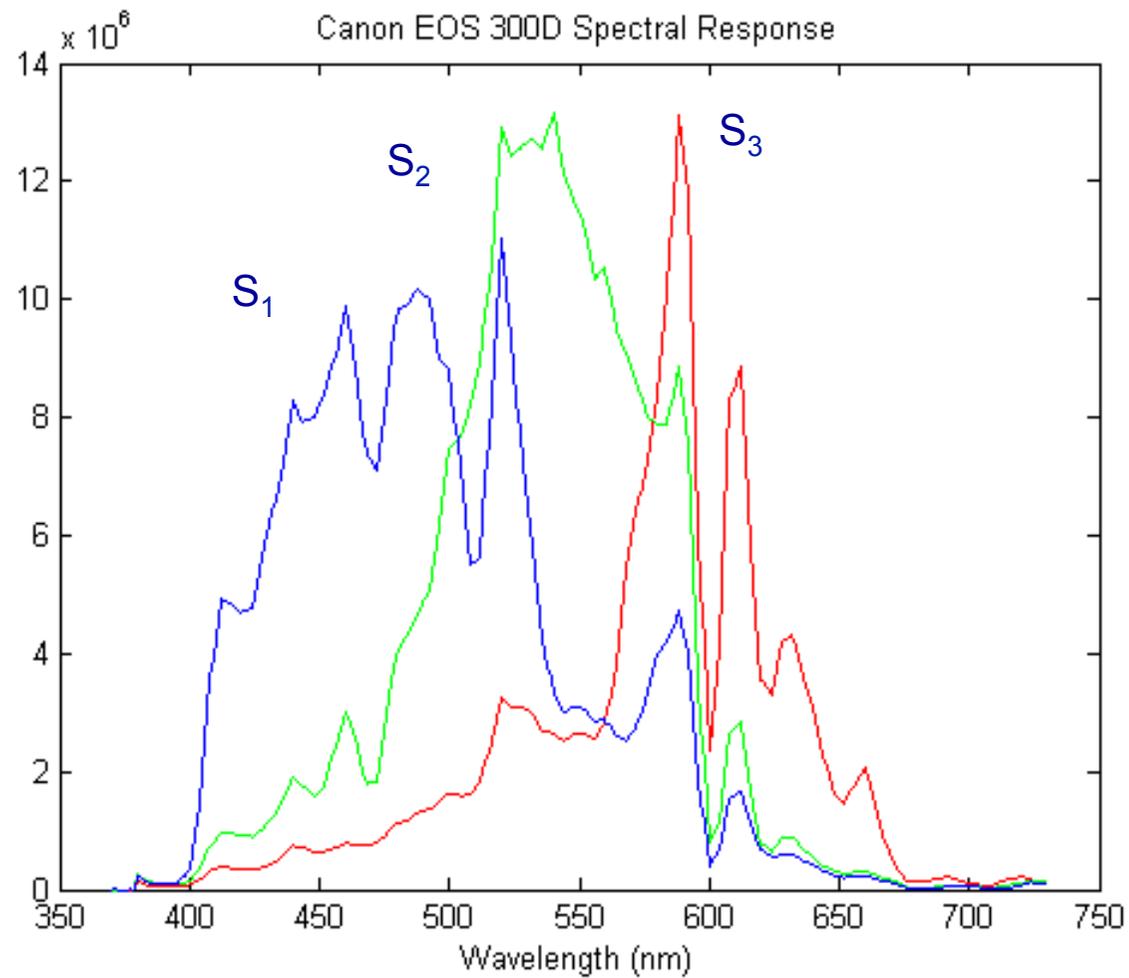
Target: (normalized) spectral sensitivities of the eye



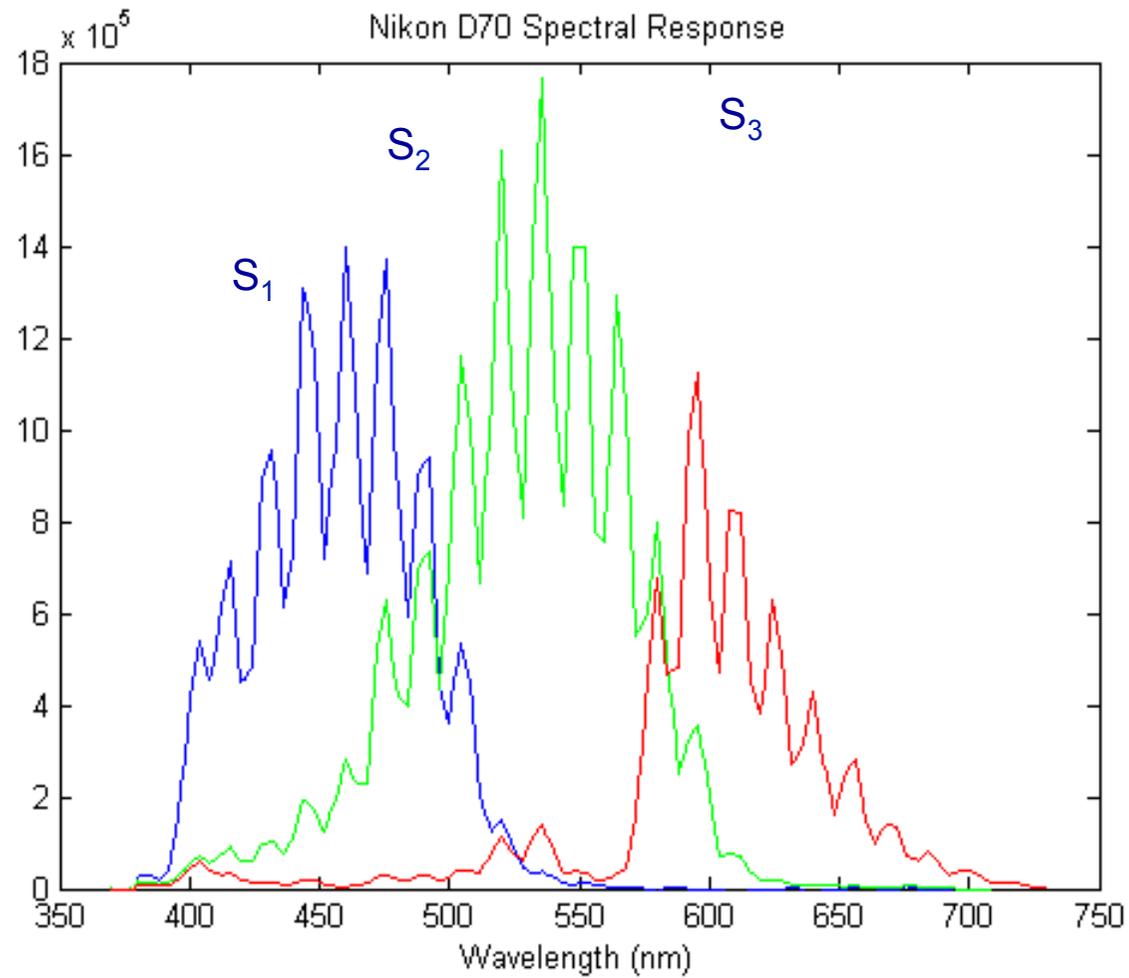
Broad range sensitivity



Sensor sensitivity: Ex. 1



Spectral sensitivity: Ex. 2



RGB model

$$C_i = \int_{\lambda} P(\lambda) S_i(\lambda) d\lambda$$

$P(\lambda)$: PSD (Power Spectral Density of the incident light)

$S_i(\lambda)$: spectral sensitivity of the "red", "green" and "blue" sensors

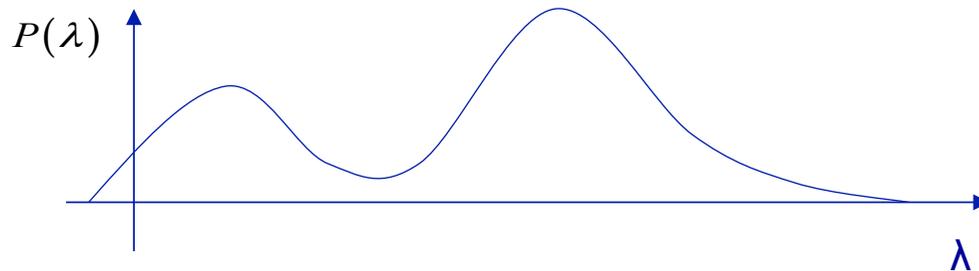
Intensity of the signals recorded by the camera in the three channels

$$R_c = k_1 \int_{\lambda} P(\lambda) S_1(\lambda) d\lambda$$

$$G_c = k_2 \int_{\lambda} P(\lambda) S_2(\lambda) d\lambda$$

$$B_c = k_3 \int_{\lambda} P(\lambda) S_3(\lambda) d\lambda$$

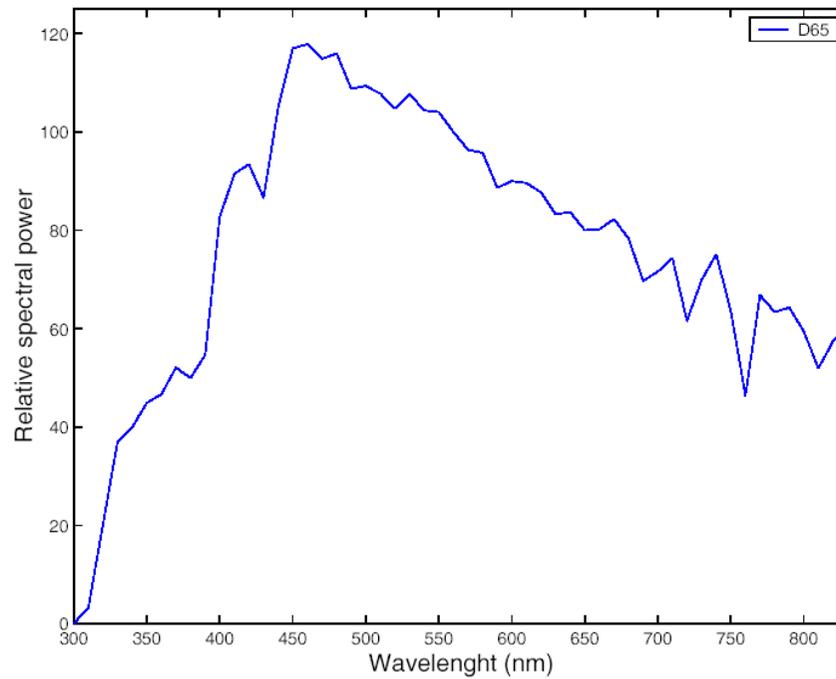
relative to the camera



We need a PSD representing the "white" to calculate k_1 , k_2 and k_3 such that for that PSD ($P(\lambda)=E(\lambda)$) $R_c=G_c=B_c=1$ (255). This is called the *reference white*

Reference white

- The reference white is the light source that is chosen to approximate the white light
 - D65, D50



Reference white

- The reference white, $E(\lambda)$, will be given the maximum tristimulus values in all channels ($R_c=G_c=B_c=255$)
- The numerical values of the R,G,B coordinates of a generic PSD $P(\lambda)$ will depend on the choice of $E(\lambda)$
 - We neglect the pedices for easyness of notations

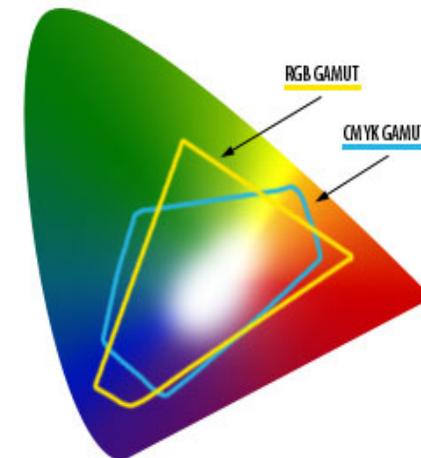
$$R_{Ec} = k_1 \int_{\lambda} E(\lambda) S_1(\lambda) d\lambda = 255$$

$$G_{Ec} = k_2 \int_{\lambda} E(\lambda) S_2(\lambda) d\lambda = 255 \rightarrow k_1, k_2, k_3$$

$$B_{Ec} = k_3 \int_{\lambda} E(\lambda) S_3(\lambda) d\lambda = 255$$

RGB tristimulus values

- The R,G,B coordinates does not have an *absolute* meaning, as their values depend on
 - The spectral sensitivity of the sensors that are used in the capture device
 - The reference white
- Thus, R,G,B values of the *same physical stimulus* (image) acquired with *different cameras* are different, in general
- Gamut: set of colors that is “manageable” by the device
 - Acquisition devices: set of colors that are represented by the device
 - → gamut mapping



