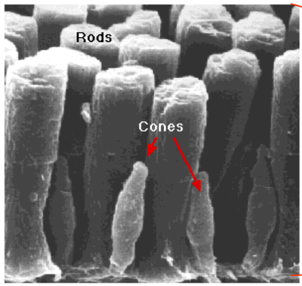
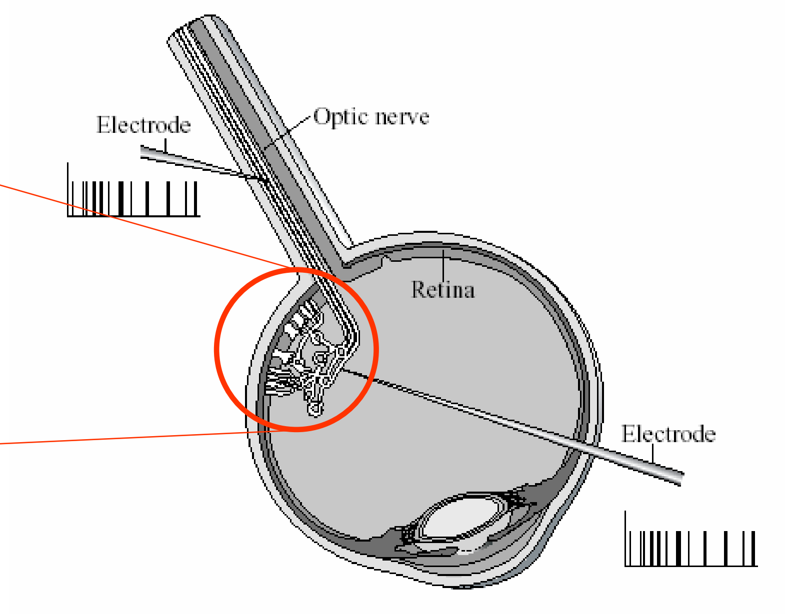
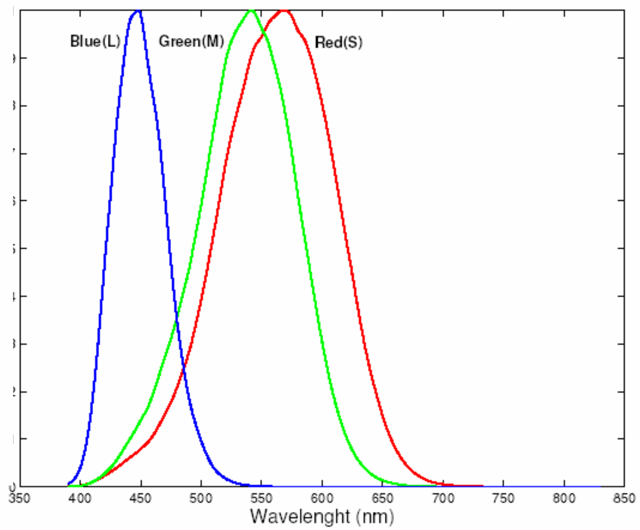
# Color Vision and Colorimetry

# Color encoding (physiology)

photosensors responses




photosensors



photons

# Color perception (psychology)

- Color is subjective!
  - Attribute of visual perception consisting of any combination of chromatic and achromatic content. This attribute can be described by **chromatic** color names such as *yellow, orange, brown, red, pink, green, blue, purple*, etc., or by **achromatic** color names such as *white, grey, black*, etc., and qualified by bright, dim, light, dark, etc., or by combinations of such names.
- *Perceived* color depends on
  - the spectral distribution of the color stimulus
  - the size, shape, structure and surround of the stimulus area
  - the state of adaptation of the observer's visual system
  - the observer's experience

  - [CIE Publication 17.4, International Lighting Vocabulary]

# Foundations of color vision

# Foundations of color vision

- Receptoral mechanisms
  - Color encoding
- Post-receptoral mechanisms
  - Opponent color model
- Cognitive aspects
  - Color and linguistics

# Photoreceptor types

## Rods

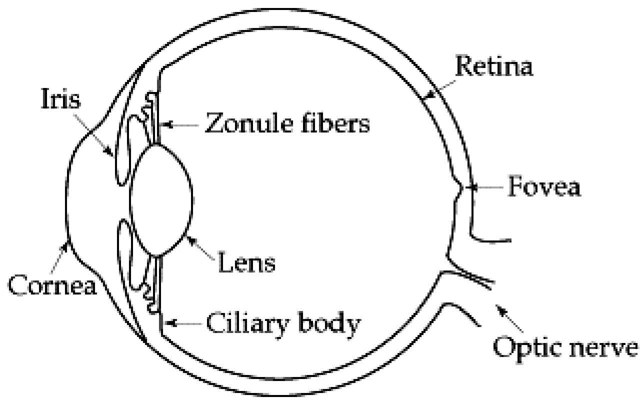
- *Scotopic* vision (low illumination)
- Do not mediate color perception
- High density in the periphery to capture many quanta
- *Low spatial resolution*
- Many-to-one structure
  - The information from many rods is conveyed to a single neuron on the retina
- Very *sensitive* light detectors
  - Reaching high quantum efficiency could be the reason behind the integration of the signals from many receptors to a single output. The price for this is a *low spatial resolution*
- About 120 millions

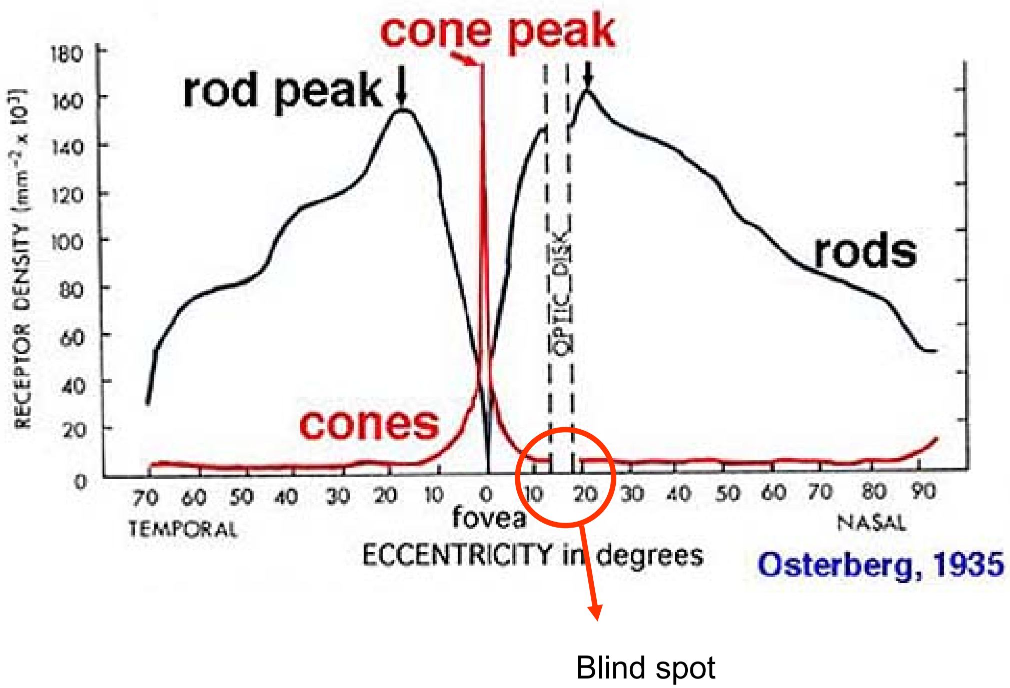
## Cones

- *Photopic* vision (high illumination)
- Mediate color perception
- High density in the fovea
- One-to-one structure
  - Do not converge into a different single neuron but are communicated along private neural channels to the cortex
- *High spatial resolution*
  - The lower sensitivity is compensated by the high spatial resolution, providing the eye with good acuity
- About 6-7 millions
  - 50000 in the central fovea

