## Simulation

EECS 20

Lecture 15 (February 21, 2001)

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Deterministic Reactive System:

for every input signal, there is exactly one output signal.

Function:

DetSys : [ Time $\rightarrow$ Inputs ] $\rightarrow$ [ Time $\rightarrow$ Outputs ]

Nondeterministic Reactive System:

for every input signal, there is one or more output signals.

Binary relation:

$$NondetSys \subseteq [\, Time \to Inputs \,] \times [\, Time \to Outputs \,]$$

such that $\forall\, x \in [\, Time \to Inputs \,],$
$\exists\, y \in [\, Time \to Outputs \,],\ (x,y) \in NondetSys$

Every pair $(x,y) \in NondetSys$ is called a behavior.

S1 is a more detailed description of S2;

S2 is an abstraction or property of S1.

System S1 refines system S2

iff

1. Time [S1] = Time [S2] ,
2. Inputs [S1] = Inputs [S2] ,
3. Outputs [S1] = Outputs [S2] ,
4. Behaviors [S1] $\subseteq$ Behaviors [S2] .

Systems S1 and S2 are equivalent

iff

1. Time [S1] = Time [S2] ,
2. Inputs [S1] = Inputs [S2] ,
3. Outputs [S1] = Outputs [S2] ,
4. Behaviors [S1] = Behaviors [S2] .

Nondeterministic causal discrete-time reactive systems
can be implemented by
nondeterministic state machines.

## Nondeterministic State Machine

Inputs

Outputs

States

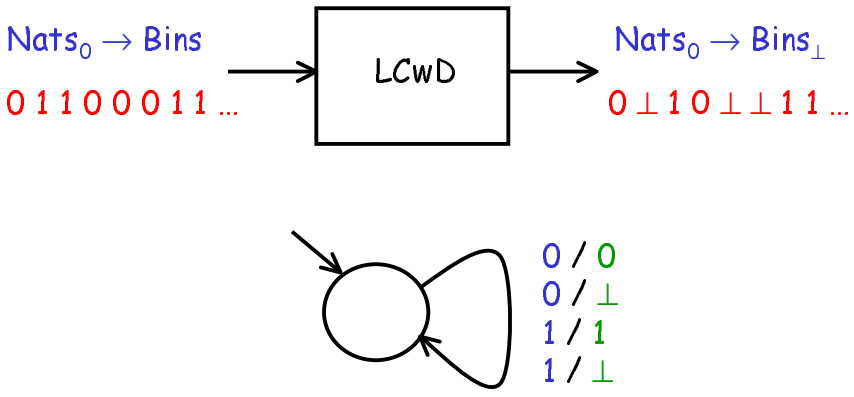
possibleInitialStates $\subseteq$ States

possibleUpdates :

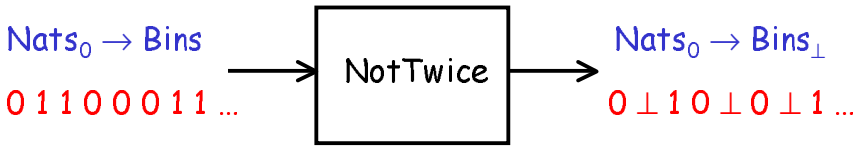
$$\text{States} \times \text{Inputs} \rightarrow \mathcal{P}(\text{States} \times \text{Outputs}) \setminus \emptyset$$

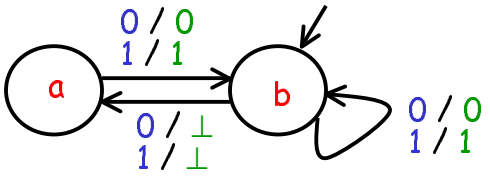
receptiveness (i.e., machine must be prepared to accept every input)

## Lossy Channel without Delay



## Channel that never drops two in a row




## Typical Exam Questions

A. Convert between three (possibly nondeterministic) system representations:

1. Input-output definition

   Deterministic case:
   define S: $[Nats_0 \to Inputs] \to [Nats_0 \to Outputs]$ such that
   for all $x \in [Nats_0 \to Inputs]$ and all $y \in Nats_0$, $(S(x))(y) = \ldots$

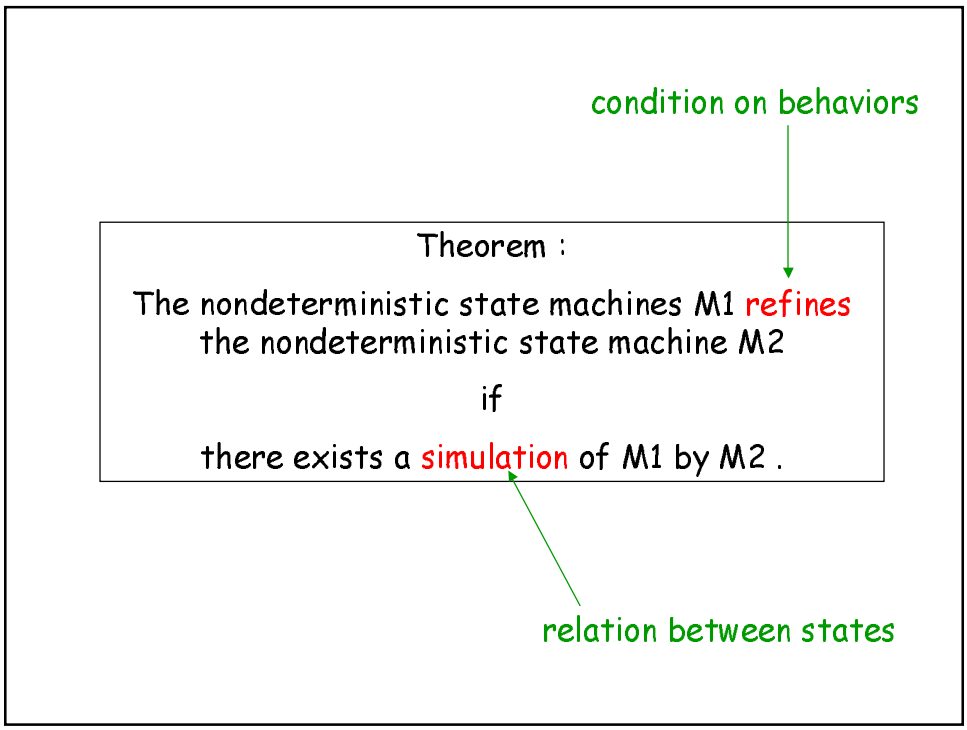
   Nondeterministic case:
   define $S \subseteq [Nats_0 \to Inputs] \times [Nats_0 \to Outputs]$ such that
   for all $x \in [Nats_0 \to Inputs]$ and all $y \in [Nats_0 \to Outputs]$,
   $(x,y) \in S$ iff ...