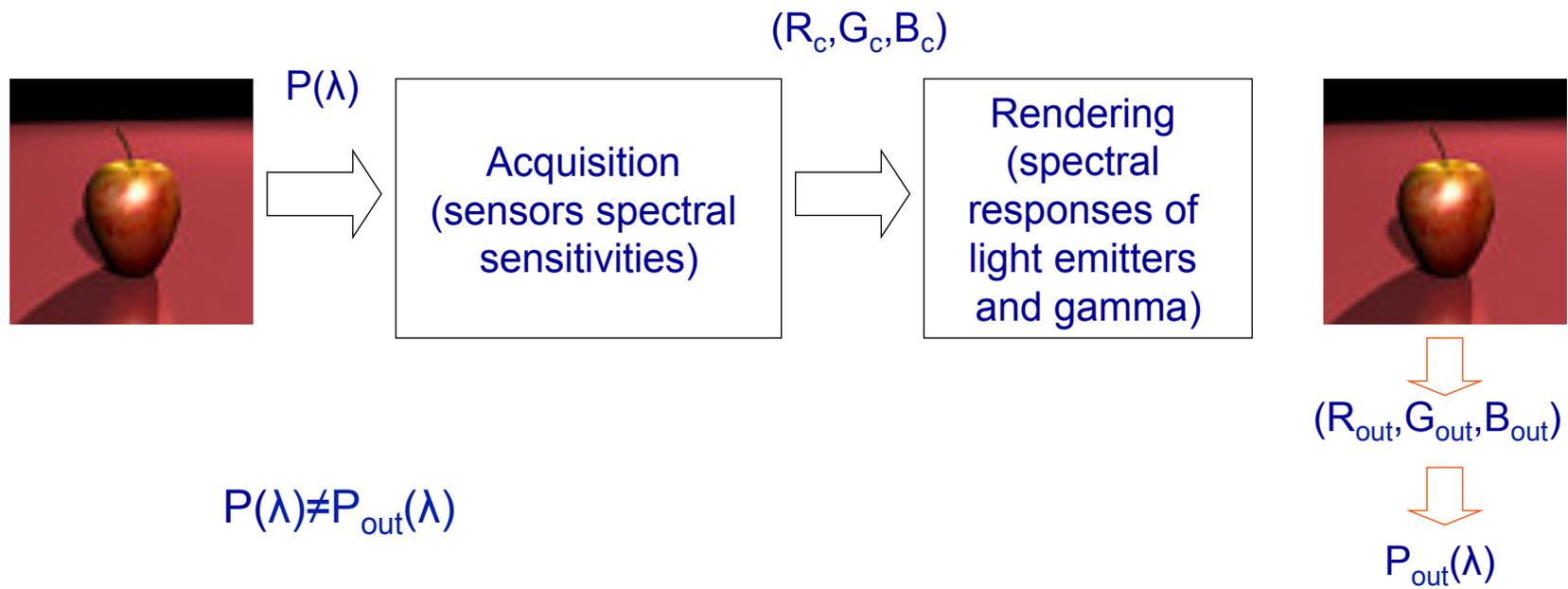
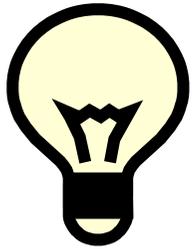
RGB model

- Similar considerations apply to rendering devices: the rendering of a color with given tristimulus coordinates (R,G,B) will depend on
 - The spectral responses of the emitters
 - phosphors for a CRT
 - color filters in a LCD
 - The calibration of the device
 - As for the acquisition devices, the color corresponding to the rendered white must be set
 - To define the entire gamut for a monitor, you only need mark the points on the diagram that represent the colors the monitor actually produces. You can measure these colors with either a colorimeter or a photospectrometer along with software that ensures the monitor is showing 100 percent red for the red measurement, 100 percent green for the green measurement, and 100 percent blue for the blue measurement.
 - The linearity of the monitor transfer function (gamma)

RGB model: rendering ex.

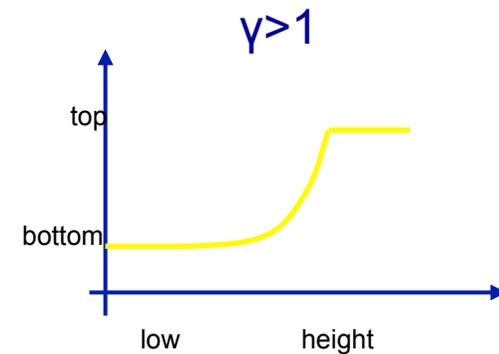
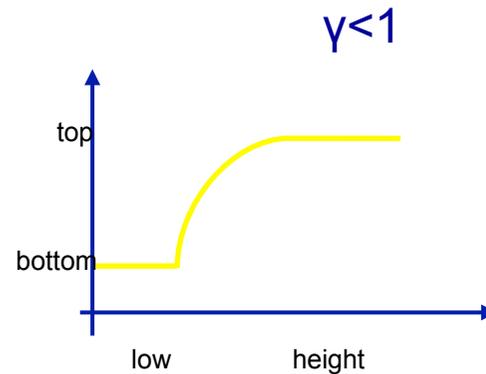
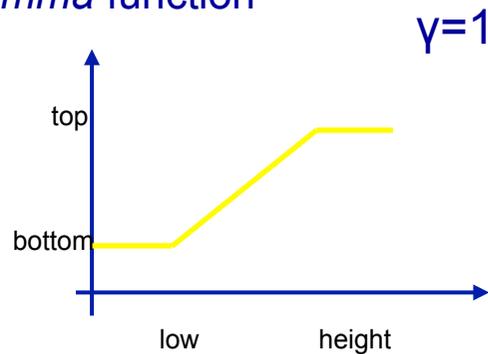
- The RGB values depend on the phosphores
- Different for the different reproduction media (CRT, television displays)
 - Example:
 - Red phosphore: $x=0.68, y=0.32$
 - Green phosphore: $x=0.28, y=0.60$
 - Blue phosphore: $x=0.15, y=0.07$
 - Given the x,y coordinates of the phosphores, the reference white point and the illuminant (D65), the RGB coordinates can be calculated
 - Calibration
 - the $R=G=B=100$ points must match in appearance with the white color as observed by 10 deg observer under the D65 illuminant
 - The brightness of the three phosphores is non linear with the RGB values. A suitable correction factor must be applied (Gamma correction)

RGB model



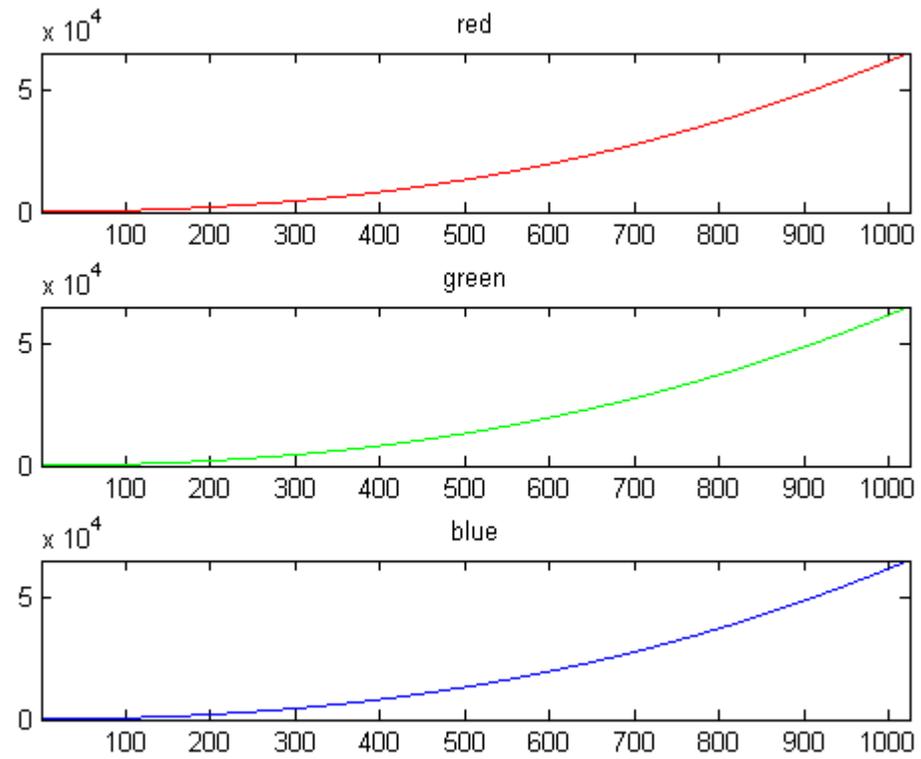
Gamma function

Gamma function



- Typical CRT monitors: gamma=2.2
- The non-linearity of the monitor can be compensated by non-uniform scaling of the RGB coordinates at input (*RGB linearization*)
- This led to the definition of the sRGB color model

sRGB



CIE-RGB

Colorimetric standard observer

RGB standard observer

- Spectral sensitivities for the human eye have been measured in reference conditions by a very large number of observers
- Performed by the CIE (Commission Internationale d'Éclairage) standardization committee
- Such curves are called Color Matching Functions (CMFs) after the type of experiment
- The so-derived tristimulus values
 - Are not device dependent
 - Are still relative as they depend on (1) the choice of the red, green and blue monochromatic primaries that were used (2) the reference white and (3) the experimental conditions

Wavelength encoding

- Scotopic matching experiment → Scotopic luminosity function $V'(\lambda)$
 - Characterizes vision at low illumination conditions
 - Rod responses
 - One primary light and one test light
 - The intensity of the light beam is the parameter

- Photopic color matching experiment → Color matching functions (CMF), photopic luminosity function $V(\lambda)$
 - Characterizes vision under high illumination conditions
 - Cones responses
 - Three primary lights and one test light
 - The intensities of each primary lights are the parameters

Brightness matching

Wavelength encoding

The photometric principle

- Basic postulate

Whatever the visual stimulus, fixed in all respects, of one patch, and whatever the fixed relative spectral distribution of the stimulus on the second patch, a brightness match can always be achieved by varying the absolute value of the second stimulus

- Basic laws of brightness matching

- Symmetry

- If A matches B then B matches A

- Transitivity

- If A matches B and B matches C then A matches C

- Proportionality

- If A matches B then kA matches kB

- Additivity

- If A matches B and C matches D then $(A+C)$ matches $(B+D)$

Brightness match

- **Definition**

- *Similar uniform light patches, producing visual stimuli defined by $\{P_\lambda d\lambda\}$ and $\{P'_\lambda d\lambda\}$, respectively, are in brightness match for the standard photopic observer if*

$$\int_{\lambda} P_{\lambda} V(\lambda) d\lambda = \int_{\lambda} P'_{\lambda} V(\lambda) d\lambda$$

- For brightness matches, the photopic luminous flux entering the eye per unit solid angle must be the same for the two patches

Matching experiments

- Scotopic matching experiment (brightness matching)
 - Low illumination conditions
 - Rod responses
 - One primary light and one test light
 - The *intensity* of the primary light beam is the parameter
- Measures the scotopic spectral sensitivity function $V'(\lambda)$
- Photopic color matching experiment
 - High illumination conditions
 - Brightness matching
 - Rods
 - Color matching
 - Cones
 - Three primary lights and one test light
 - The intensities of each primary lights are the parameters
- Measures the photopic spectral sensitivity function $V(\lambda)$
- Measures the Color Matching Functions (CMFs)

Brightness matching

Necessary and sufficient condition for a brightness match between two stimuli of radiant power distribution $\{P_\lambda d\lambda\}$ and $\{P'_\lambda d\lambda\}$, respectively

$$\int_{\lambda} P_\lambda \beta(\lambda) d\lambda = \int_{\lambda} P'_\lambda \beta(\lambda) d\lambda$$

Where $\beta(\lambda)$ is a fixed function characterizing the brightness-matching process depending on

- the spectral radiance power distributions (relative or absolute) of the matching stimuli
- the observational conditions
 - field size, eccentricity of the field of view, state of adaptation as modified by previous or surrounding stimuli
- *Quantum efficiency* of the human visual system

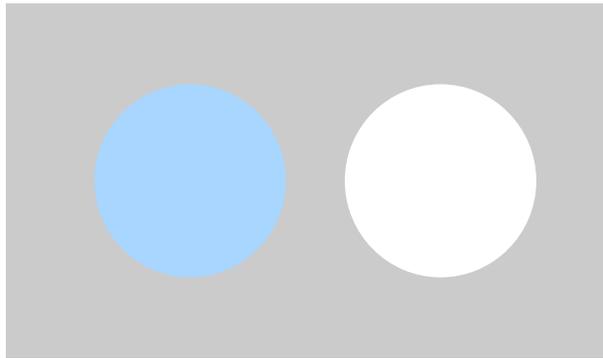
➤ *Ideal photometric observer*

➤ ~~defined by the CIE by the specification of two fixed functions~~

- *Scotopic* matching $\beta(\lambda) \rightarrow V'(\lambda)$

Scotopic brightness matching

Bipartite field



Primary light

Test light

The **primary** light has a fixed relative spectral distribution and only the *intensity* can vary

The **test** light can have any spectral distribution. It is common to use an equal-energy spectral light

Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable

Scotopic spectral sensitivity function

$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

\mathbf{r} : system vector (transfer function)

\mathbf{t} : spectral distribution of the test light

e : response of the observer

Assuming that the system is linear (homogeneity and superposition hold), the system vector can be measured by feeding it with n_λ monochromatic lights.

It is common to choose an **equal-energy spectral** light (*reference white*) as test light.