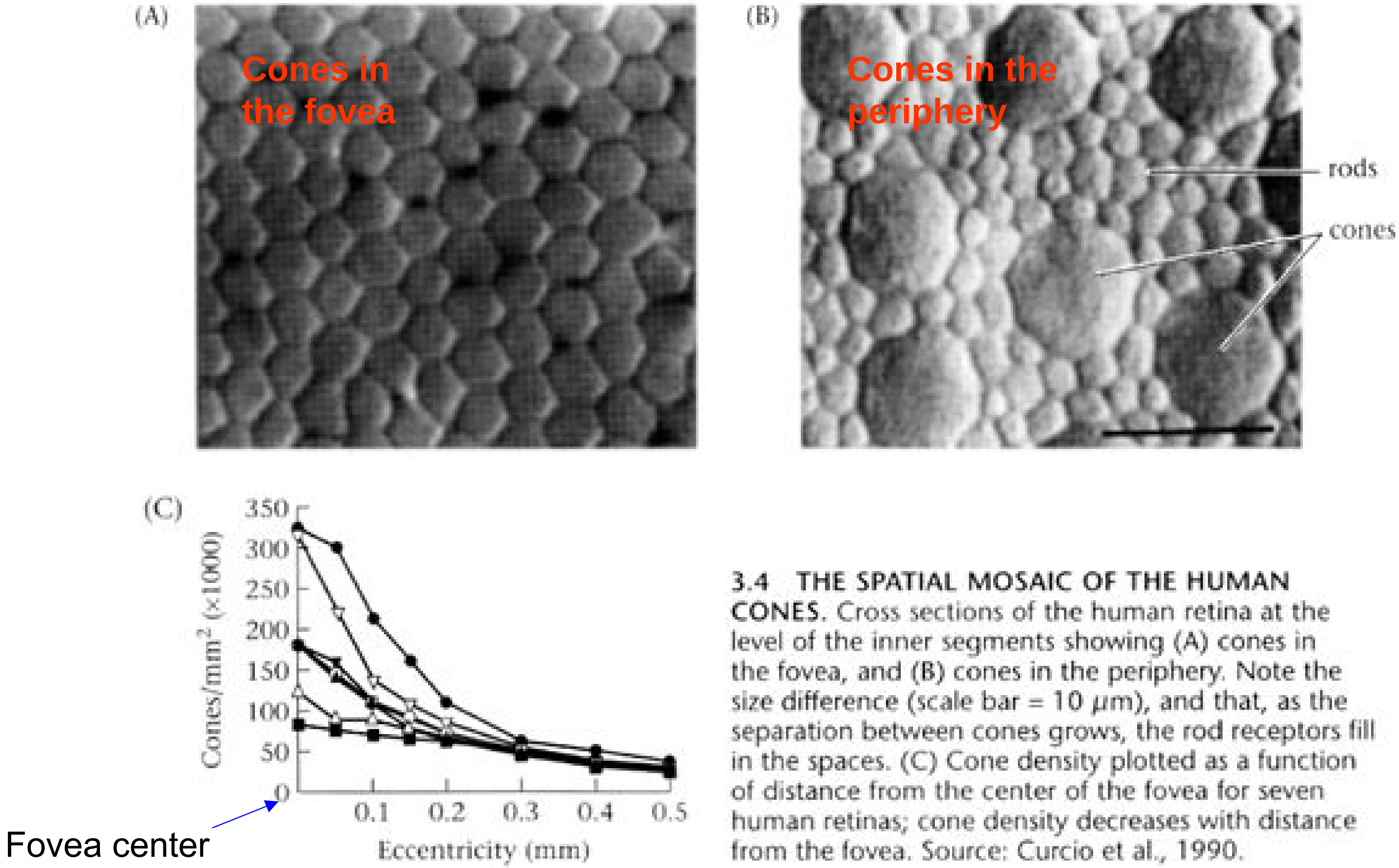
# The fovea

- The fovea is the region of the highest visual acuity. The central fovea contains no rods but does contain the highest concentration of cones.



# Cones and Rods mosaic

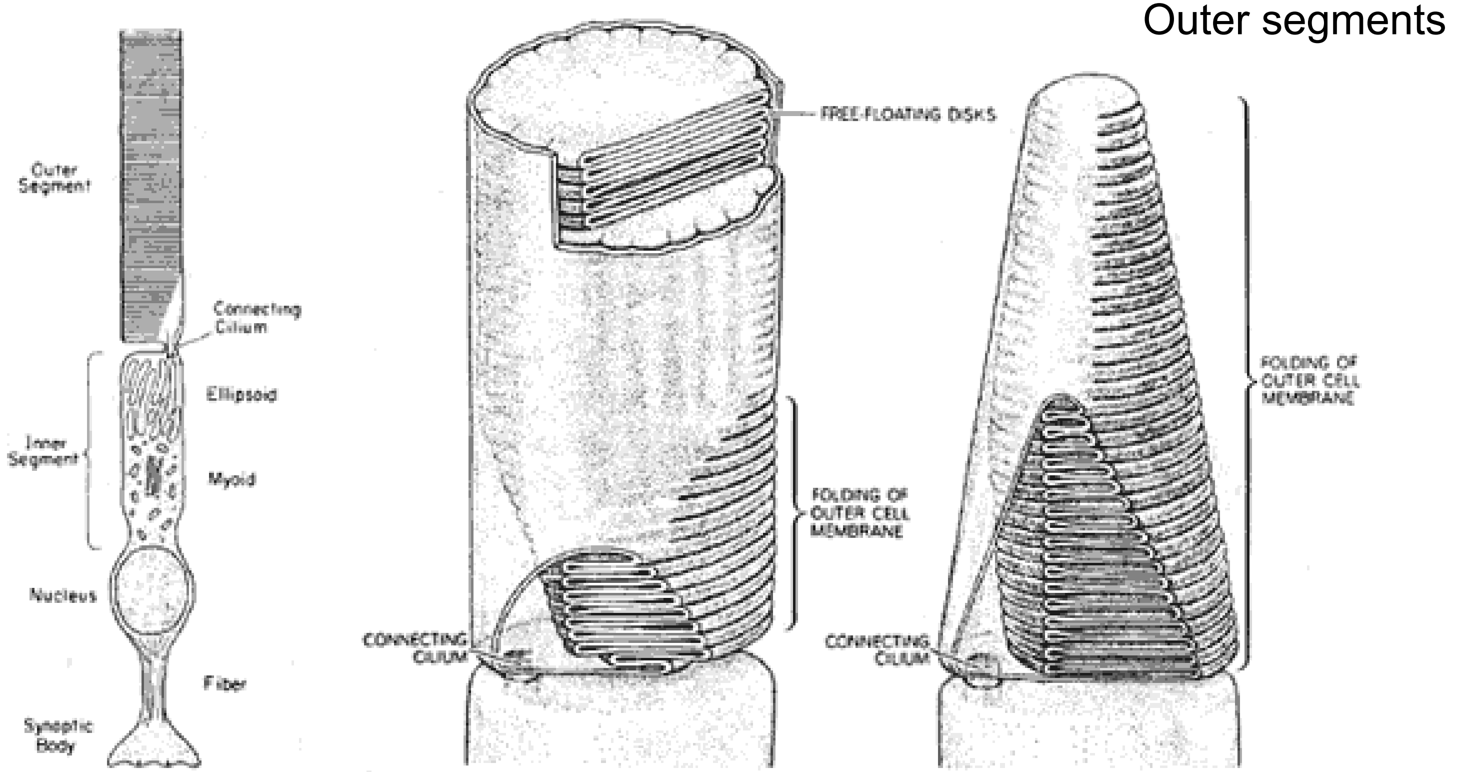



**3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES.** Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 $\mu$m), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.