2. Transition diagram

3. Block diagram

B. Equivalence, bisimulation, minimization of deterministic state machines

Refinement, simulation, and minimization
of nondeterministic state machines



Theorem :

The nondeterministic state machines M1 refines the nondeterministic state machine M2

if

there exists a simulation of M1 by M2 .

Not "if-and-only-if" !

Not symmetric !

(We say that "M2 simulates M1".)

In the following, assume

Inputs [M1] = Inputs [M2]

Outputs [M1] = Outputs [M2]

A binary relation $S \subseteq States\,[M1] \times States\,[M2]$ is a simulation of M1 by M2

iff

1. $\forall\, p \in possibleInitialStates\,[M1]$,

   $\exists\, q \in possibleInitialStates\,[M2]$, $(p, q) \in S$ and

2. $\forall\, p \in States\,[M1]$, $\forall\, q \in States\,[M2]$,

   if $(p, q) \in S$,

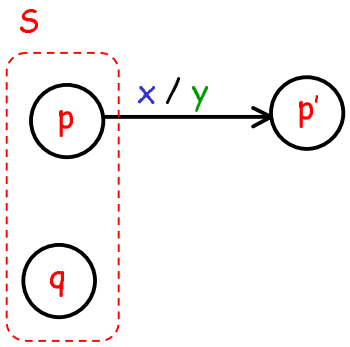
   then $\forall\, x \in Inputs$, $\forall\, y \in Outputs$, $\forall\, p' \in States\,[M1]$,

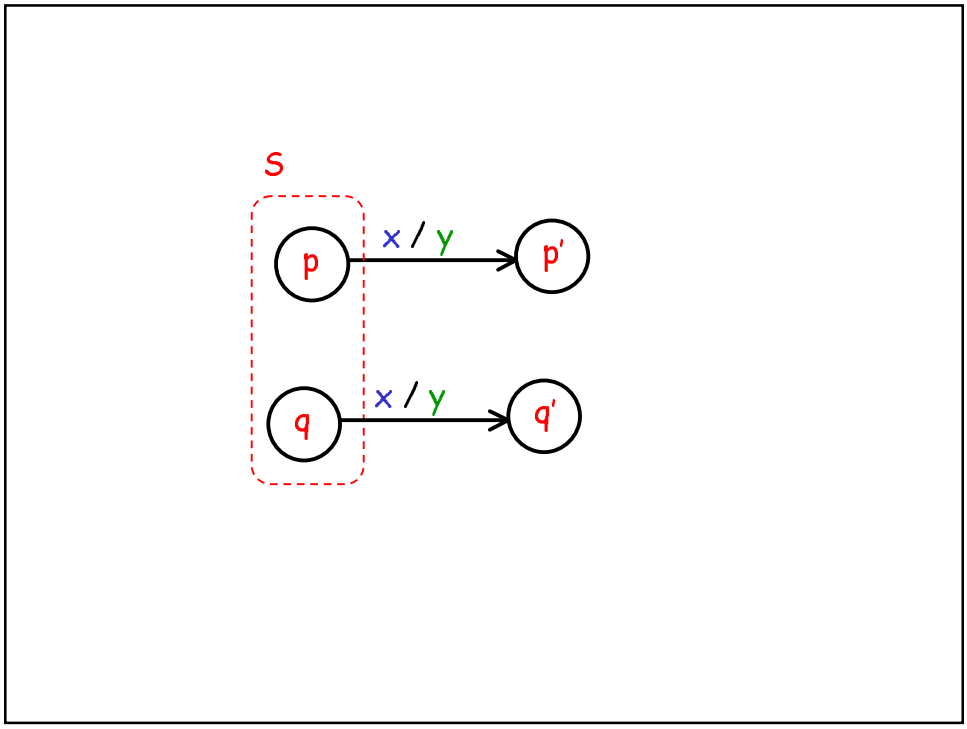
   if $(p', y) \in possibleUpdates\,[M1]\,(p, x)$

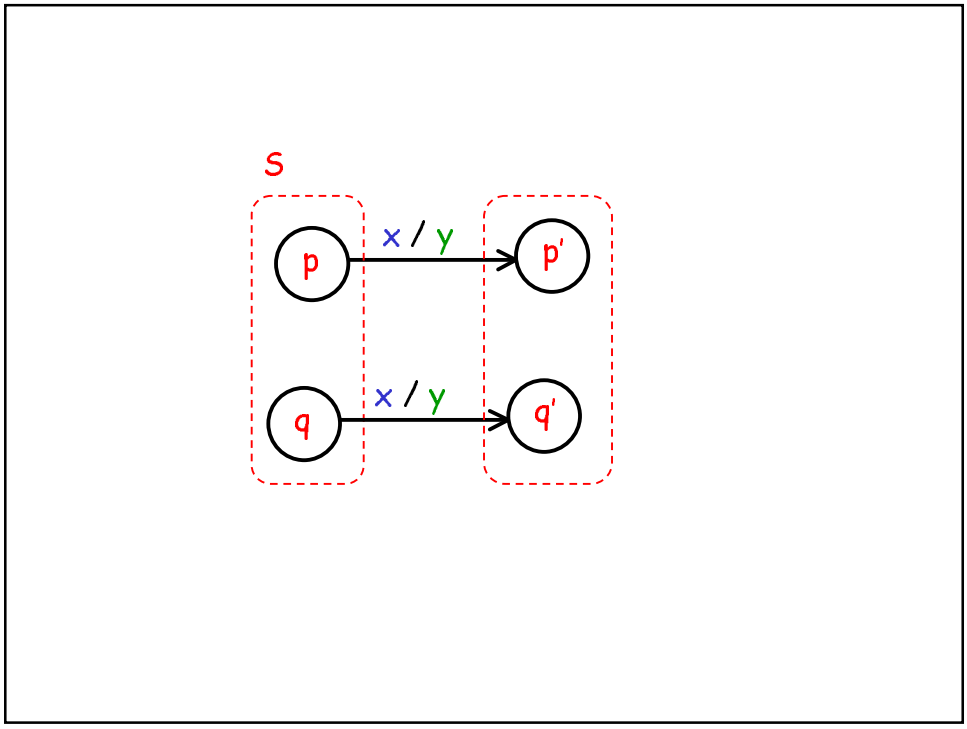
   then $\exists\, q' \in States\,[M2]$,