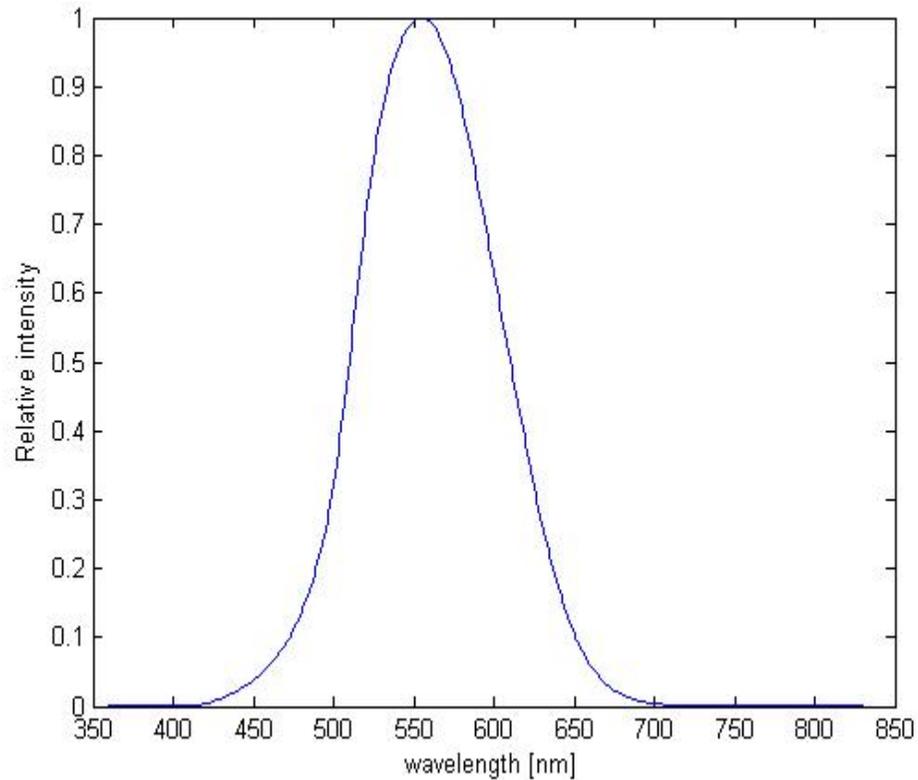
$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = r_1$$

Each monochromatic light will determine one entry of the system vector, resulting in the *Scotopic Spectral Sensitivity function*.

Scotopic spectral sensitivity function

- In scotopic conditions, the eye is sensible only to *relative intensities* of the two lights. The spectral distribution is immaterial (in low illumination conditions, color is not “perceivable”).
 - Physiological interpretation: the rhodopsin absorption coefficient depends on the wavelength, but the response is the same for any wavelength. Once a photon is absorbed, the information about its wavelength is lost. Hence, **the appearance of the stimulus is independent of its spectrum.**
 - **The shape of $V'(\lambda)$ reflects the dependence of the absorption coefficients from the wavelength.**
- In order to measure the spectral sensitivity at each wavelength a set of equal energy spectral (monochromatic lights) are used as test lights
- Relative intensities are recorded ($I_{REF}/I_{test}=I_{REF}$ since $I_{test}=\text{const.}=1$) and the normalized (values between zero and one)
 - Prior to normalization, due to the linearity of the system, the system vector is unique up to a scale factor.
- *$V'(\lambda)$ was adopted by CIE in 1951 in a field of 10 degrees (Crawford 1949) and eccentrically with more than 5 degrees (Wald 1945) with complete darkness adaptation*

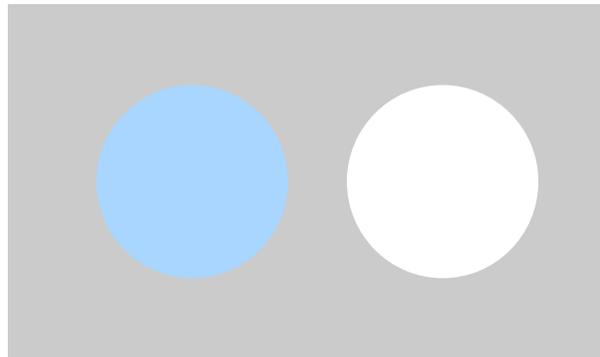
Scotopic spectral sensitivity function $V'(\lambda)$



Data available at <http://cvision.ucsd.edu/cie.htm>

Photopic brightness matching

Bipartite field



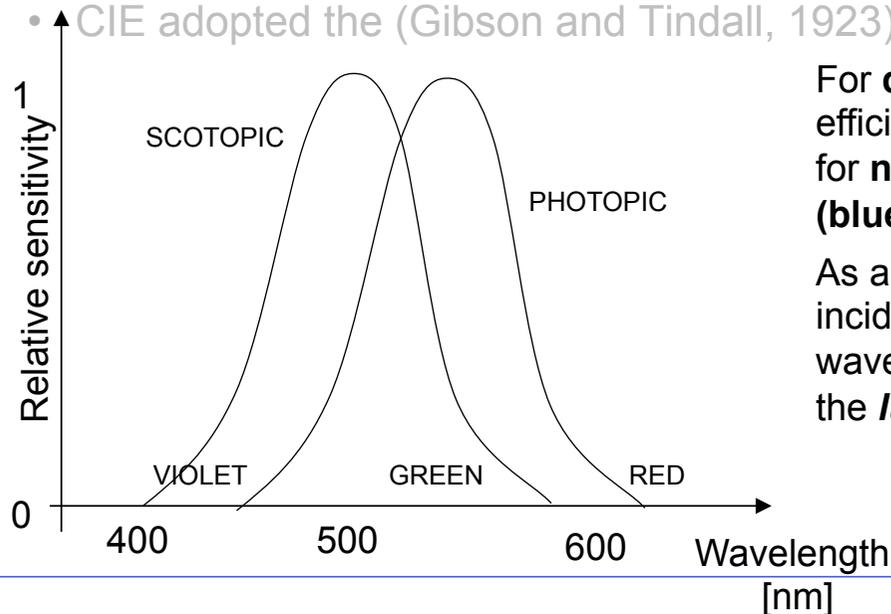
Primary light: only
the *intensity* can
vary

Test light

Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable, following an ad-hoc paradigm

Photopic curve $V(\lambda)$

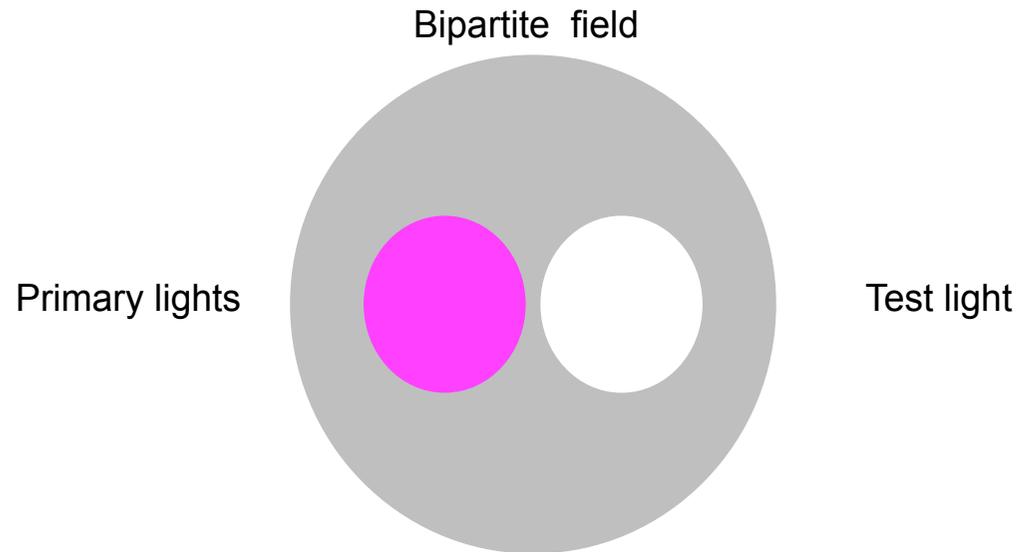
- High illumination levels
- Different paradigms
 - The direct comparison of the brightness leads to unreliable results due to the difference in color
 - Flickering method (Coblentz and Emerson, 1918)
 - Step-by-step method of heterochromatic photometry (Hyde et al. 1918)
 - CIE adopted the (Gibson and Tindall, 1923)



For **daylight** vision the maximum efficiency is at **555 nm (yellow)** while for **night** vision it shifts to **505 nm (blue)**

As a measure of the sensitivity of the eye to incident monochromatic light at each wavelength, it corresponds to a measure of the **luminous efficacy** of the eyes.

Color matching

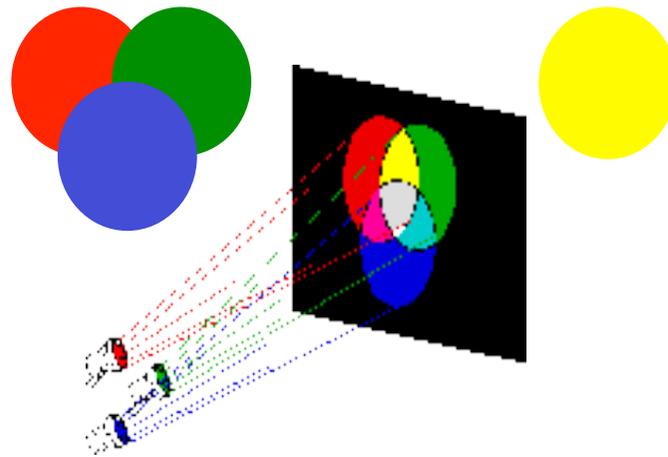
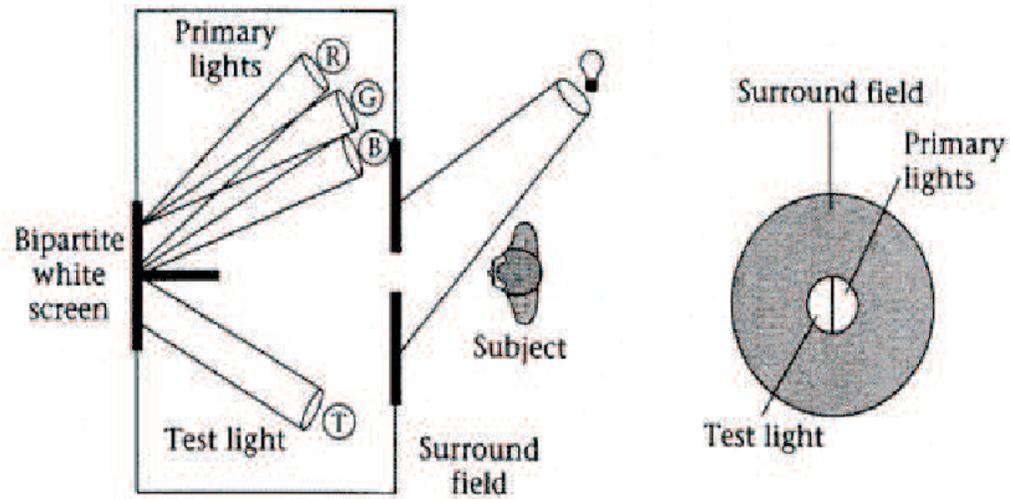


There are **three primary lights** with fixed relative spectral distribution and only the intensity can vary. These are chosen to be **monochromatic**

The test light can have any spectral distribution. It is common to choose a equal energy light and decompose it into the monochromatic components for testing the entire set of wavelengths.

Task: Adjust the intensities of the primary lights so that the primary and test lights appear indistinguishable

Color matching



Measuring the CMFs

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

R : system matrix (transfer function). Each line represents the *Color Matching Function* (CMF) for the corresponding primary light

t : spectral distribution of the test light

e : response of the observer

Assuming a equal-energy test light

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix} = t \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

since we are measuring relative intensities we can choose $t=1$

Color Matching Functions (CMFs)

Assuming that the symmetry, transitivity and homogeneity hold (*Grassmann's laws of additive color mixtures*), the system matrix can be measured by feeding it with n_λ monochromatic lights

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} r_1^1 \\ r_1^2 \\ r_1^3 \end{bmatrix}$$

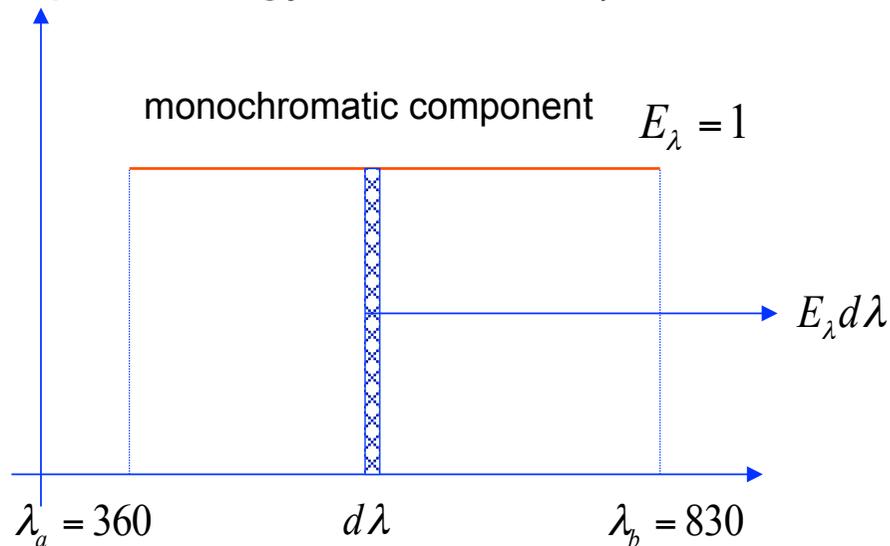
The response to each monochromatic light will determine one *column* of the system matrix, so one entry of each CMF.