# Cones and Rods shape

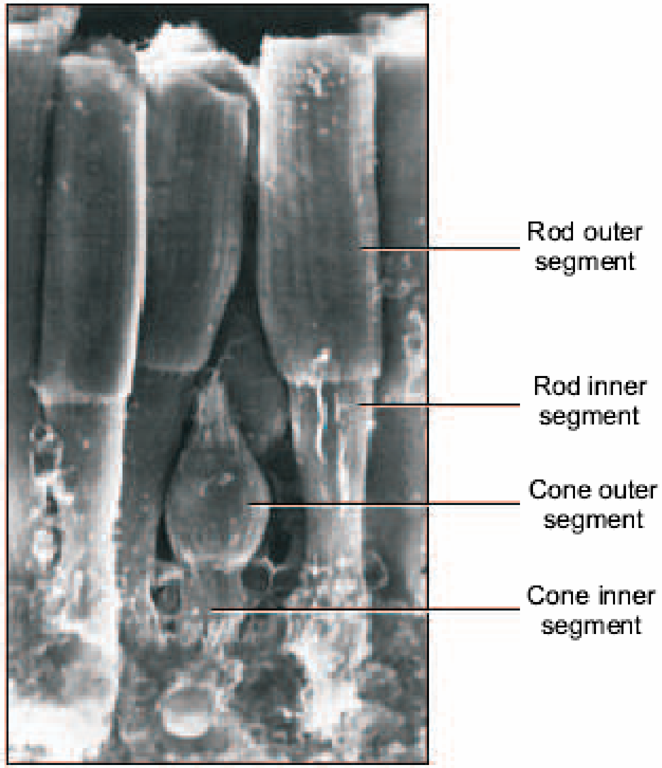
Photoreceptor cell

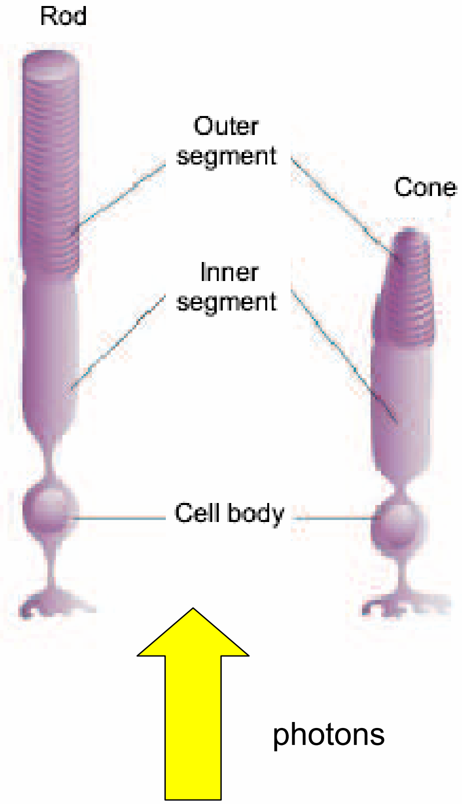
Outer segments




At the left is a generalized conception of the important structural features of a vertebrate photoreceptor cell. At the right are shown the differences between the structure of rod (left) and cone (right) outer segments. These diagrams are from Young (1970) and Young (1971).

# Cones and rods shapes

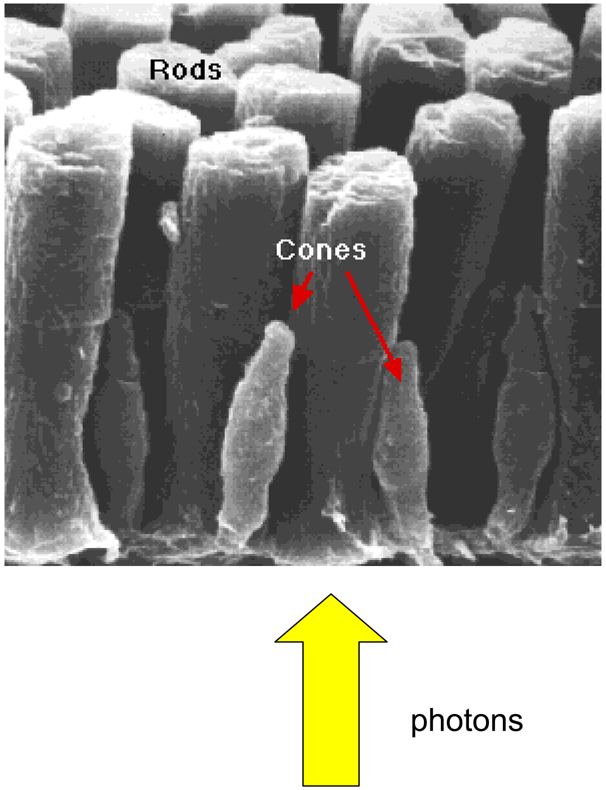


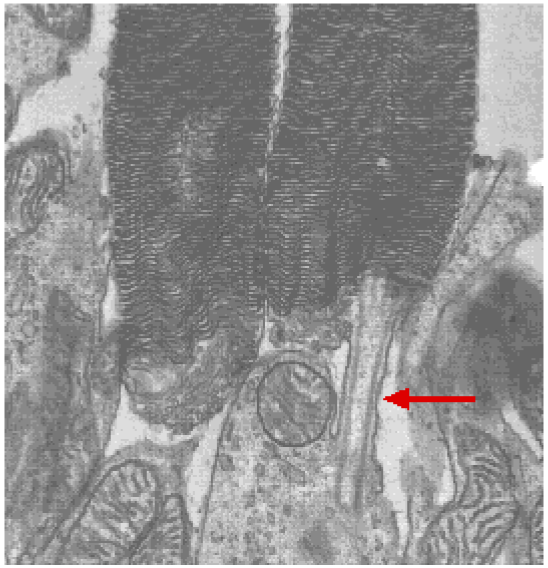




The light enters the **inner** segment and passes into the outer segment which contains light absorbing photopigments. Less than 10% photons are absorbed by the photopigments [Baylor, 1987].

The rods contain a photopigment called rhodopsin.

# Cone and rods



# Photoreceptor mosaic

- The retinal image is sampled by the photo-receptors of the retina
  - Discrete sampling grid → signal processing issues (sampling, aliasing…)