   $(q', y) \in possibleUpdates\,[M2]\,(q, x)$ and

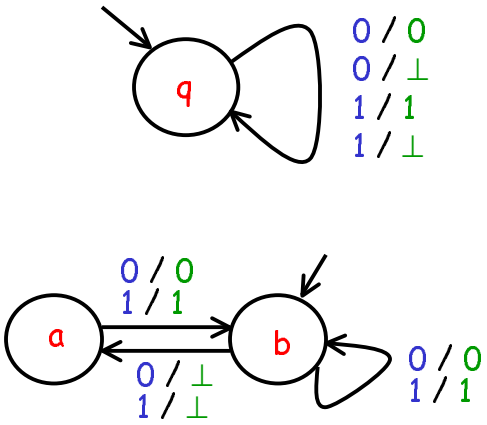
   $(p', q') \in S$.



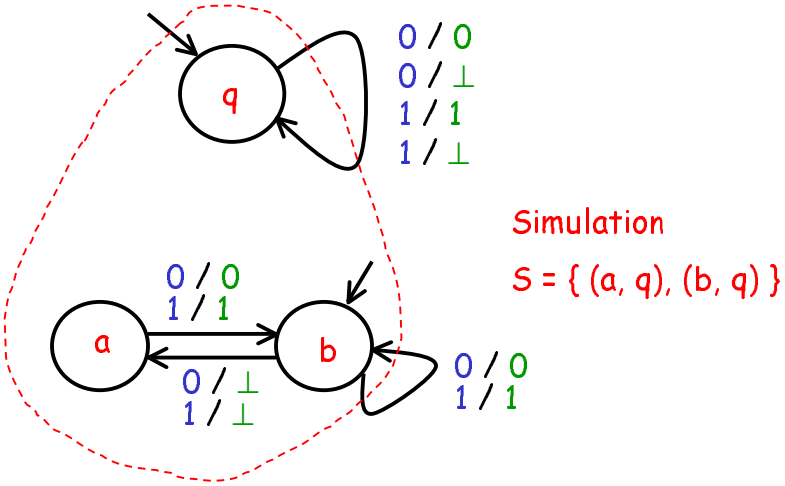

S
p
x / y
p'
q
x / y
q'


S
p
x / y
p'
q
x / y
q'

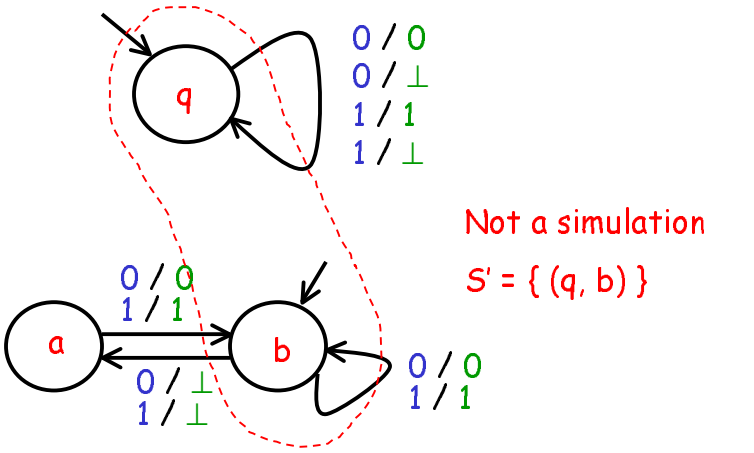
NotTwice refines Lossy Channel without Delay




NotTwice is simulated by Lossy Channel without Delay



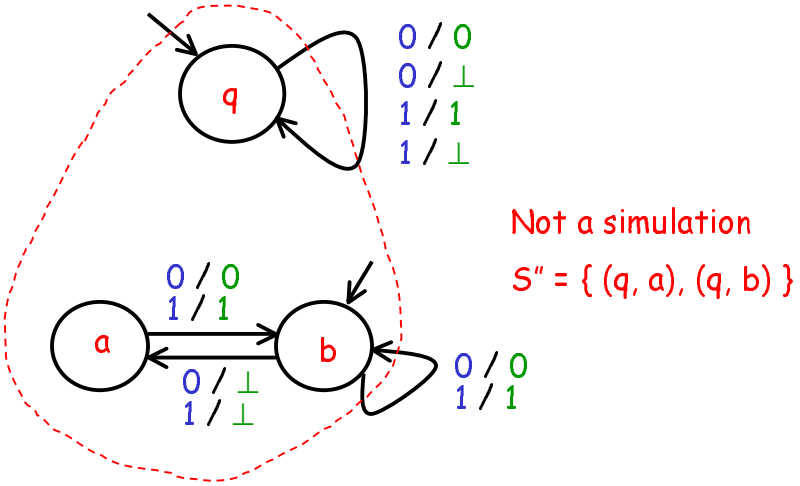
Lossy Channel without Delay is not simulated by NotTwice