It can be shown that the system matrix is not unique. Using **different sets of primaries** leads to different CMFs. Though, different sets of CMFs are related by a **linear transformation**

→ **Need to choose one set of primaries**

Color Matching Functions

- In other words, the CMFs are the *spectral* tristimulus values of the *equal energy* stimulus E (reference white)



$$E_\lambda = \bar{r}(\lambda)R + \bar{g}(\lambda)G + \bar{b}(\lambda)B$$

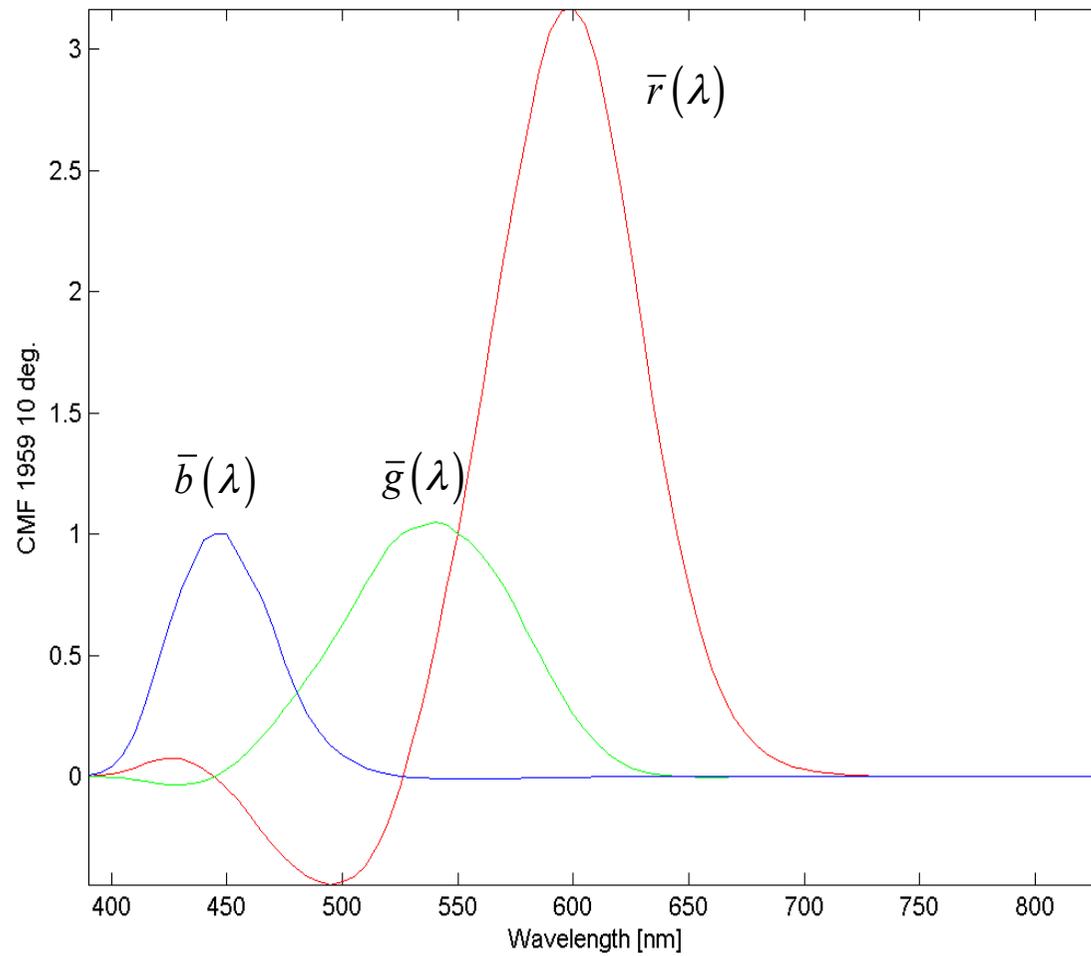
$$\lambda_R = 700nm$$

$$\lambda_G = 546.1nm$$

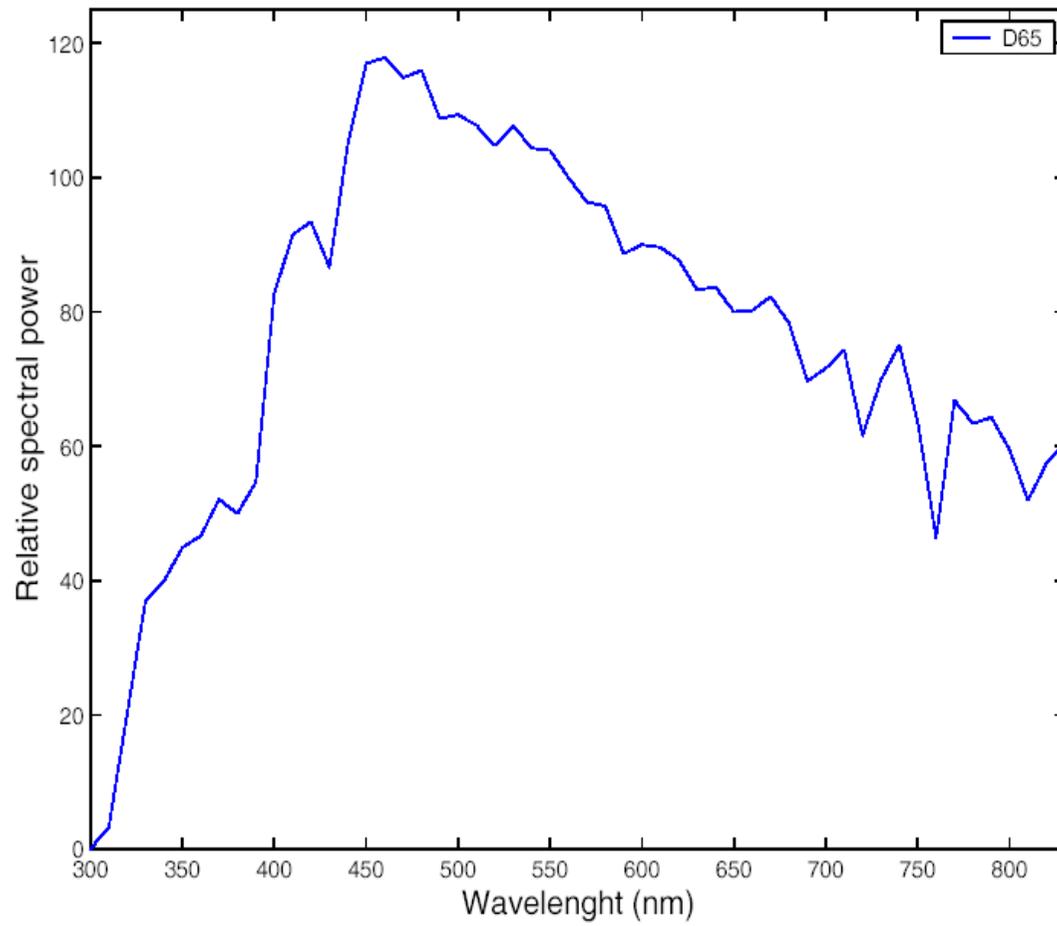
$$\lambda_B = 435.8nm$$

$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ are called color matching functions

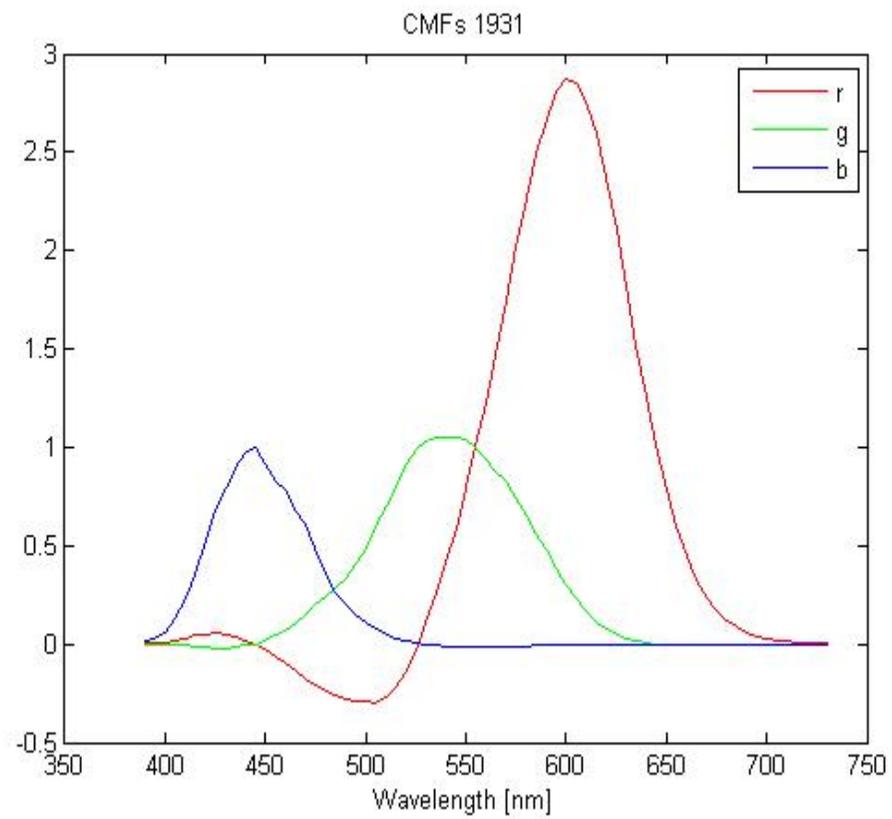
CMFs



D65



CMF rgb 1931



Stiles and Burch 10deg (1959)

Primary lights: monochromatic

$$\lambda_R = 645.16 \text{ nm}$$

$$\lambda_G = 526.32 \text{ nm}$$

$$\lambda_B = 444.44 \text{ nm}$$

$\bar{r}_{10}(\lambda)$, $\bar{g}_{10}(\lambda)$, $\bar{b}_{10}(\lambda)$ CMFs

$$t(\lambda) = R \cdot \bar{r}_{10}(\lambda) + G \cdot \bar{g}_{10}(\lambda) + B \cdot \bar{b}_{10}(\lambda)$$

t : monochromatic test light

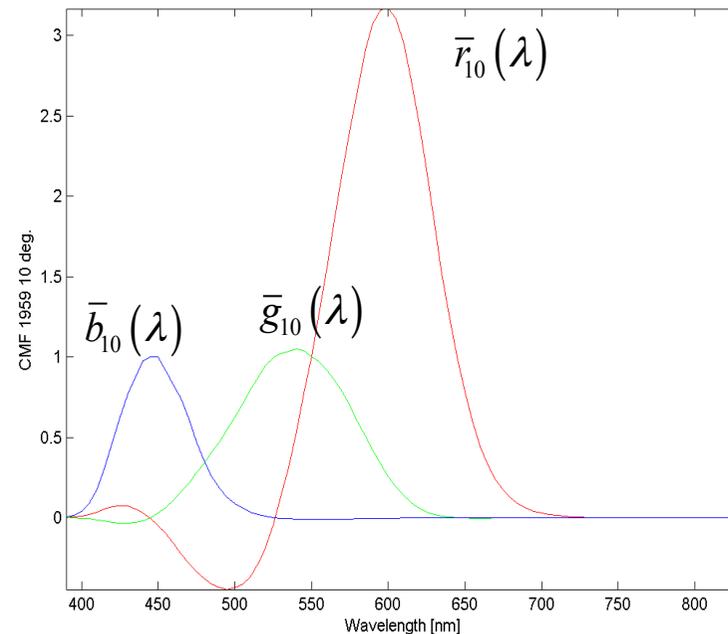
(R,G,B) : **tristimulus values** of t

A **10 degrees bipartite field** was

used. Negative values for the tristimulus value mean that the corresponding primary was added to the test light in order to match the color appearance.

This outlines that not every test color can be matched by an additive mixture of the three primaries.

The presence of negative values could be impractical, so another color coordinate system was chosen as the reference by the *Commission Internationale d'Éclairage (CIE)* in 1931.