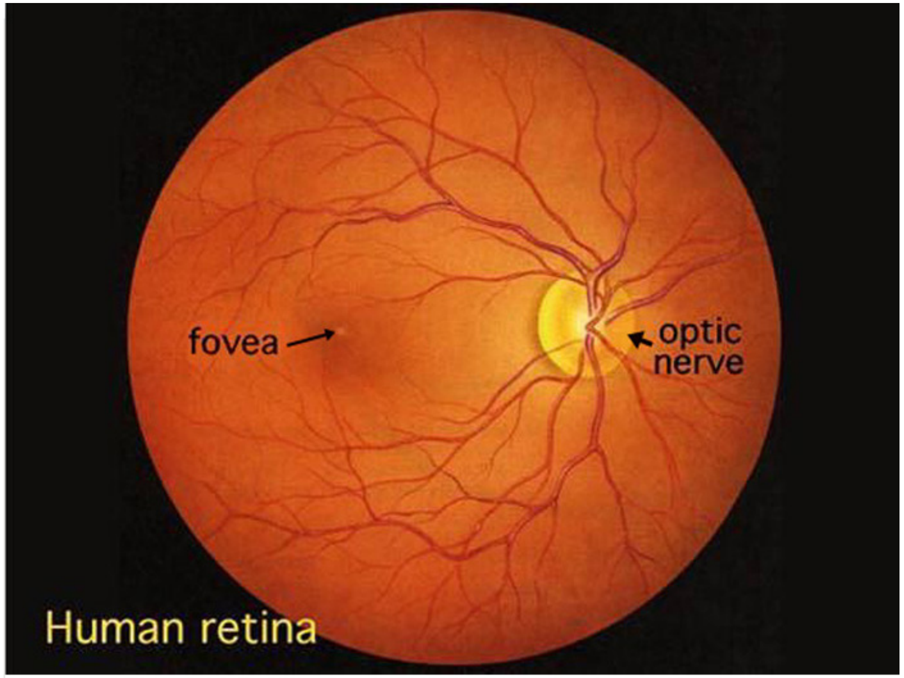



*Fig. 1. Human retina as seen through an opthalmoscope.*

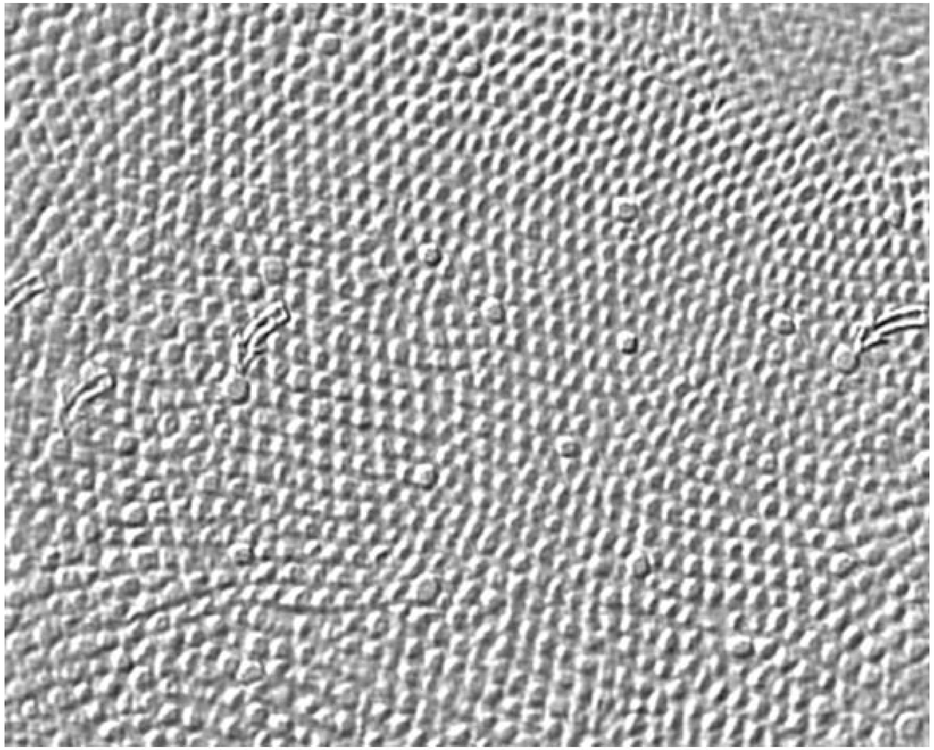
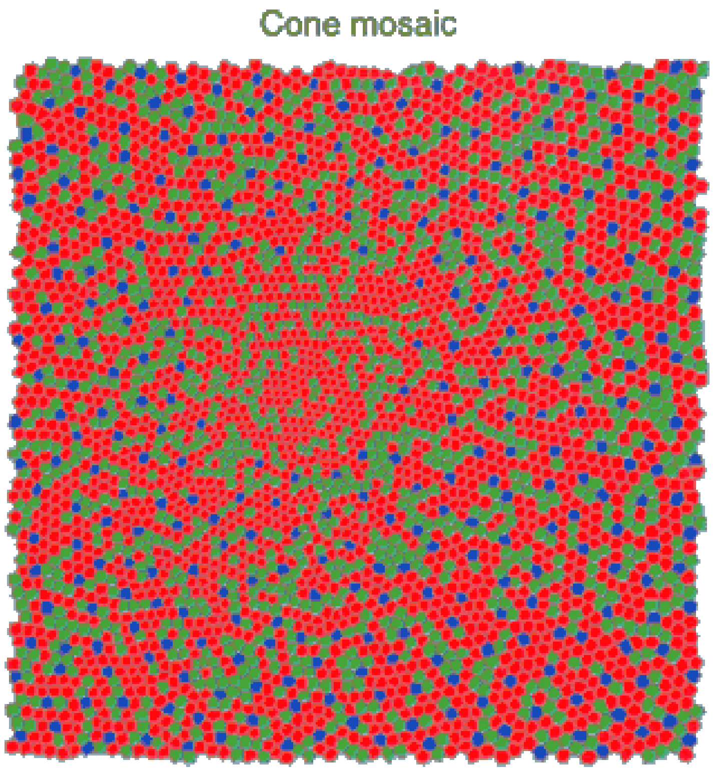


*Fig. 13. Tangential section through the human fovea. Larger cones (arrows) are blue cones.*

# Cone mosaic



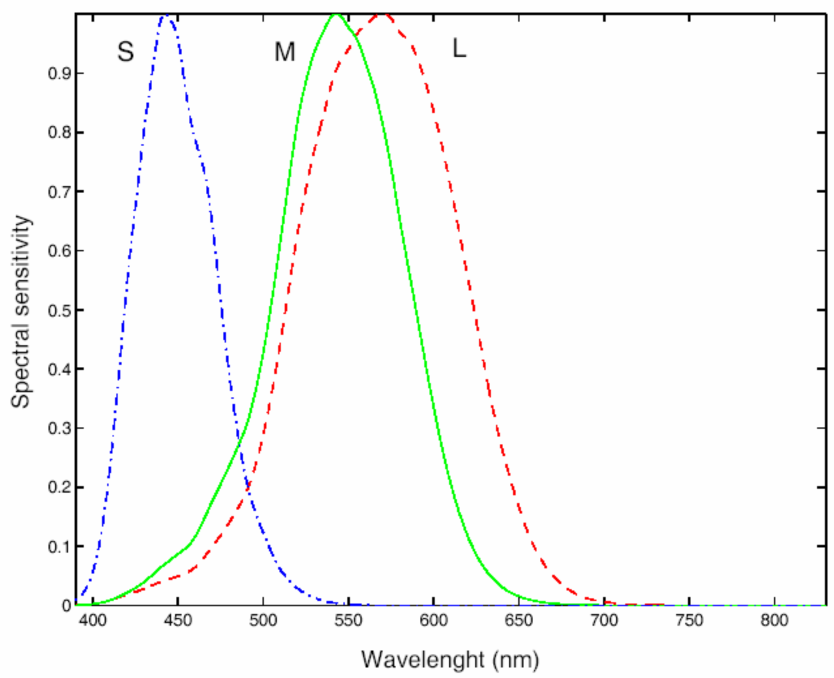

Williams (1985) measured the sampling density of the mosaic of the L- and M-cones together. His results are consistent with a sampling frequency of 60 cpd at the central fovea, consistent with a center-to-center spacing of the cones of 30 minutes of degree.

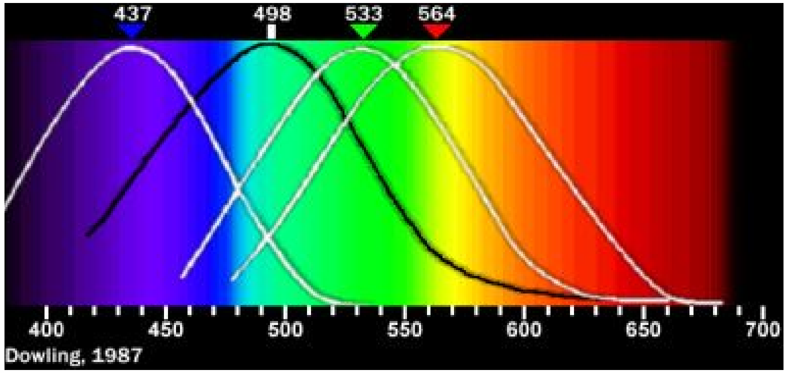
The sampling frequency than decreases when increasing the visual angle, consistently with the decrease in cone density.

This diagram was produced based on histological sections from a human eye to determine the density of the cones. The diagram represents an area of about ***1° of visual angle***. The number of **S-cones was set to 7%** based on estimates from previous studies. The **L-cone:M-cone ratio was set to 1.5**. This is a reasonable number considering that recent studies have shown wide ranges of cone ratios in people with normal color vision. In the central fovea an area of approximately 0.34° is S-cone free. The S-cones are semi-regularly distributed and the M- and L-cones are randomly distributed.
Throughout the whole retina ***the ratio of L- and M- cones to S-cones is about 100:1***.

# Types of cones

normalized cone responses




The cones are classified based on their **wavelength selectivity as L (long), M (medium) and S (short)** wavelength sensors.

L,M and S cones have different sensitivity and spatial distributions.

The S cones are far less numerous and more sensitive than the others.

# Cone Spectral Sensitivity

Stimulus (light pulse) → cone → Response (cone photocurrent)

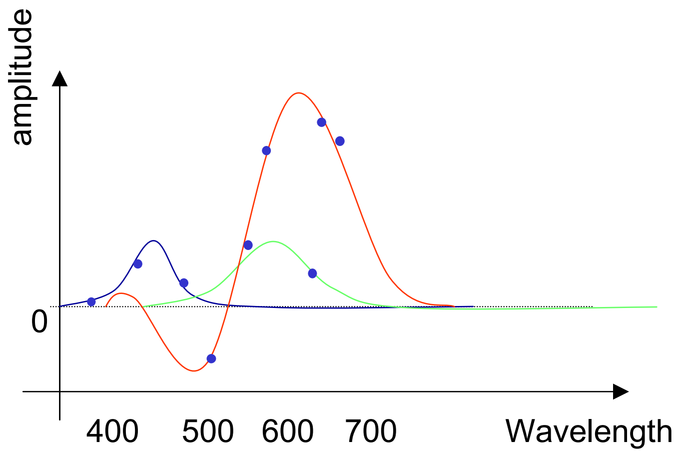
The amplitude of the photocurrent depends on both the **intensity** and the **wavelength** of the stimulus