Not a simulation

S' = { (q, b) }

Lossy Channel without Delay is not simulated by NotTwice




Not a simulation

S'' = { (q, a), (q, b) }

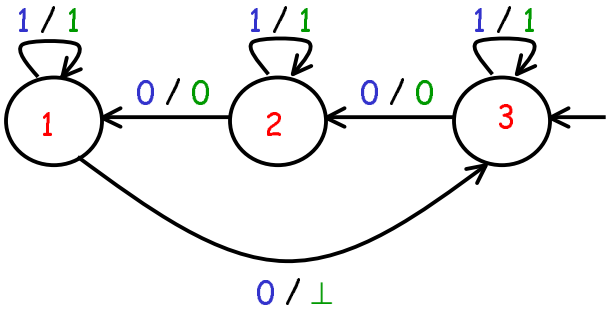
Channel that drops every third zero

Channel that drops every third zero

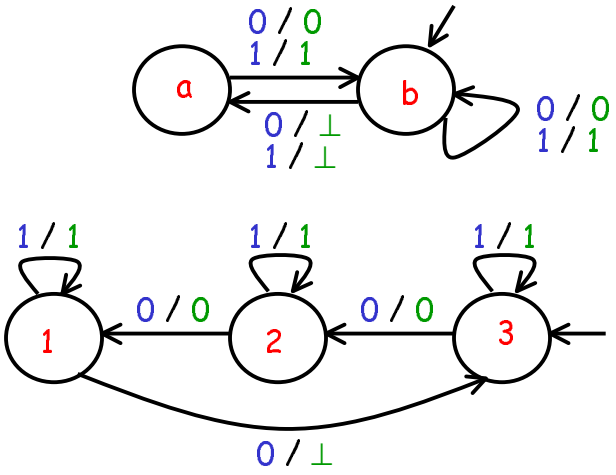
State between time t-1 and time t:

- 3 third zero from now will be dropped
- 2 second zero from now will be dropped
- 1 next zero will be dropped