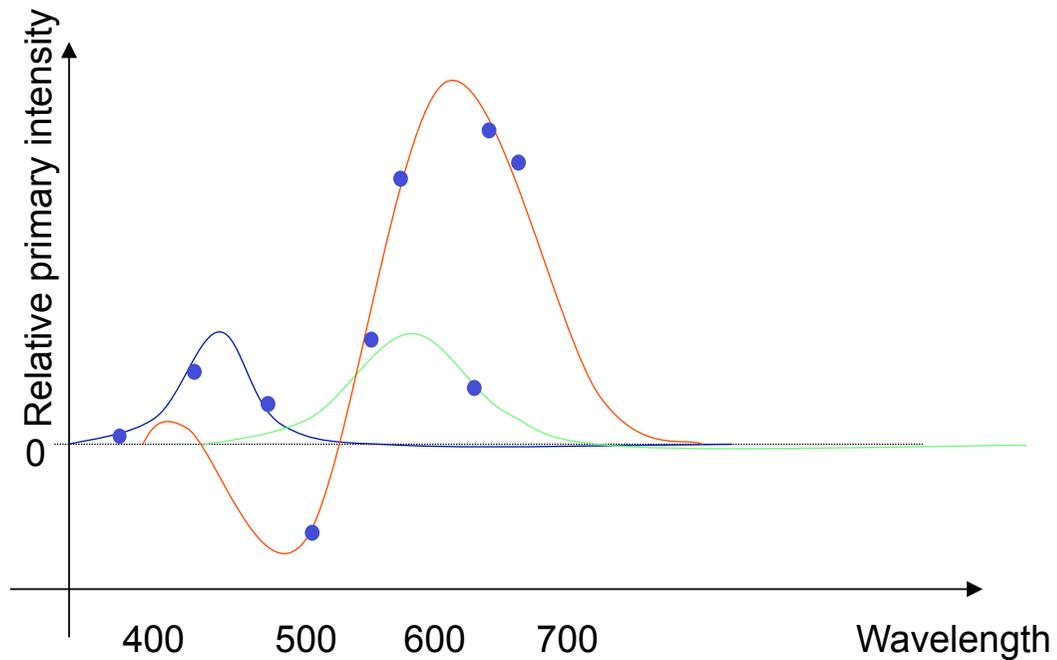
Cone photopigments and CMF

- How well do the spectral sensitivities of the cone photopigments predict performance on the photopic color matching experiment?

$$\begin{array}{c} \text{Biological} \\ \text{measurements} \end{array}
 \begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \text{Spectral sensitivity of L photopigments} \\ \text{Spectral sensitivity of M photopigments} \\ \text{Spectral sensitivity of S photopigments} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}
 \quad
 \begin{array}{c} \text{Psychophysical measurements} \end{array}
 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \text{CMF of primary 1} \\ \text{CMF of primary 2} \\ \text{CMF of primary 3} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

- There should be a linear transformation that maps the cone absorption curves to the system matrix of the color matching experiment
- Linking hypothesis*

Cone photopigments and CMFs



From the agreement between these two datasets one can conclude that the photopigment spectral responsivities provide a satisfactory biological basis to explain the photopic color matching experiments

Tristimulus values for complex stimuli

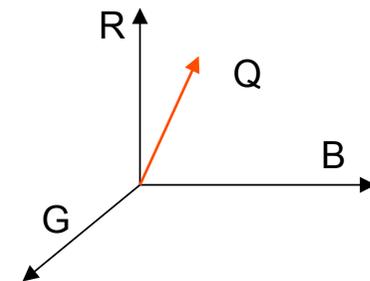
- Color stimuli are represented by vectors in a three-dimensional space, called the *tristimulus space*
 - Let Q be an arbitrary **monochromatic** color stimulus and **R,G and B the fixed primary stimuli** chosen for the color matching experiment

$$Q = R_Q \vec{R} + G_Q \vec{G} + B_Q \vec{B}$$

- R_Q, G_Q, B_Q : *tristimulus values* of Q
- The scalar multipliers R_Q, G_Q, B_Q are measured in terms of the *assigned respective units of the corresponding primaries*
- It is customary to choose these units such that when additively mixed yield a complete color match with a specified *achromatic* stimulus, usually one with an *equal-energy spectrum* on a wavelength basis

The **units** of these primaries was chosen in the radiant power ratio of **72.1:1.4:1.0**, which places the chromaticity coordinates of the equal energy stimulus E at the center of the (r,g) chromaticity diagram

→ $R_W = G_W = B_W = 1$ for the reference white

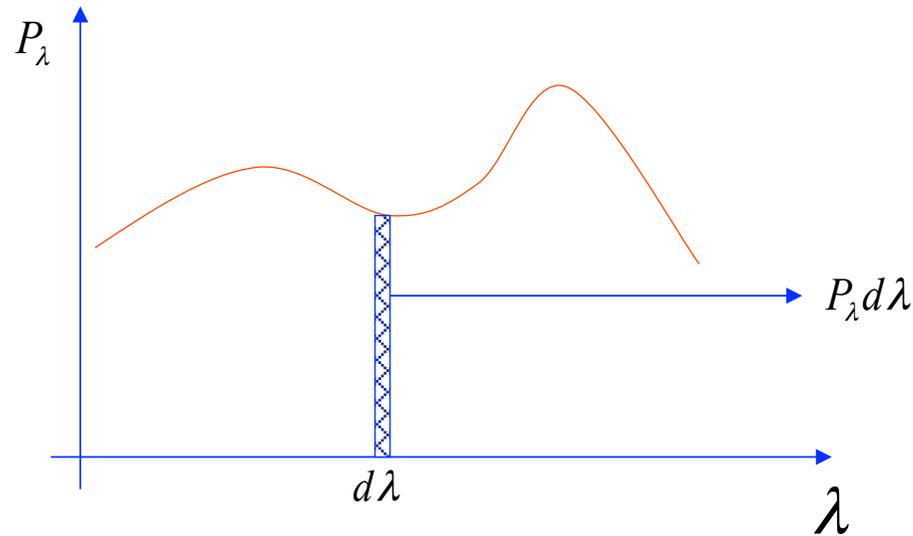


Complex stimuli

- A given complex stimuli Q with spectral power density (SPD) $\{P_\lambda d\lambda\}_Q$ can be seen as an *additive mixture* of a set of monochromatic stimuli Q_i with SPD $\{P_\lambda d\lambda\}_{Q_i}$
 - For each monochromatic stimulus

$$\vec{P}_\lambda = R_\lambda \vec{R} + G_\lambda \vec{G} + B_\lambda \vec{B}$$

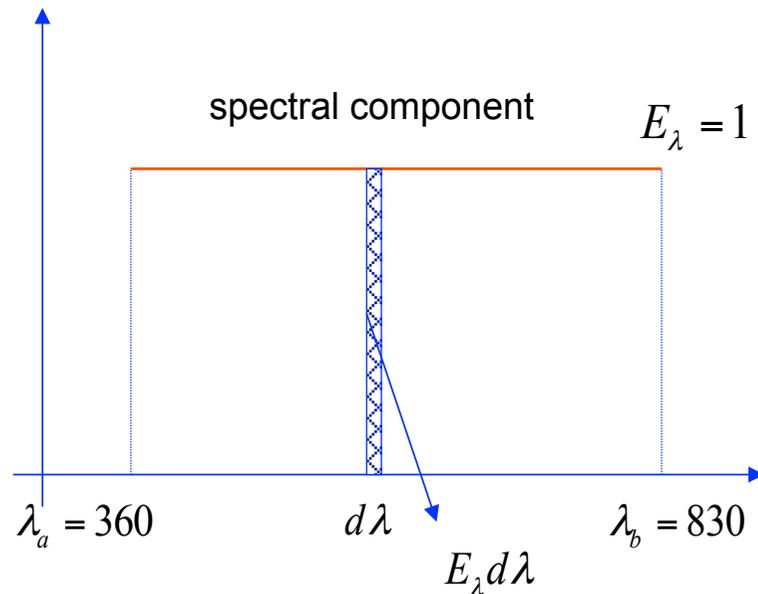
$R_\lambda, G_\lambda, B_\lambda$ spectral tristimulus values



Special case: reference white

- The *reference white* is used to express the complex spectrum in a different form

$$P_\lambda = E_\lambda$$



The Color Matching Functions are normalized such that the tristimulus values of the reference white are equal.

$$\begin{aligned} \vec{E}_\lambda &= \bar{r}(\lambda)\vec{R} + \bar{g}(\lambda)\vec{G} + \bar{b}(\lambda)\vec{B} \Rightarrow \\ \int_\lambda \vec{E}_\lambda d\lambda &= \int_\lambda (\bar{r}(\lambda)\vec{R} + \bar{g}(\lambda)\vec{G} + \bar{b}(\lambda)\vec{B}) d\lambda = \\ &= \int_\lambda (\bar{r}(\lambda)\vec{R}) d\lambda + \int_\lambda (\bar{g}(\lambda)\vec{G}) d\lambda + \int_\lambda (\bar{b}(\lambda)\vec{B}) d\lambda = \\ &= \int_\lambda \bar{r}(\lambda) d\lambda \times \vec{R} + \int_\lambda \bar{g}(\lambda) d\lambda \times \vec{G} + \int_\lambda \bar{b}(\lambda) d\lambda \times \vec{B} \end{aligned}$$

Normalization conditions

$$E_R = \int_{-\infty}^{+\infty} \bar{r}(\lambda) d\lambda = 1$$

$$E_G = \int_{-\infty}^{+\infty} \bar{g}(\lambda) d\lambda = 1$$

$$E_B = \int_{-\infty}^{+\infty} \bar{b}(\lambda) d\lambda = 1$$

Then

$$\vec{E} = 1\vec{R} + 1\vec{G} + 1\vec{B}$$

Tristimulus values of a complex stimulus

$$\begin{aligned}Q_\lambda &= (P_\lambda d\lambda)E_\lambda = (P_\lambda d\lambda)\bar{r}(\lambda)\vec{R} + (P_\lambda d\lambda)\bar{g}(\lambda)\vec{G} + (P_\lambda d\lambda)\bar{b}(\lambda)\vec{B} \rightarrow \\R_Q &= \int (P_\lambda d\lambda)\bar{r}(\lambda) = \int P_\lambda \bar{r}(\lambda) d\lambda \\G_Q &= \int (P_\lambda d\lambda)\bar{g}(\lambda) = \int P_\lambda \bar{g}(\lambda) d\lambda \\B_Q &= \int (P_\lambda d\lambda)\bar{b}(\lambda) = \int P_\lambda \bar{b}(\lambda) d\lambda\end{aligned}$$

Metameric stimuli different SPD, same color appearance

$$\begin{aligned}R_Q &= \int P_\lambda^1 \bar{r}(\lambda) d\lambda = \int P_\lambda^2 \bar{r}(\lambda) d\lambda \\G_Q &= \int P_\lambda^1 \bar{g}(\lambda) d\lambda = \int P_\lambda^2 \bar{g}(\lambda) d\lambda \\B_Q &= \int P_\lambda^1 \bar{b}(\lambda) d\lambda = \int P_\lambda^2 \bar{b}(\lambda) d\lambda\end{aligned}$$

Chromatic coordinates

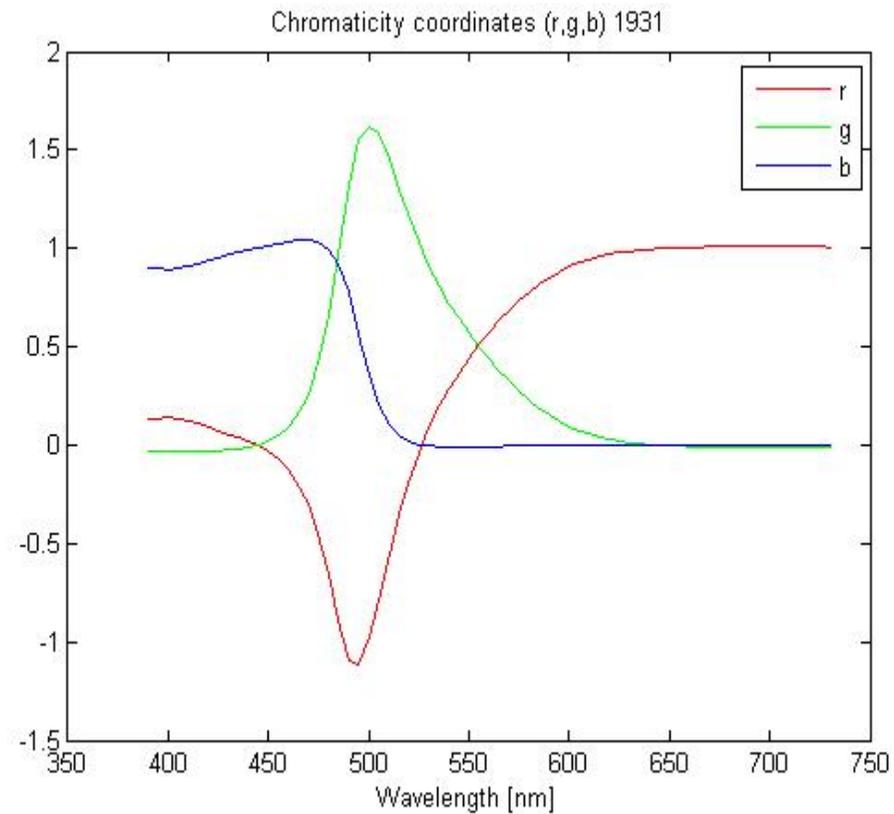
- Spectral chromaticity coordinates

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$r(\lambda) + g(\lambda) + b(\lambda) = 1$$

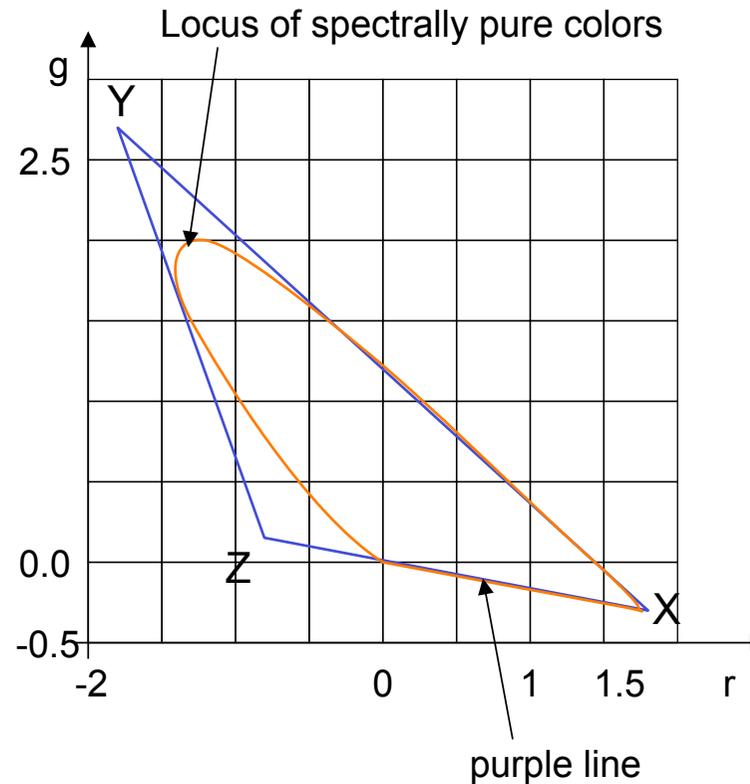


(r,g) chromaticity diagram

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$
$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$
$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

r, g, b : chromaticity coordinates

$\bar{r}, \bar{g}, \bar{b}$ color matching functions (tristimulus values of the reference white)