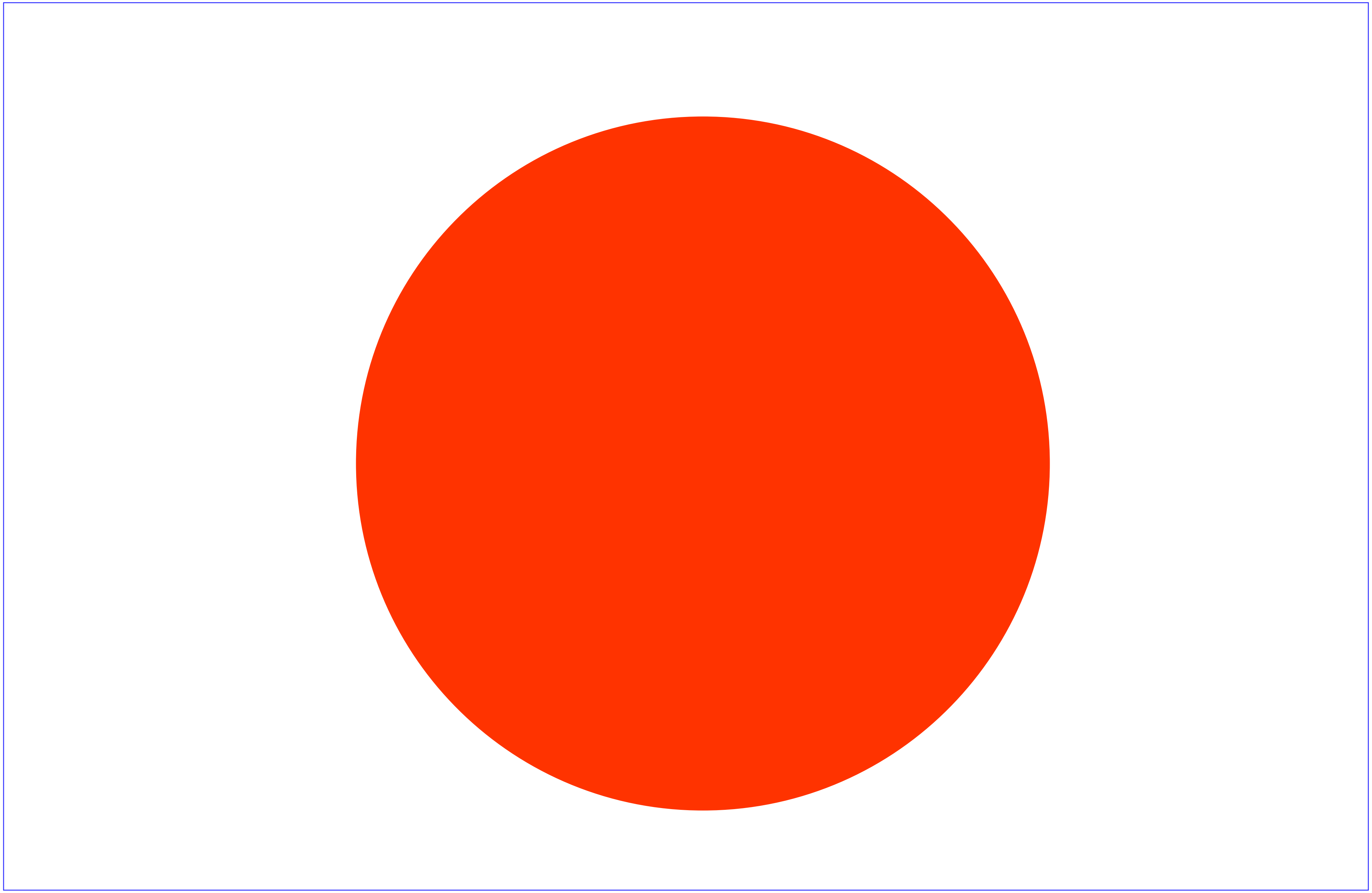
Color matching is obtained when: $A(I_1(\lambda_1)=A(I_2(\lambda_2))$

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \text{Spectral sensitivity of L photopigments} \\ \text{Spectral sensitivity of M photopigments} \\ \text{Spectral sensitivity of S photopigments} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

Cone response (Amplitude $A(I(\lambda))$)

Stimulus ($I(\lambda)$)

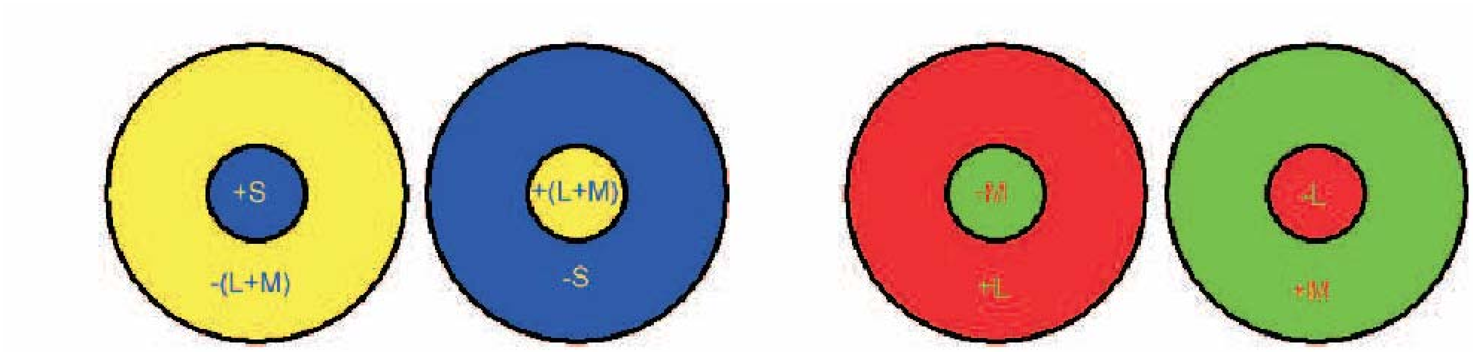




Gloria Menegaz

# Opponent Color Model

- Perception is mediated by *opponent color channels*
  - *Evidences*
    - Afterimages
    - Certain colors cannot be perceived simultaneously (i.e. no *reddish-green* or *bluish-yellow*)



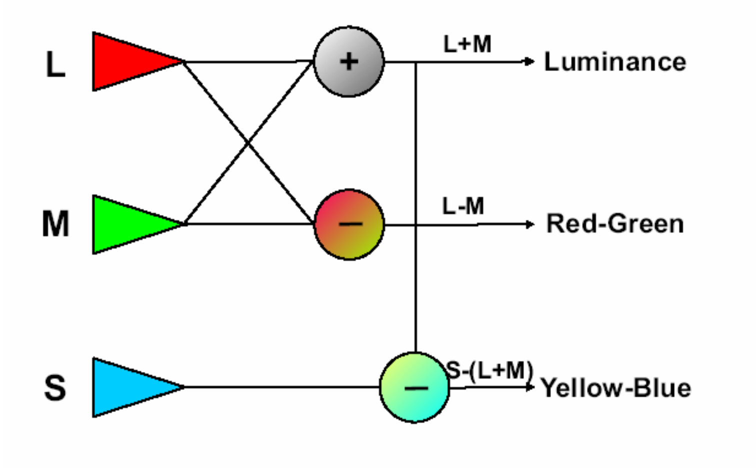

(a) (b)

Example of typical center-surround antagonistic receptive fields: (a) on-center yellow-blue receptive fields; (b) on-center red-green receptive fields.

Because of the fact that the L, M and S cones have different spectral sensitivities, are in different numbers and have different spatial distributions across the retina, the respective receptive fields have quite different properties.

# Opponent color channels

Cone interconnections in the retina leading to opponent color channels




- As a convenient simplification, the existence of three types of color receptive fields is assumed, which are called *opponent channels*.
- The black-white or *achromatic* channel results from the sum of the signals coming from L and M cones (L+M). It has the highest spatial resolution.
- The *red-green* channel is mainly the result of the M cones signals being subtracted from those of the L cones (L-M). Its spatial resolution is slightly lower than that of the achromatic channel (L+M).
- Finally the *yellow-blue* channel results from the addition of L and M and subtraction of S cone signals. It has the lowest spatial resolution.