Channel that drops every third zero



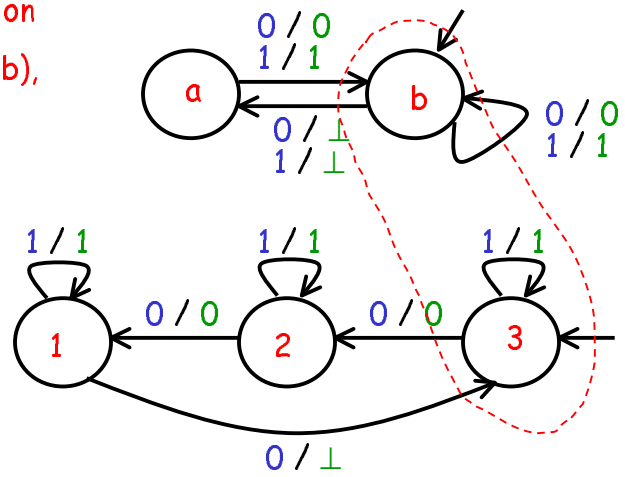

ThirdZero refines NotTwice



ThirdZero is simulated by NotTwice

Simulation

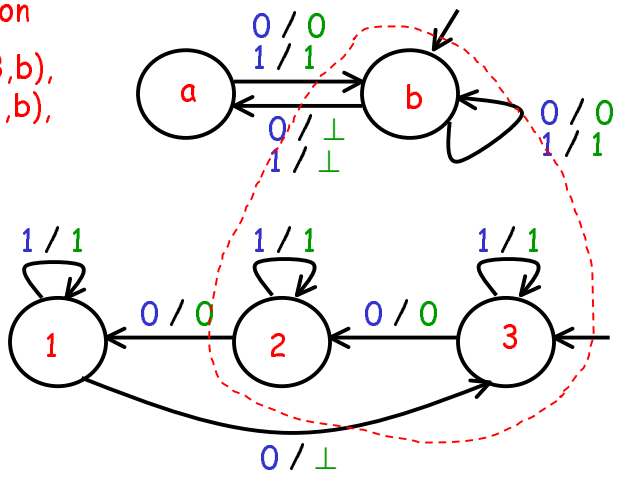
S = { (3,b),




ThirdZero is simulated by NotTwice

Simulation

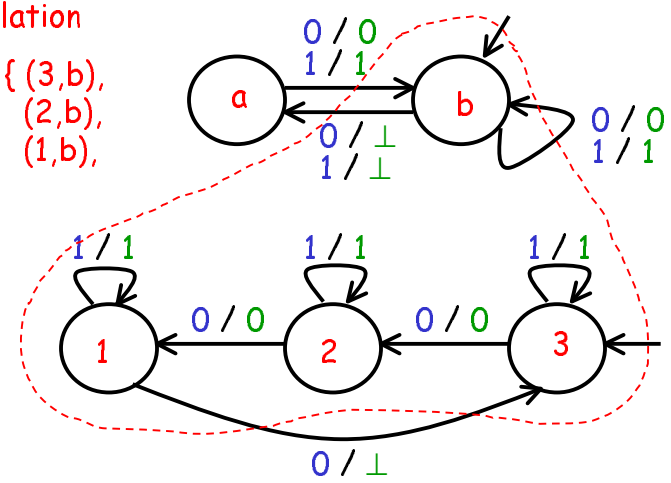
S = { (3,b),
(2,b),



ThirdZero is simulated by NotTwice

Simulation

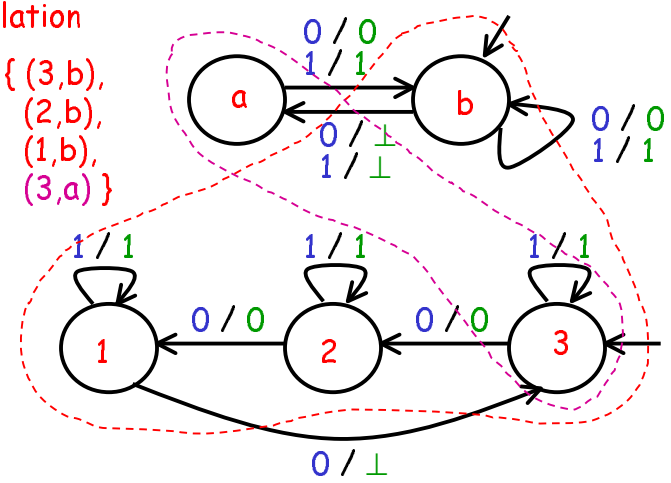
S = { (3,b),
(2,b),
(1,b),



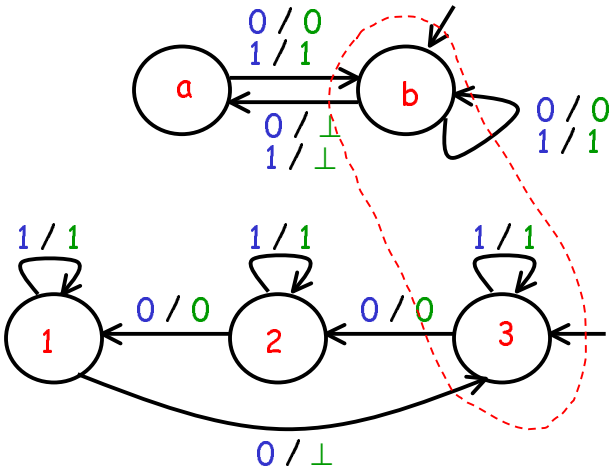

ThirdZero is simulated by NotTwice

Simulation

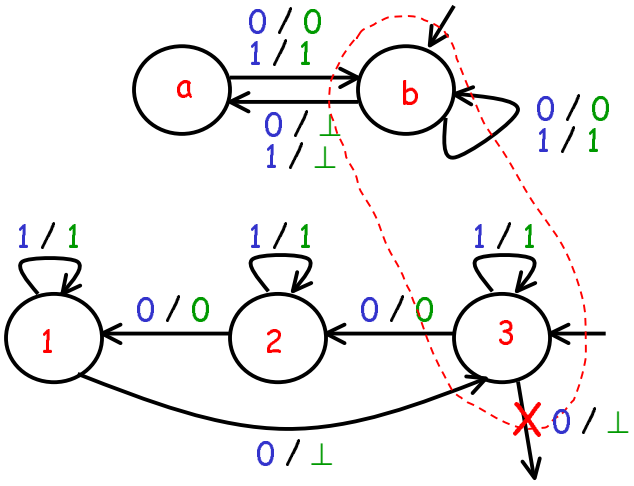
S = { (3,b),
(2,b),
(1,b),
(3,a) }



NotTwice is not simulated by ThirdZero




NotTwice is not simulated by ThirdZero



Special cases of nondeterministic state machines

1. Deterministic
2. Output-deterministic ("almost deterministic")

For these machines, simulations can be found easily (provided they exist).

A state machine is output-deterministic

iff

1. there is only one initial state, and
2. for every state and every input-output pair, there is only one successor state.

Deterministic implies output-deterministic, but not vice versa.

For example, LCwD and NotTwice are output-deterministic; ThirdZero is deterministic.

Deterministic: for every input signal x, there is exactly one run of the state machine.

Output-deterministic: for every behavior (x,y), there is exactly one run.

If M2 is an output-deterministic state machine, then a simulation S of M1 by M2 can be found as follows:

1. If $p \in$ possibleInitialStates [M1] and
   possibleInitialStates [M2] = { q },
   then $(p,q) \in S$.
2. If $(p,q) \in S$ and
   $(p',y) \in$ possibleUpdates [M1] (p,x) and
   possibleUpdates [M2] (q,x) = { q' },
   then $(p',q') \in S$.

If M2 is a deterministic state machine, then

M1 is simulated by M2

iff

M1 is equivalent to M2.

If M2 is a deterministic state machine, then

M1 is simulated by M2

iff

M1 is equivalent to M2.

"if and only if"

M1 refines M2, and M2 refines M1

If M2 is a nondeterministic state machine, then

M1 is simulated by M2

implies

M1 refines M2,

but M2 may not refine M1.

( Recall M1 = ThirdZero and M2 = NotTwice .)

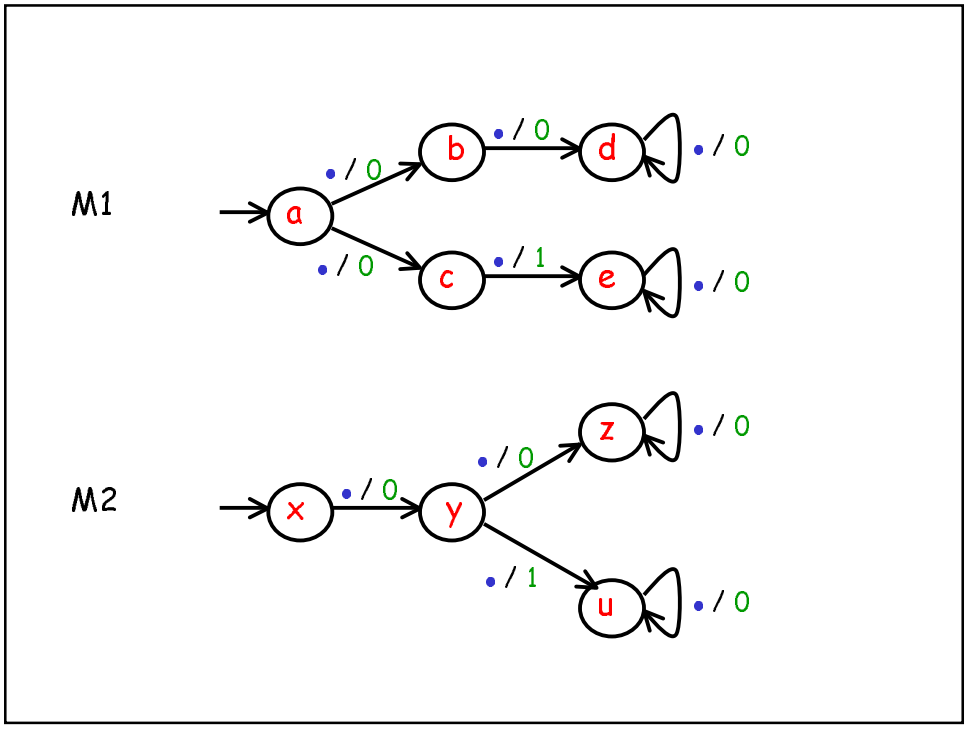
If M2 is a nondeterministic state machine, then