Chromaticity coordinates

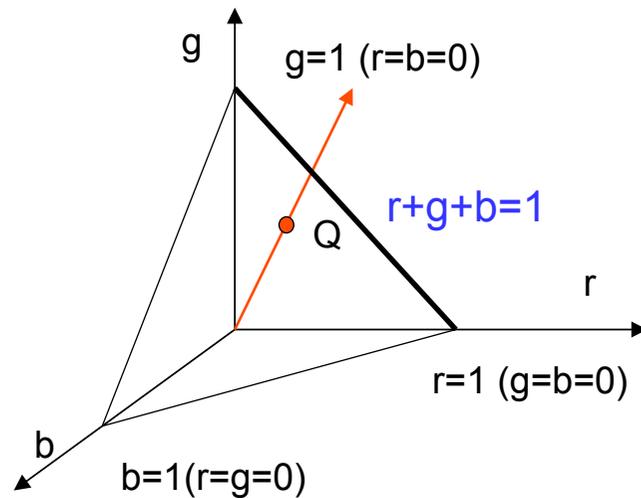
Chromaticity coordinates

$$r = \frac{R}{R+G+B}$$

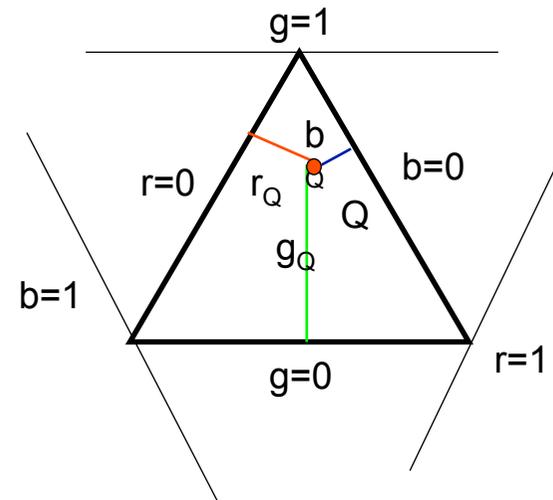
$$g = \frac{G}{R+G+B} \quad \Longrightarrow \quad r + g + b = 1$$

$$b = \frac{B}{R+G+B}$$

R,G,B: tristimulus value of the generic color



Maxwell color triangle



(r,g) specify the *hue and saturation* of the color while the information about the luminance is lost

A step in colorimetry...

CIE 1931 Standard Observer

- In colorimetric practice, the main objective is to obtain results valid for the group of normal trichromats. To this end, the color matching properties of an *ideal trichromatic observer* are defined by specifying three independent functions of λ which are identified with the ideal observer CMFs.
- The CIE 1931 SO also embodies the additivity law for brightness ($V(\lambda)$ photopic luminous efficiency function)
 - For an observer who makes brightness matches that *conform to the additivity law for brightness*, and who also *makes color matches* that are trichromatic in the stronger sense, it can be shown that $V(\lambda)$ is a combination of the CMFs, provided all the pairs of metameric stimuli are also in brightness match.

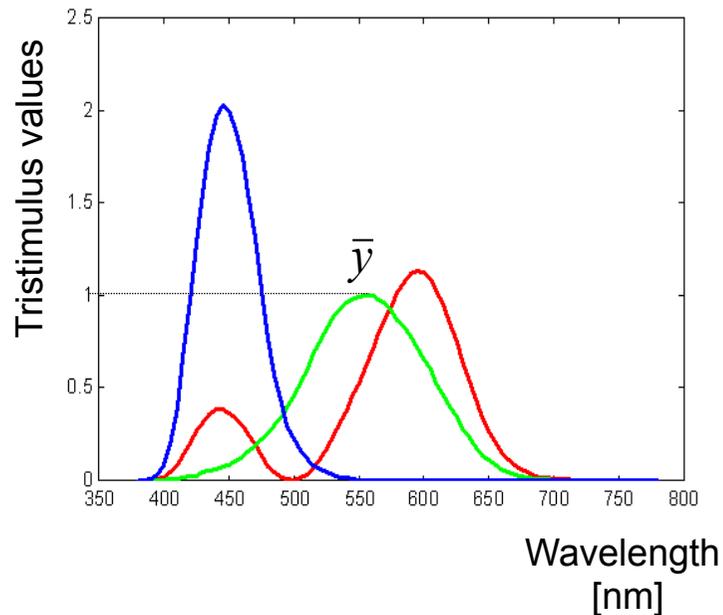
For such an observer, it is possible to select from the infinitely many equivalent sets of CMFs one set for which one of the three CMFs, usually taken to be the central one

coincides with $V(\lambda)$.

In this way, the CIE 1931 SO combines both color matching and heterochromatic brightness matching properties in a single quantitative scheme.

CIE 1931 Standard Colorimetric Observer

- Standard system for color representation: X,Y,Z tristimulus coordinate system $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$
- Color matching functions



Features

- $\lambda=380$ to 780 nm, $\Delta\lambda=5$ nm
- Measured at 2 degrees
- Always non negative
- \bar{y} is a rough approximation of the *brightness* of monochromatic lights of equal size and duration (*Standard photopic luminosity function $V(\lambda)$*)
- They cannot be measured by color matching experiments
- Quite inaccurate at low wavelengths

Improvements

- In 1959 a new set of CIE XYZ coordinates was derived based on the CMFs measured by Stiles&Burch at 10 degrees (CIE 1964 Supplementary Standard Colorimetric Observer).

Guidelines for the derivation of CIE 1931 SO

- Projective transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

(r_x, g_x, b_x) : coordinates of (1,0,0) as measured in the {r,g,b} system

....

- Need to determine the matrix A of the transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \dots & \\ & & a_{3,3} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \dots & \\ & & a_{3,3} \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1}$$

- This is accomplished by imposing some conditions

Guidelines for the derivation of CIE 1931 SO

1. The function $\bar{y}(\lambda)$ must be equal to the luminosity function of the eye $V(\lambda)$
 $\bar{y}(\lambda) = V(\lambda)$

this sets a relation among 3 coefficients

$$a_{11} + a_{12} + a_{13} = \text{const.}$$

2. The constant spectrum of white, $E(\lambda)=1$, should have equal tristimulus values

$$\sum_{i=1}^N \bar{x}(\lambda_i) = \sum_{i=1}^N \bar{y}(\lambda_i) = \sum_{i=1}^N \bar{z}(\lambda_i)$$

$$\sum_{i=1}^N \bar{x}(\lambda_i) = a_{11} \sum_{i=1}^N \bar{r}(\lambda_i) + a_{12} \sum_{i=1}^N \bar{g}(\lambda_i) + a_{13} \sum_{i=1}^N \bar{b}(\lambda_i)$$



$$(a_{11} + a_{12} + a_{13}) =$$

$$= (a_{21} + a_{22} + a_{23}) =$$

$$= (a_{31} + a_{32} + a_{33}) = \sigma$$

but

$$\sum_{i=1}^N \bar{r}(\lambda_i) = \sum_{i=1}^N \bar{g}(\lambda_i) = \sum_{i=1}^N \bar{b}(\lambda_i) = S$$

thus

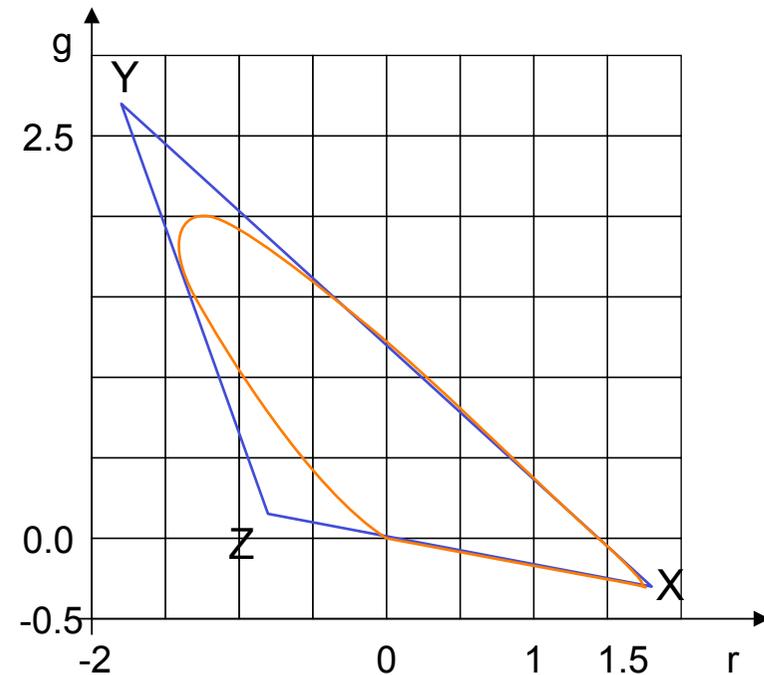
$$\sum_{i=1}^N \bar{x}(\lambda_i) = (a_{11} + a_{12} + a_{13})S$$

[Ref: Color vision and colorimetry, D. Malacara]

.....

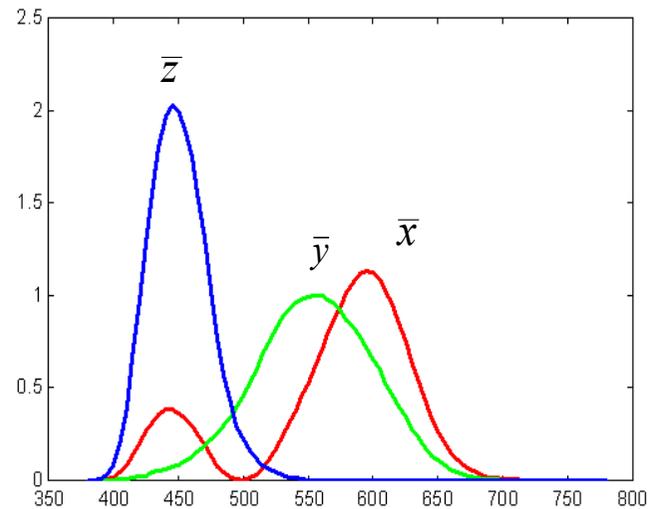
Guidelines for the derivation of CIE 1931 SO

3. The line joining X and Y be tangent to the curve on the red side
 - In this way, a linear combination of X and Y is sufficient to describe those colors without any Z
 - This introduces other conditions on a_{31} , a_{32} , a_{33} and sets their values
4. No values of $\bar{x}(\lambda)$ is negative
 - This adds a condition relating a_{11} , a_{12} and a_{13} , whose sum must be equal to known constant σ . This leaves one degree of freedom that is used to set the area of the XYZ triangle at its minimum



From rgb to xyz

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} 0.41846 & -0.1586 & -0.08283 \\ -0.09117 & 0.25243 & 0.01571 \\ 0.00092 & -0.00255 & 0.17860 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



rgb2xyz

- Chromaticity coordinates

$$x = \frac{0.49r + 0.31g + 0.2b}{0.66697r + 1.1324g + 1.20063b}$$
$$y = \frac{0.17697r + 0.81240g + 0.01063b}{0.66697r + 1.1324g + 1.20063b}$$
$$z = \frac{0.0r + 0.01g + 0.99b}{0.66697r + 1.1324g + 1.20063b}$$