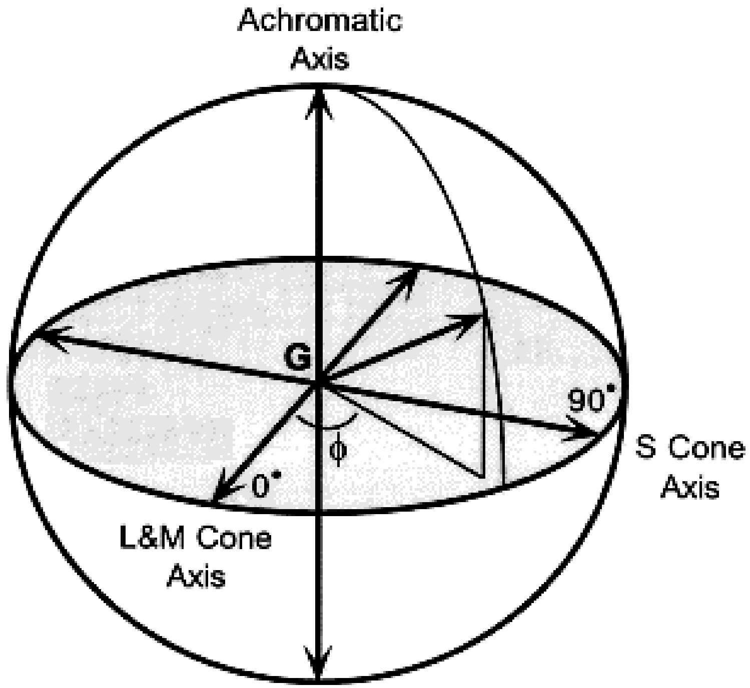
# The DKL Color Space

- Differential space [Derrington et al. 1984]
- Implements the opponent color model:

L&M Axis ➡ Red-Green (L-M)

S Axis ➡ Yellow-Blue (S-(L+M))

Achrom. Axis ➡ Achromatic (L+M)

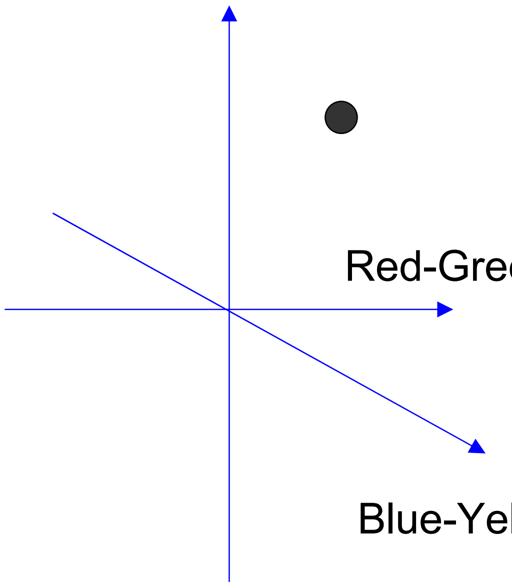



- L&M and S Cone Axes define an *equiluminant* plane.
- A color in the equiluminant plane can be specified by its azimuth Φ.

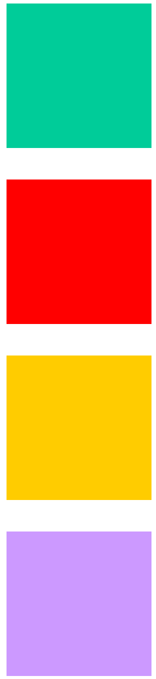
- DKL color space explicitly represents the responses of retinal processing.

# Cognitive aspects

**Perceptual uniformity**




**Color naming**

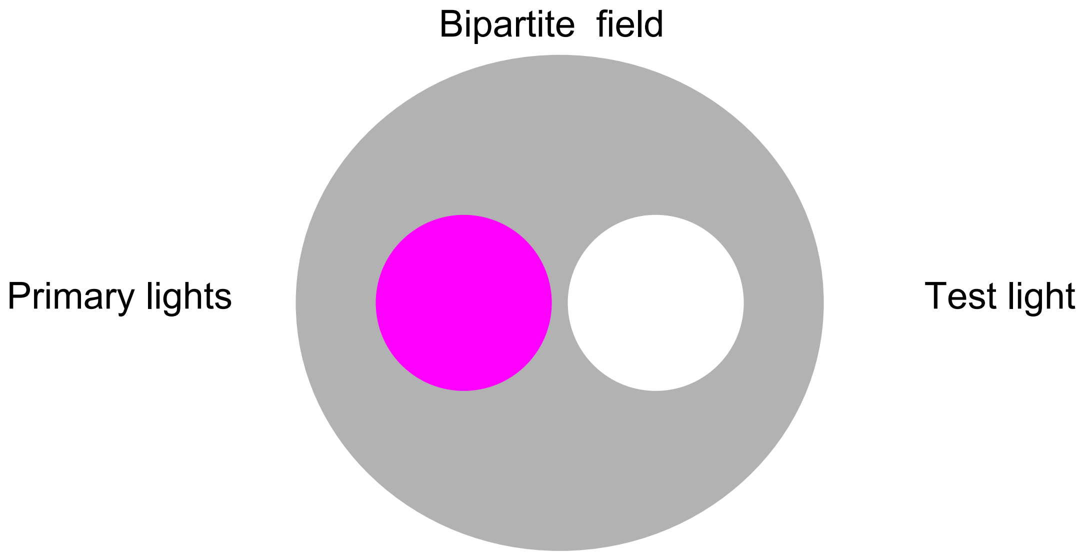



Problem 1:
Which color model is most representative of color appearance?

Problem 2:
Which names are given to the different colors?

# Color matching functions

# Color matching

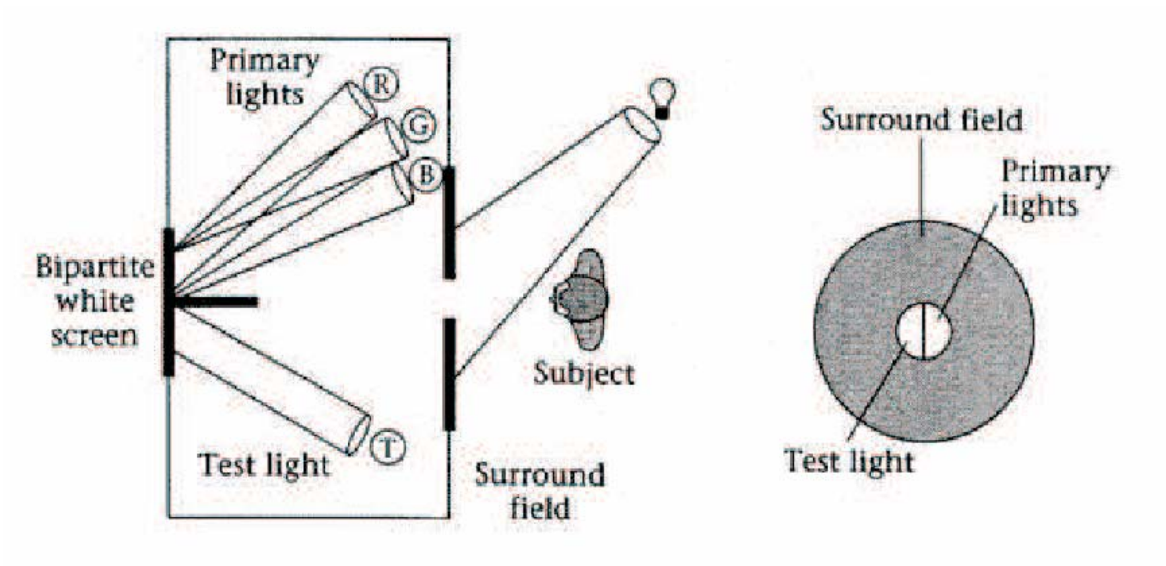


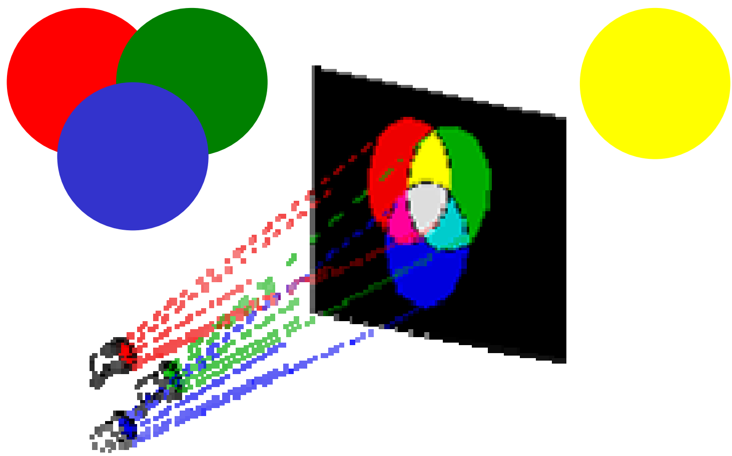

There are **three primary lights** with fixed relative spectral distribution and only the intensity can vary. These are chosen to be **monochromatic**

The test light can have any spectral distribution. It is common to chose a equal energy light and decompose it into the monochromatic components for testing the entire set of wavelengths.