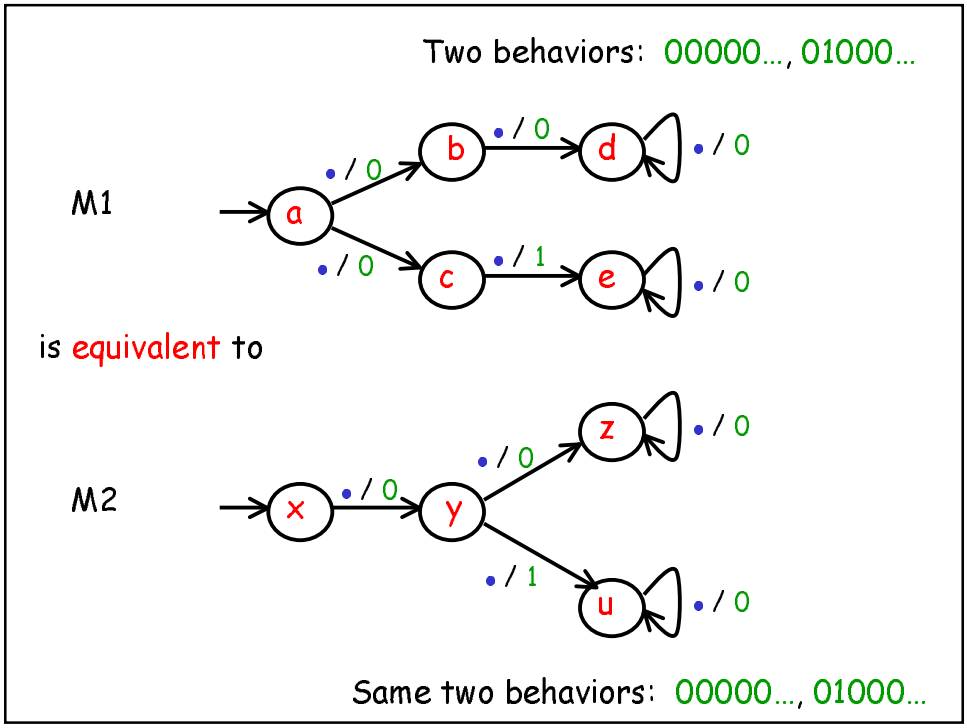
M1 is simulated by M2

implies

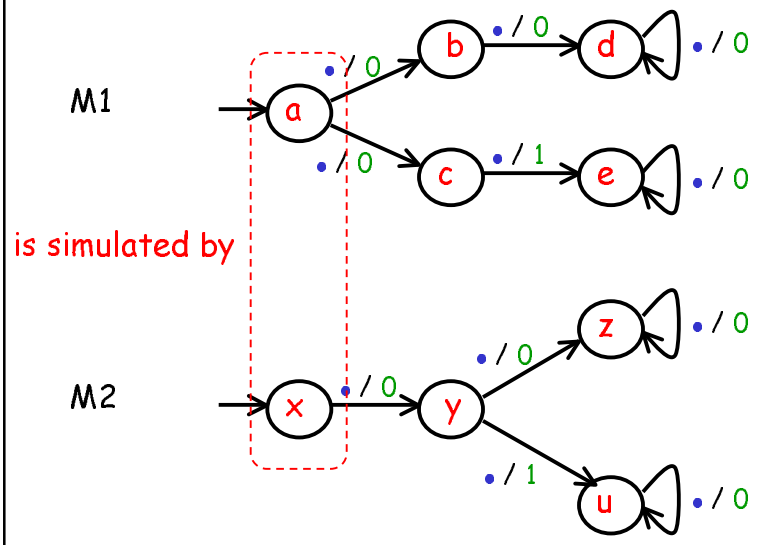
M1 refines M2,

but M1 refine M2 even if M1 is not simulated by M2.

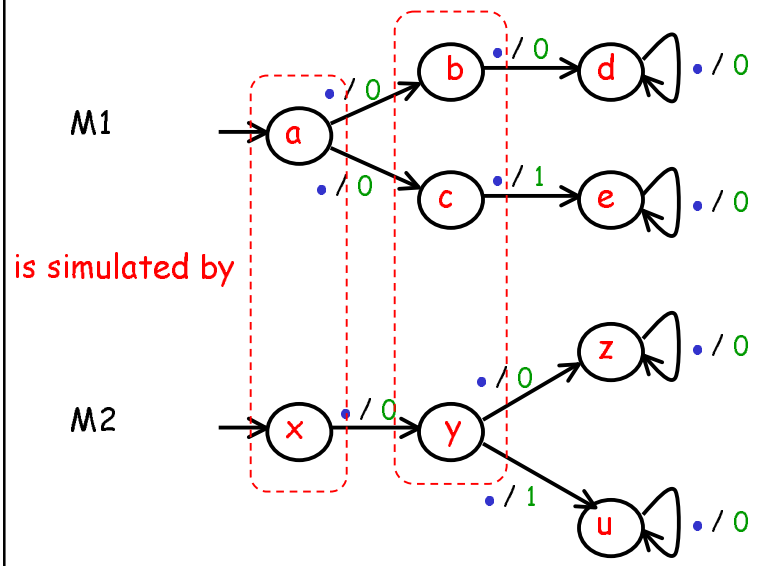

M1
a
b
c
d
e
/ 0
/ 0
/ 0
/ 0
/ 1
/ 0
M2
x
y
z
u
/ 0
/ 0
/ 1
/ 0


Two behaviors: 00000…, 01000…
M1
a
b
c
d
e
/ 0
/ 0
/ 0
/ 0
/ 1
/ 0
is equivalent to
M2
x
y
z
u
/ 0
/ 0
/ 1
/ 0
Same two behaviors: 00000…, 01000…