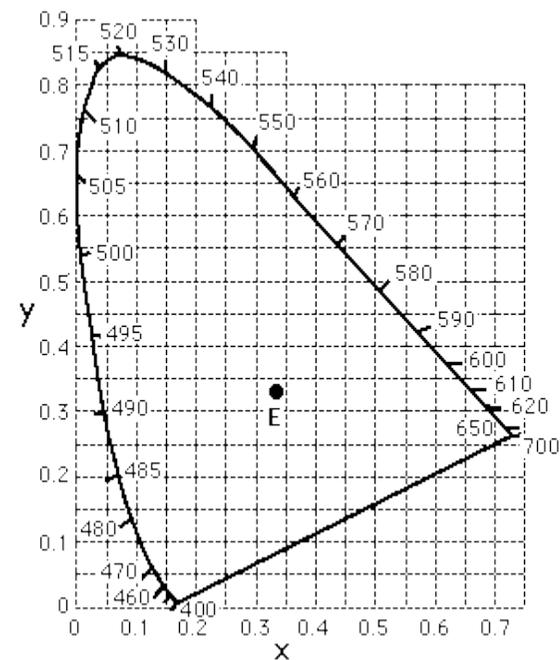
- Tristimulus values

$$X = \frac{x}{y}V \quad Y = V \quad Z = \frac{z}{y}V$$

- CMF

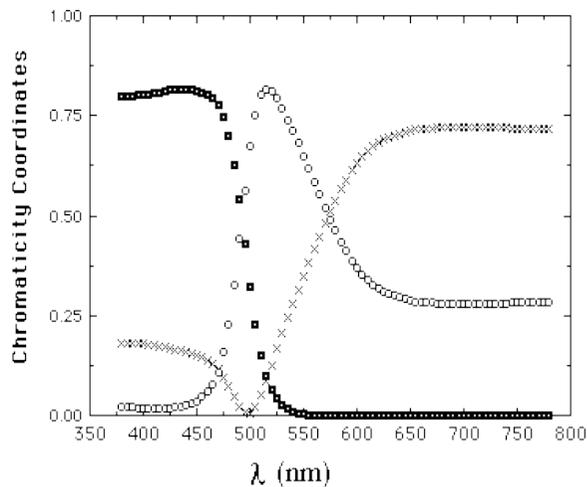
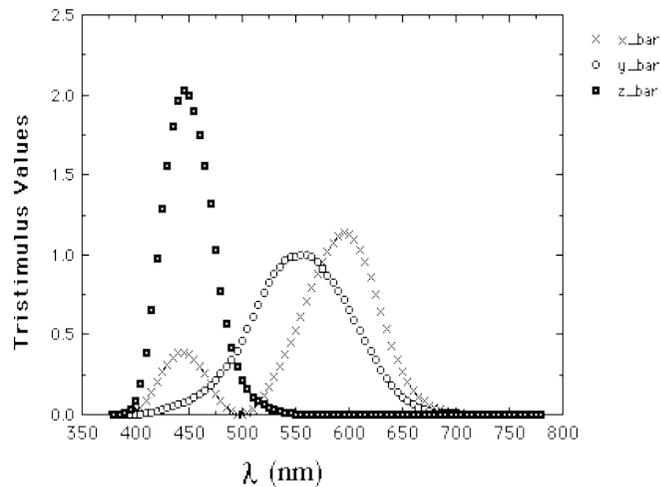
$$\bar{x}(\lambda) = \frac{x(\lambda)}{y(\lambda)}V(\lambda)$$
$$\bar{y}(\lambda) = V(\lambda)$$
$$\bar{z}(\lambda) = \frac{z(\lambda)}{y(\lambda)}V(\lambda)$$

(x,y) chromaticity diagram



$$x_E = y_E = \frac{1}{3}$$

CIE 1964 SO



Features

- 10 degrees field
- Extended set of wavelengths (390 to 830 nm)
- r,g,b CMFs obtained directly from the observations
 - Measures of the radiant power of each monochromatic test stimulus
- High illumination intensity
 - To minimize rods intrusion
- Data extrapolated at 1nm resolution

$$\bar{x}_{10}(\lambda) = 0.341080\bar{r}_{10}(\lambda) + 0.189145\bar{g}_{10}(\lambda) + 0.387529\bar{b}_{10}(\lambda)$$

$$\bar{y}_{10}(\lambda) = 0.139058\bar{r}_{10}(\lambda) + 0.837460\bar{g}_{10}(\lambda) + 0.073316\bar{b}_{10}(\lambda)$$

$$\bar{z}_{10}(\lambda) = 0.0\bar{r}_{10}(\lambda) + 0.039553\bar{g}_{10}(\lambda) + 2.026200\bar{b}_{10}(\lambda)$$

CIE Chromaticity Coordinates

- (X,Y,Z) tristimulus values

$$X = \int P_{\lambda} \bar{x}(\lambda) d\lambda$$

$$Y = \int P_{\lambda} \bar{y}(\lambda) d\lambda$$

$$Z = \int P_{\lambda} \bar{z}(\lambda) d\lambda$$

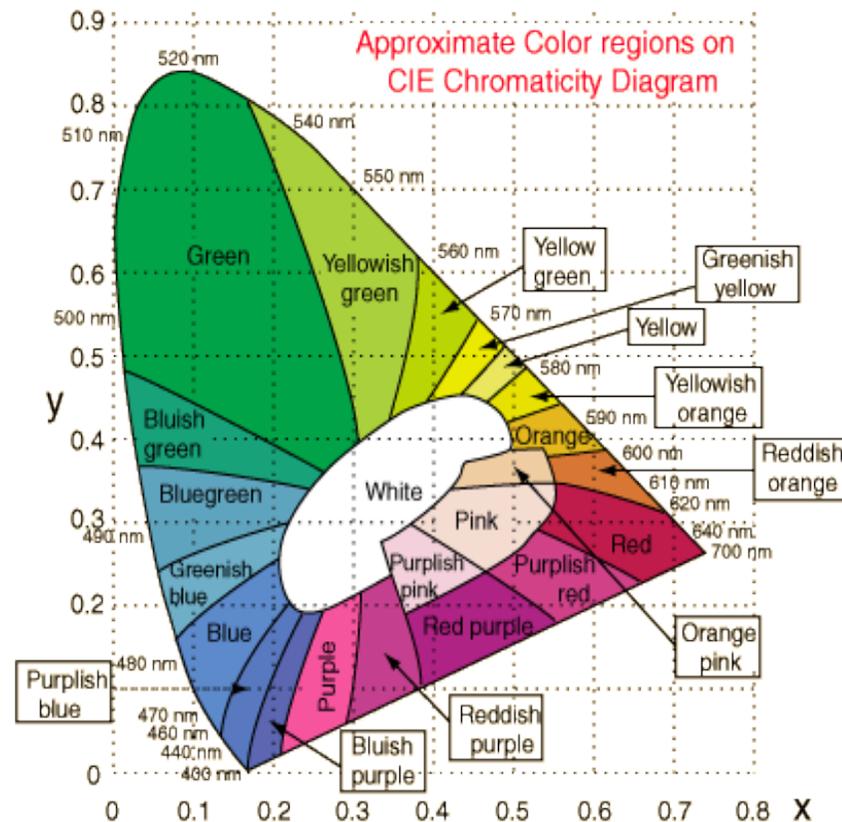
- Chromaticity coordinates

$$x(\lambda) = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

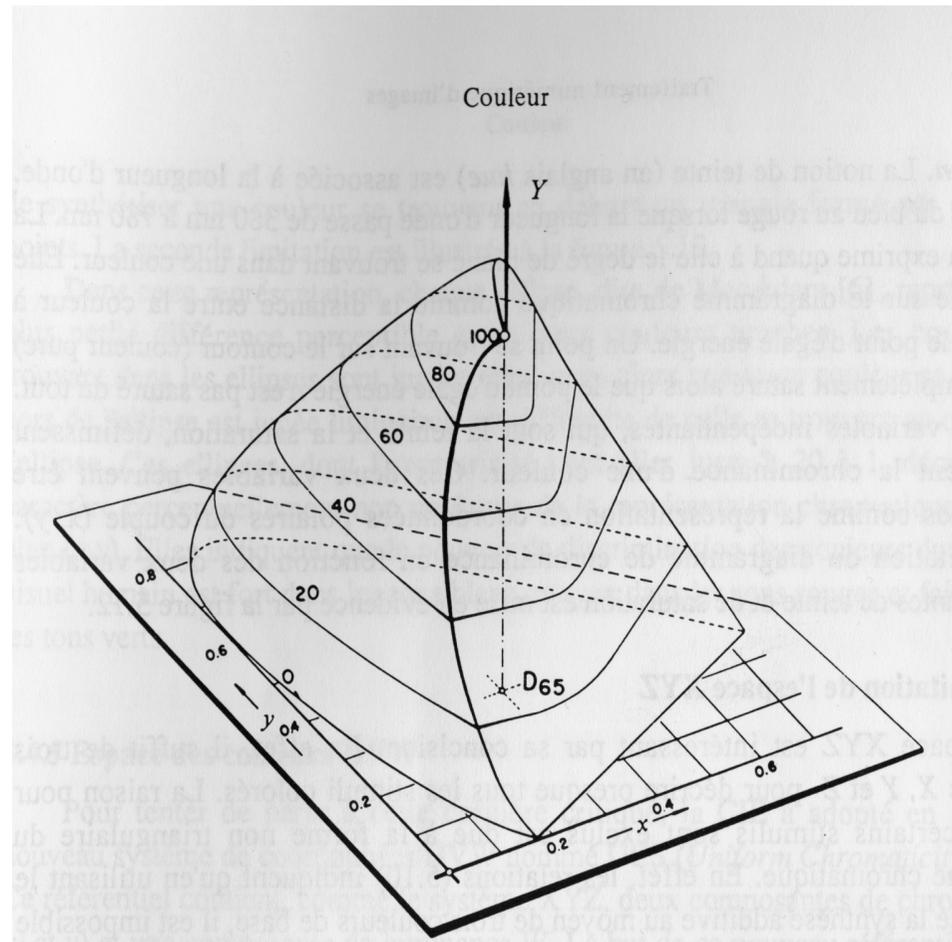
$$y(\lambda) = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

$$z(\lambda) = \frac{\bar{z}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

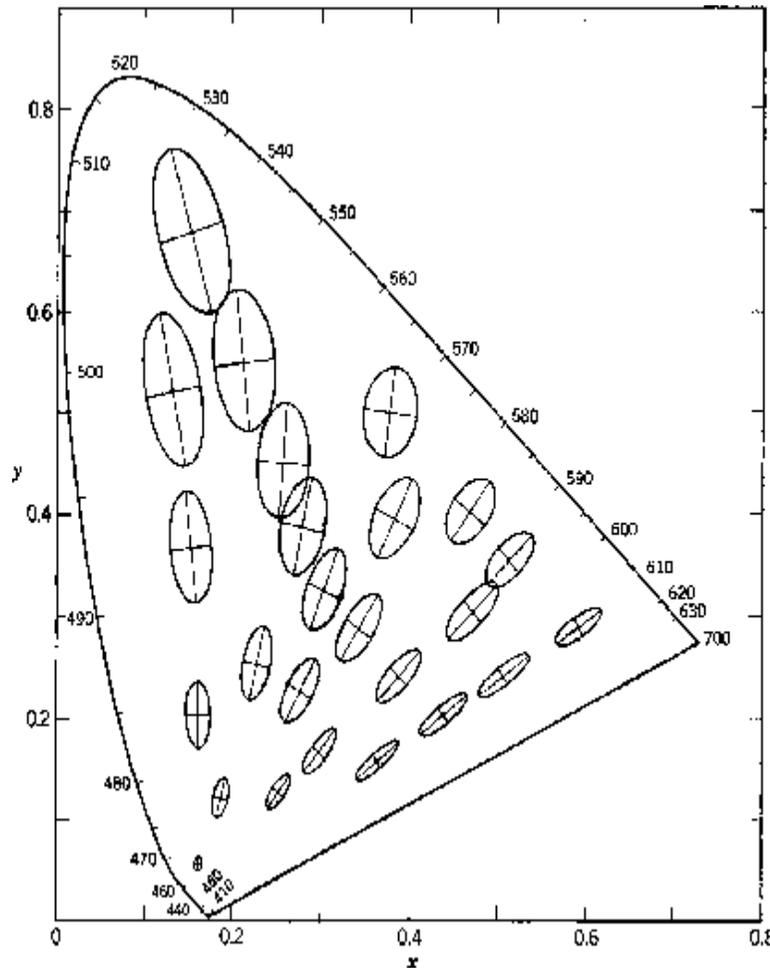
$$x(\lambda) + y(\lambda) + z(\lambda) = 1 \quad \text{x - y chromaticity diagram}$$



xyY



Mac Adams' ellipses



The ellipses represent a **constant perceptual color stimulus**, at a constant luminance, at various positions and in various directions, in the x,y diagram.

The areas of the ellipses vary greatly.

This means that the XYZ colorspace (as the RGB color space) is *not perceptually uniform*.

To avoid this nonuniformity, CIE recommended a new CIE 1964 UCS (Uniform-Chromaticity Scale) diagram, to be used with constant luminance levels.

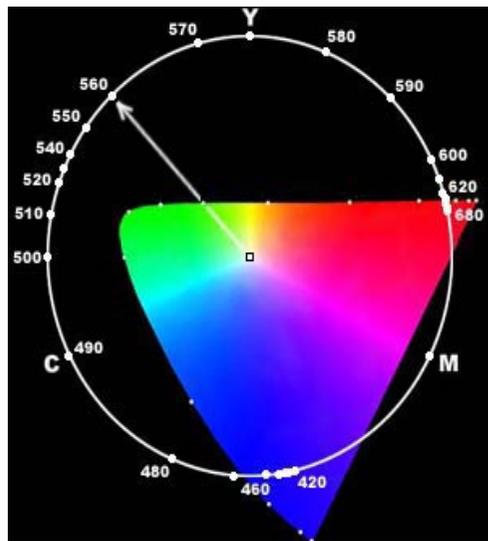
In perceptually uniform colorspace, the size of the MacAdams' ellipses are more uniform and the eccentricity is lower.