**Task**: Adjust the intensities of the primary lights so that the primary and test lights appear indistinguishable

# Color matching



# Measuring the CMFs

$$\vec{e} = \begin{bmatrix} r_1^{\,1} & r_2^{\,1} & \dots & r^1{}_{n_\lambda} \\ r_1^{\,2} & r_2^{\,2} & \dots & r^2{}_{n_\lambda} \\ r_1^{\,3} & r_2^{\,3} & \dots & r^3{}_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

**R** : system matrix (transfer function). Each line represents the *Color Matching Function* (CMF) for the corresponding primary light

**t** : spectral distribution of the test light

**e** : response of the observer

Assuming a equal-energy test light

$$\vec{e} = \begin{bmatrix} r_1^{\,1} & r_2^{\,1} & \dots & r^1{}_{n_\lambda} \\ r_1^{\,2} & r_2^{\,2} & \dots & r^2{}_{n_\lambda} \\ r_1^{\,3} & r_2^{\,3} & \dots & r^3{}_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} t \\ t \\ \vdots \\ t \end{bmatrix} = t \begin{bmatrix} r_1^{\,1} & r_2^{\,1} & \dots & r^1{}_{n_\lambda} \\ r_1^{\,2} & r_2^{\,2} & \dots & r^2{}_{n_\lambda} \\ r_1^{\,3} & r_2^{\,3} & \dots & r^3{}_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

since we are measuring relative intensities we can choose $t=1$

# Color Matching Functions (CMFs)

Assuming that the symmetry, transitivity and homogeneity hold (*Grassmann's laws of additive color mixtures*), the system matrix can be measured by feeding it with $n_\lambda$ monochromatic lights

$$\vec{e} = \begin{bmatrix} r_1^{\,1} & r_2^{\,1} & \dots & r^{1}_{n_\lambda} \\ r_1^{\,2} & r_2^{\,2} & \dots & r^{2}_{n_\lambda} \\ r_1^{\,3} & r_2^{\,3} & \dots & r^{3}_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} r_1^{\,1} \\ r_1^{\,2} \\ r_1^{\,3} \end{bmatrix}$$

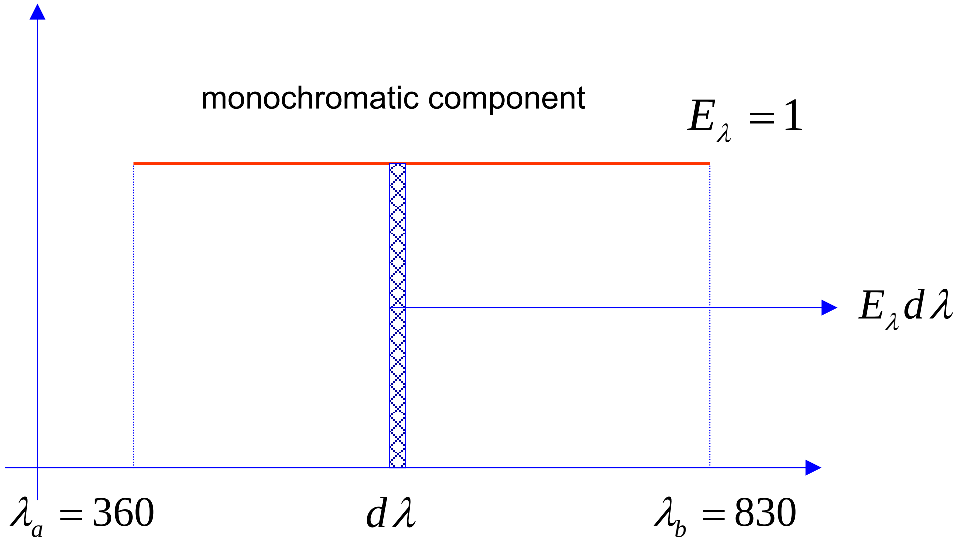
The response to each monochromatic light will determine one *column* of the system matrix, so one entry of each CMF.

It can be shown that the system matrix is not unique. Using **different sets of primaries** leads to different CMFs. Though, different sets of CMFs are related by a **linear transformation**

→ **Need to choose one set of primaries**

# Color Matching Functions

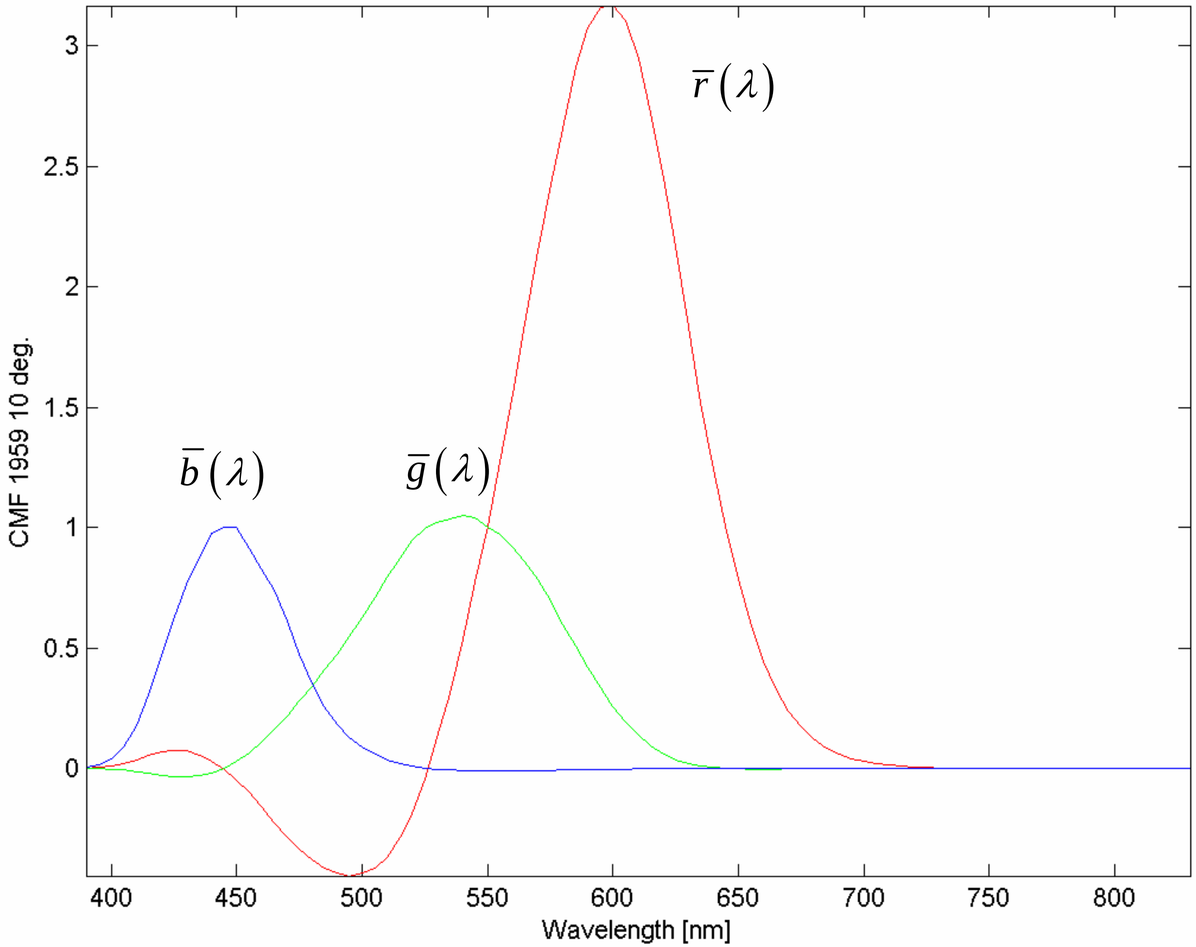
- In other words, the CMFs are the *spectral* tristimulus values of the *equal energy* stimulus E (*reference white*)




$$E_\lambda = \bar{r}(\lambda)R + \bar{g}(\lambda)G + \bar{b}(\lambda)B$$
$$\lambda_R = 700nm$$
$$\lambda_G = 546.1nm$$
$$\lambda_B = 435.8nm$$