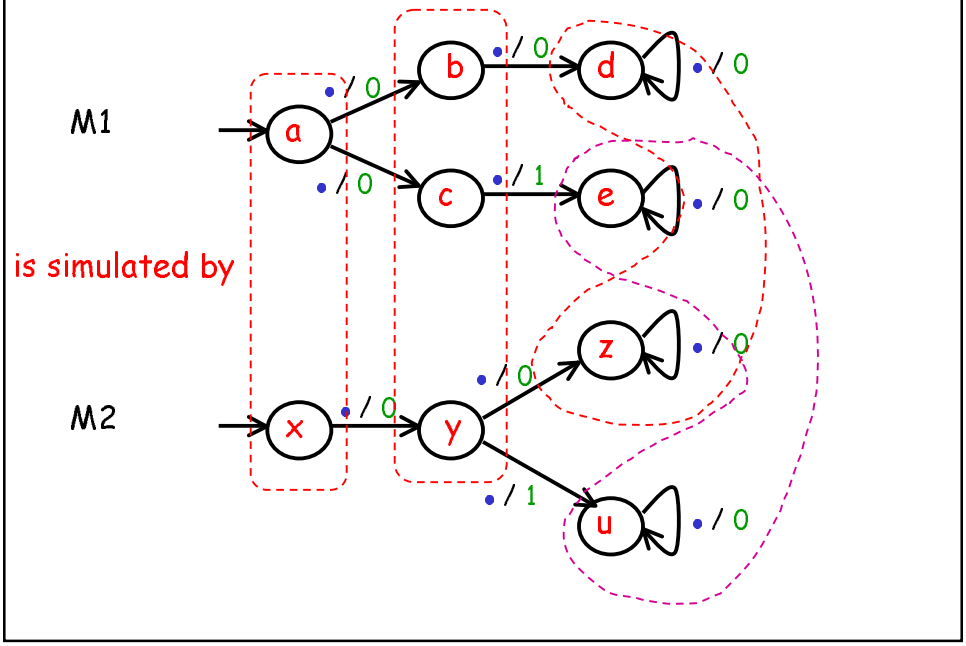
Simulation S = { (a, x),

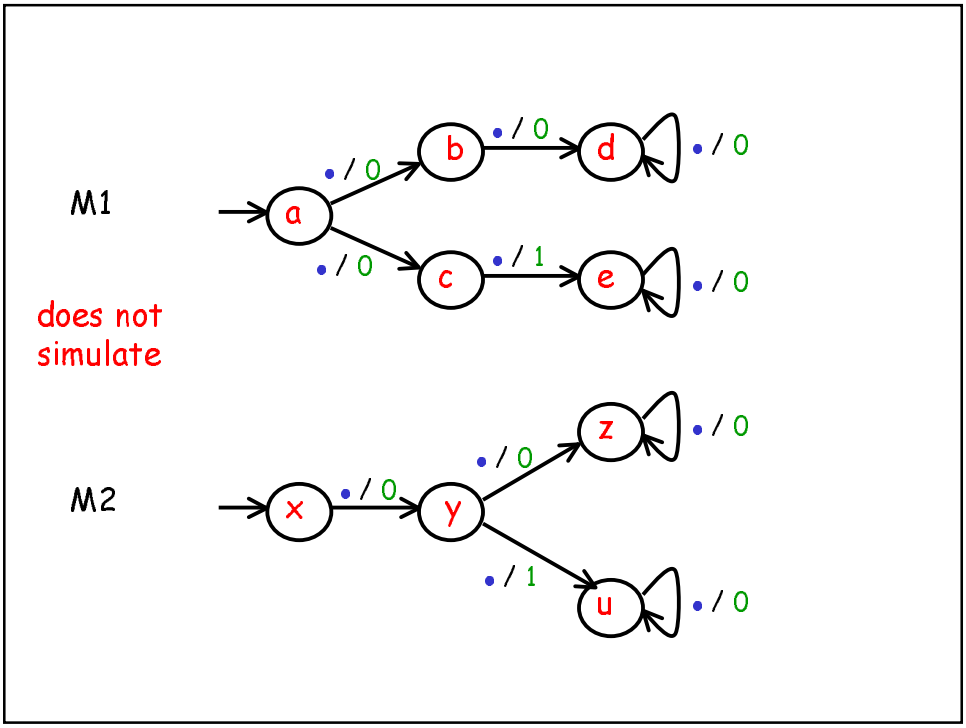


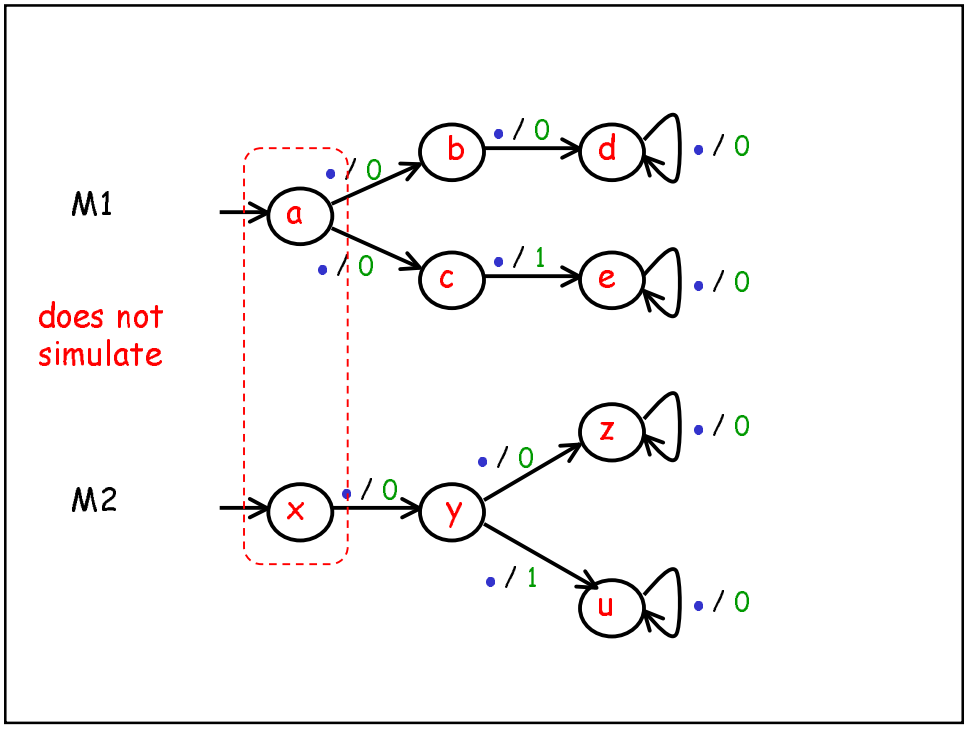

Simulation S = { (a, x), (b, y), (c, y),

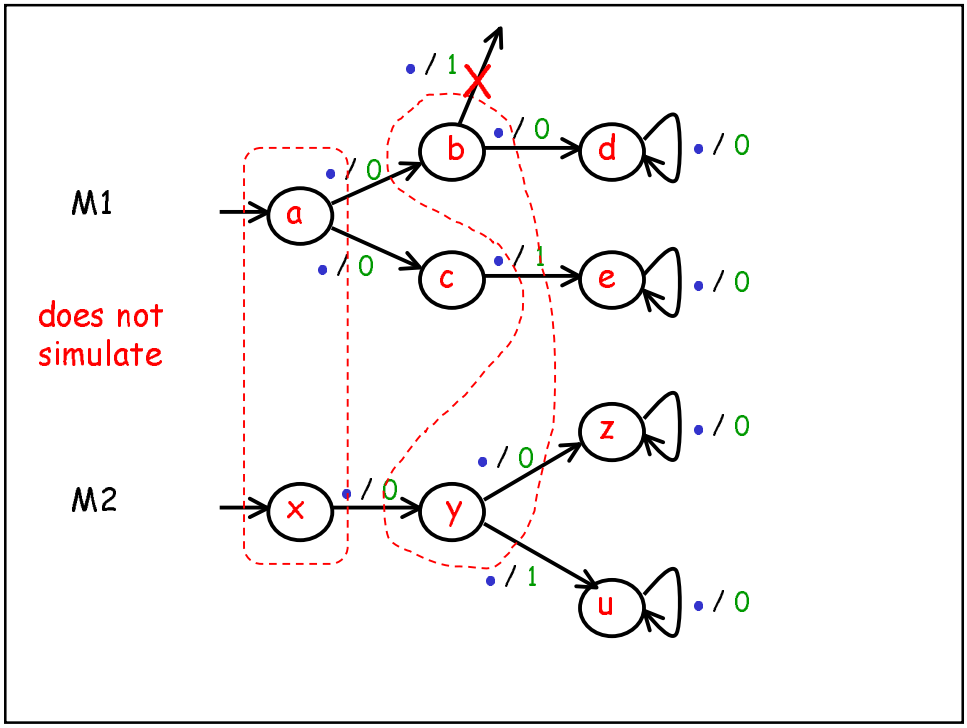


Simulation S = { (a, x), (b, y), (c, y), (d, z), (e, u) }




M1
does not