Perceptually uniform Colorspaces

- CIE 1960 Luv colorspace
 - reversible transformation

$$u = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v = \frac{6Y}{X + 15Y + 3Z} = \frac{6x}{-2x + 12y + 3}$$



- CIE 1976 L*u*v* (CIELUV)

$$u' = u$$

$$v' = 1.5v$$

L*: *perceived lightness*

$$L^* = 116 \left(\frac{Y}{Y_n} \right)^{1/3} - 16$$

$$u^* = 13L^*(u' - u'_n)$$

$$v^* = 13L^*(v' - v'_n)$$

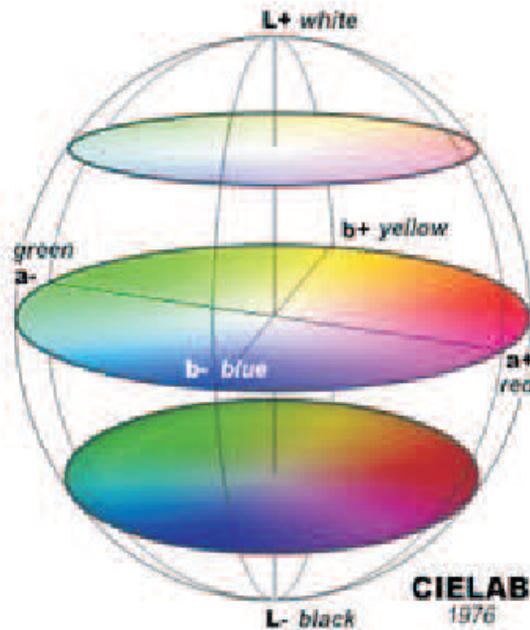
$$u' = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v' = \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3}$$

u'_n, v'_n : reference white

Perceptually uniform Color models

CIE 1976 L*a*b* (CIELAB)



X_n, Y_n, Z_n : *reference white*

Tristimulus values for a nominally white object-color stimulus. Usually, it corresponds to the spectral radiance power of one of the CIE standard illuminants (as D65 or A), reflected into the observer's eye by a perfect reflecting diffuser. Under these conditions, X_n, Y_n, Z_n are the tristimulus values of the standard illuminant with $Y_n=100$.

For: $\frac{Y}{Y_n}, \frac{X}{X_n}, \frac{Z}{Z_n} \geq 0.01$

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{1/3} - 16$$

$$a^* = 500\left[\left(\frac{X}{X_n}\right)^{1/3} - \left(\frac{Y}{Y_n}\right)^{1/3}\right]$$

$$b^* = 200\left[\left(\frac{Y}{Y_n}\right)^{1/3} - \left(\frac{Z}{Z_n}\right)^{1/3}\right]$$

otherwise

$$L^* = 116 \left[f\left(\frac{Y}{Y_n}\right) - \frac{16}{116} \right]$$

$$a^* = 500 \left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right]$$

$$b^* = 200 \left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right]$$

$$f\left(\frac{Y}{Y_n}\right) = \begin{cases} \left(\frac{Y}{Y_n}\right)^{1/3} & \text{for } \frac{Y}{Y_n} > 0.008856 \\ 7.787 \frac{Y}{Y_n} + \frac{16}{116} & \text{for } \frac{Y}{Y_n} \leq 0.008856 \end{cases}$$

Hint: the diffuse light depends on both the physical properties of the surface and the illuminant

Perceptual correlates

- Color difference formula

$$\Delta E^*_{u,v} = \left[(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2 \right]^{1/2}$$

- Perceptual correlates

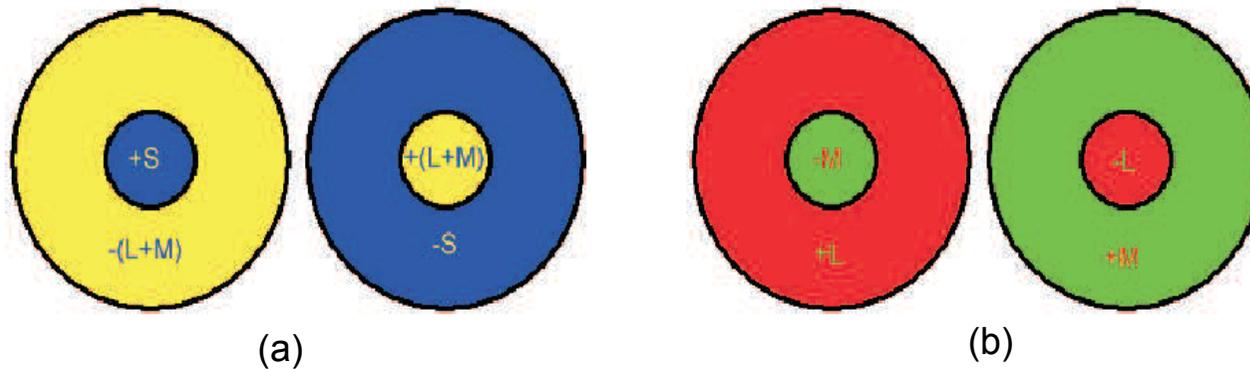
L^* : lightness

$$C^*_{u,v} = \left[(u^*)^2 + (v^*)^2 \right]^{1/2} : \text{chroma}$$

$$S^*_{u,v} = \frac{C^*_{u,v}}{L^*} : \text{saturation}$$

Opponent color models

- Underlying model: *opponent channels*



Example of typical center-surround antagonistic receptive fields: (a) on-center yellow-blue receptive fields; (b) on-center red-green receptive fields.

Because of the fact that the L, M and S cones have different spectral sensitivities, are in different numbers and have different spatial distributions across the retina, the respective receptive fields have quite different properties.

Experimental evidence: color after-image, non existence of colors like “greenish-red” or “yellowish-blue”

Summary

- References
 - B. Wandell, “Foundations of visions”
 - Wyszecki&Stiles, “Color science, concepts, methods, quantitative data and formulae”, Wiley Classic Library
 - D. Malacara, “Color vision and colorimetry, theory and applications”, SPIE Press

Color images

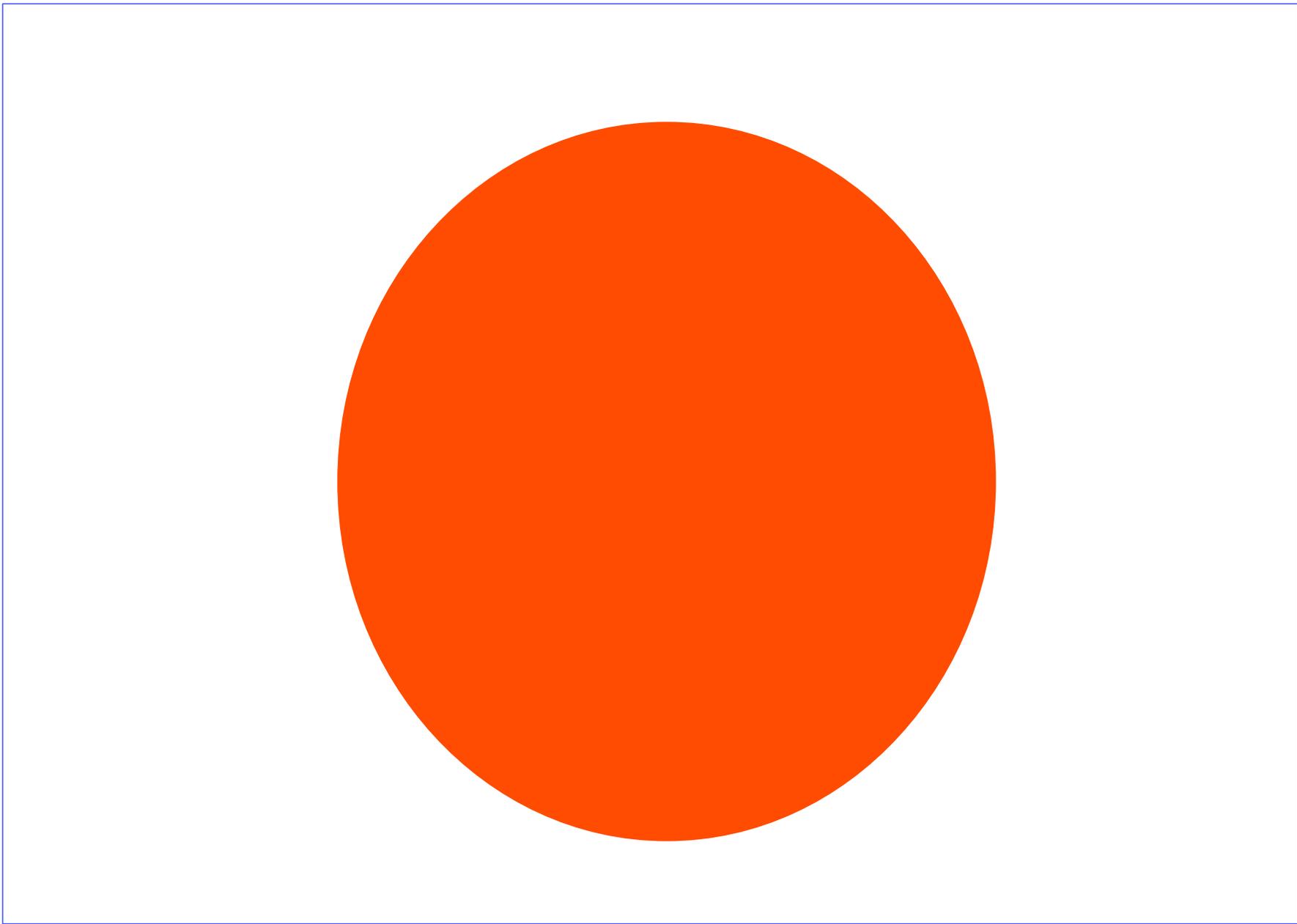
- Different approaches
 - An edge is present iff there is a gradient in the luminance
 - An edge exists if there is a gradient in any of the tristimulus components
 - “Total gradient” above a predefined threshold

$$G(j, k) = G_1(j, k) + G_2(j, k) + G_3(j, k)$$

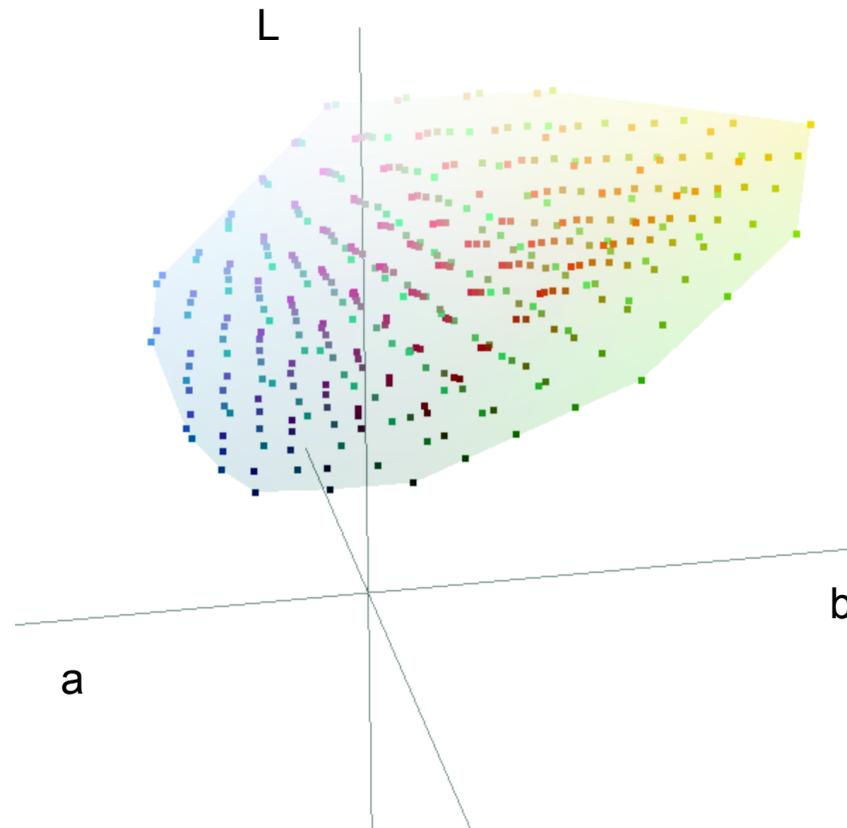
- “Vector sum gradient” above a predefined threshold

$$G(j, k) = \left\{ |G_1(j, k)|^2 + |G_2(j, k)|^2 + |G_3(j, k)|^2 \right\}^{1/2}$$

$G_i(j, k)$: i -th linear or non-linear tristimulus value

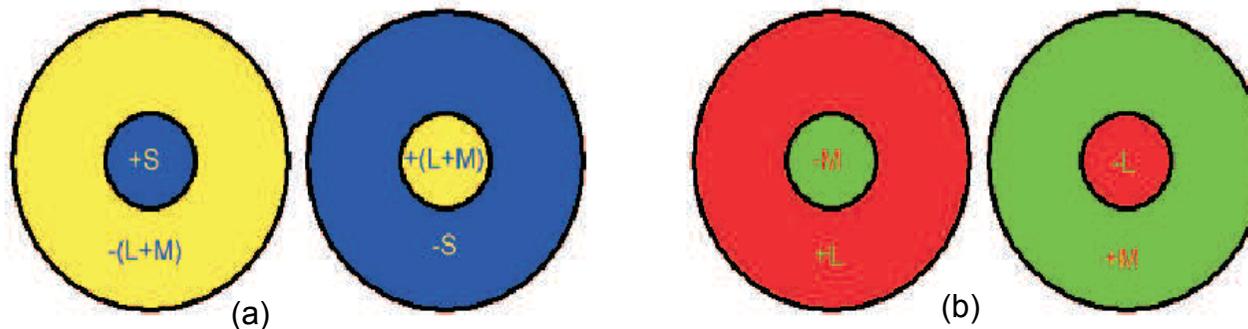


Color representation in Lab



Opponent Color Model

- Perception is mediated by *opponent color channels*
 - *Evidences*
 - Afterimages
 - Certain colors cannot be perceived simultaneously (i.e. no *reddish-green* or *bluish-yellow*)



Example of typical center-surround antagonistic receptive fields: (a) on-center yellow-blue receptive fields; (b) on-center red-green receptive fields.

Because of the fact that the L, M and S cones have different spectral sensitivities, are in different numbers and have different spatial distributions across the retina, the respective receptive fields have quite different properties.

Opponent Colors