simulate
M2
a
b
c
d
e
x
y
z
u
/ 0
/ 1


M1
does not
simulate
M2
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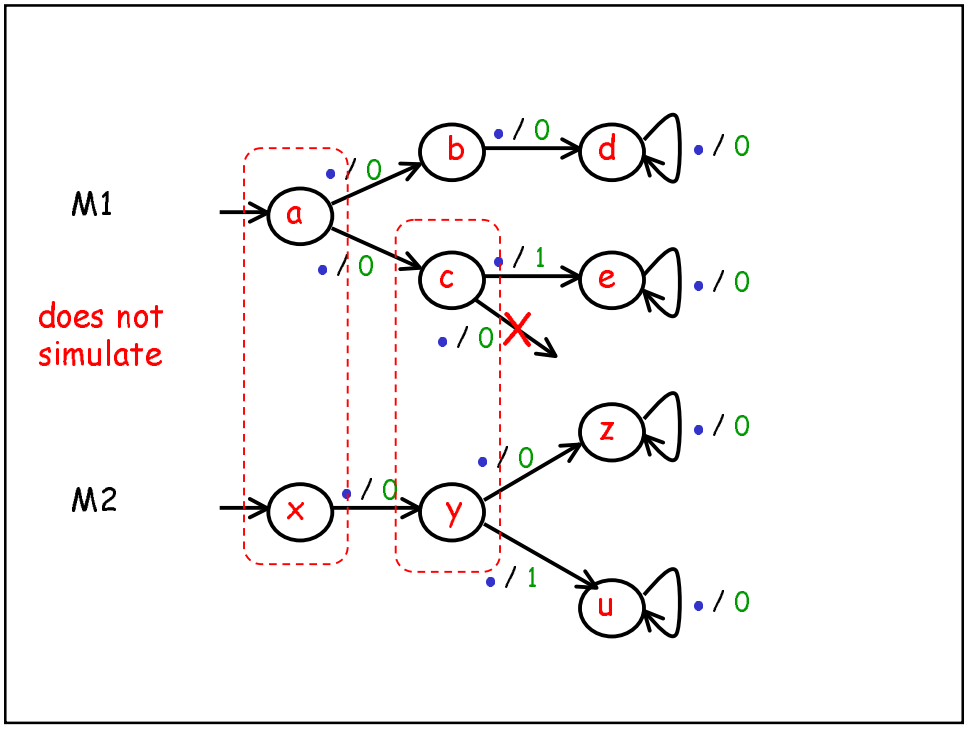
/ 0
/ 1


M1
does not
simulate
M2
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d
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/ 0