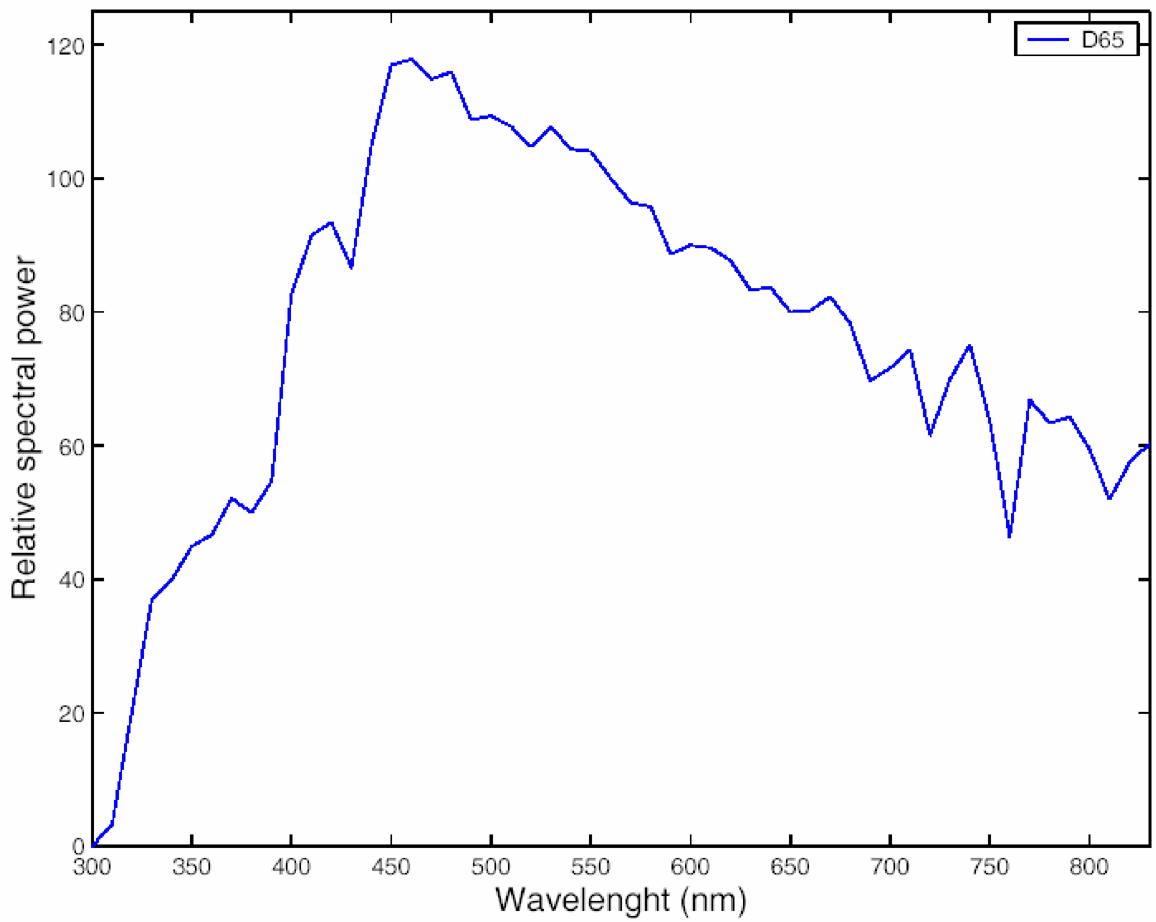
$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ are called color matching functions

# CMFs



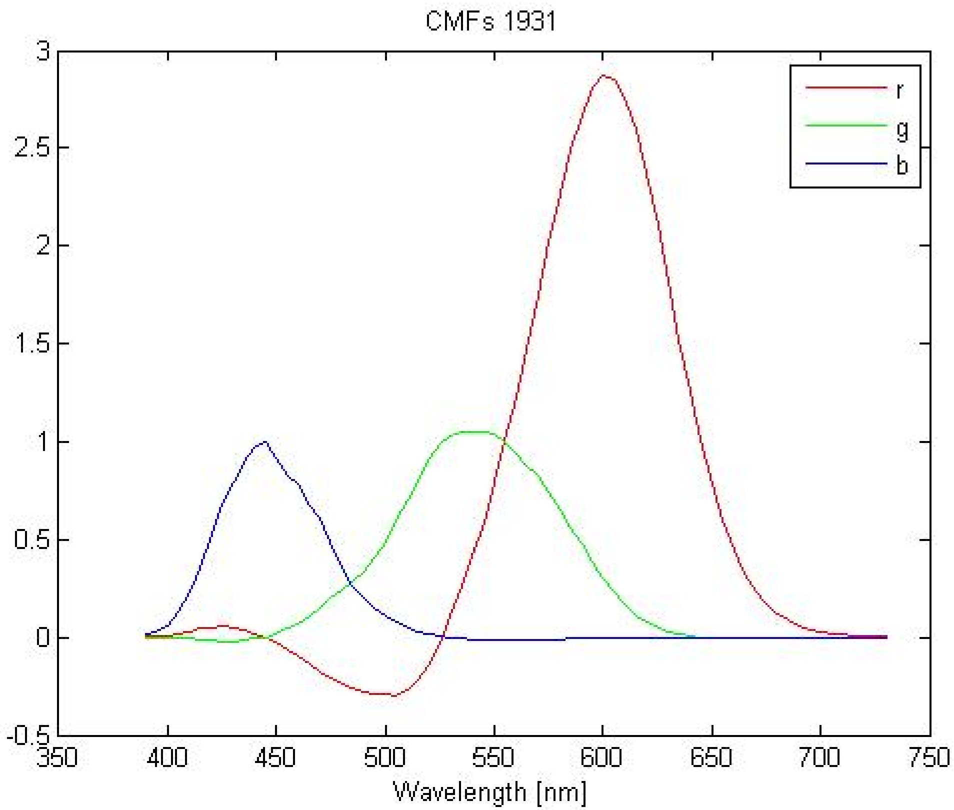
$\bar{r}(\lambda)$
$\bar{b}(\lambda)$
$\bar{g}(\lambda)$
CMF 1959 10 deg.
3
2.5
2
1.5
1
0.5
0
400
450
500
550
600
650
700
750
800
Wavelength [nm]

# D65



D65
120
100
80
60
40
20
0
300
350
400
450
500
550
600
650
700
750
800
Relative spectral power
Wavelenght (nm)

# CMF rgb 1931



CMFs 1931
3
2.5
2
1.5
1
0.5
0
-0.5
350
400
450
500
550
600
650
700
750
Wavelength [nm]
r
g
b

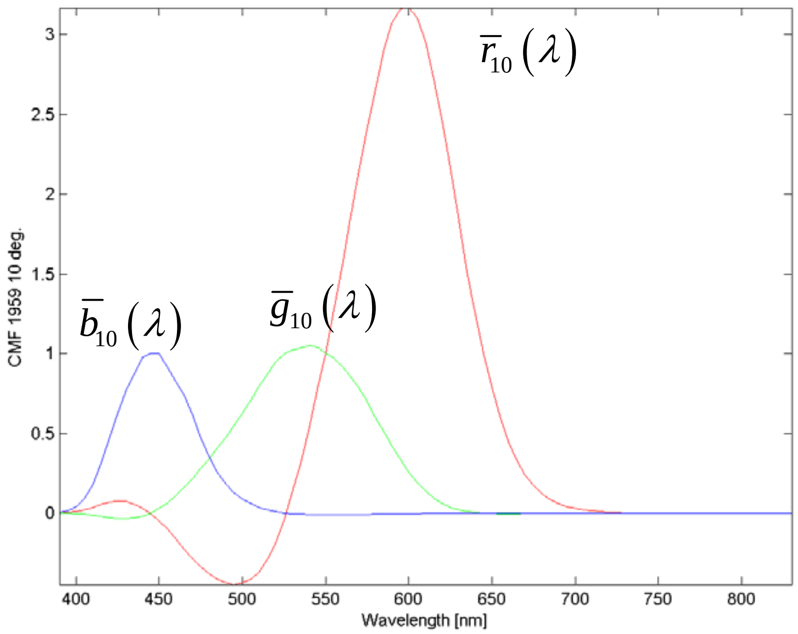
# Stiles and Burch 10deg (1959)

Primary lights: monochromatic

$\lambda_R$=645.16 nm
$\lambda_G$= 526.32 nm
$\lambda_B$= 444.44 nm

$$\bar{r}_{10}(\lambda), \bar{g}_{10}(\lambda), \bar{b}_{10}(\lambda) \qquad \text{CMFs}$$

$$t(\lambda) = R \cdot \bar{r}_{10}(\lambda) + G \cdot \bar{g}_{10}(\lambda) + B \cdot \bar{b}_{10}(\lambda)$$




***t*** : monochromatic test light

(R,G,B) : ***tristimulus values*** of ***t***

A **10 degrees bipartite field** was used

Negative values for the tristimulus value mean that the corresponding primary was added to the test light in order to match the color appearance.

This outlines that not every test color can be matched by an additive mixture of the three primaries.

The presence of negative values could be impractical, so another color coordinate system was chosen as the reference by the *Commisione Internationale d'Eclairage (CIE)* in 1931.

# Cone photopigments and CMF

- How well do the spectral sensitivities of the cone photopigments predict performance on the photopic color matching experiment?

Biological measurements

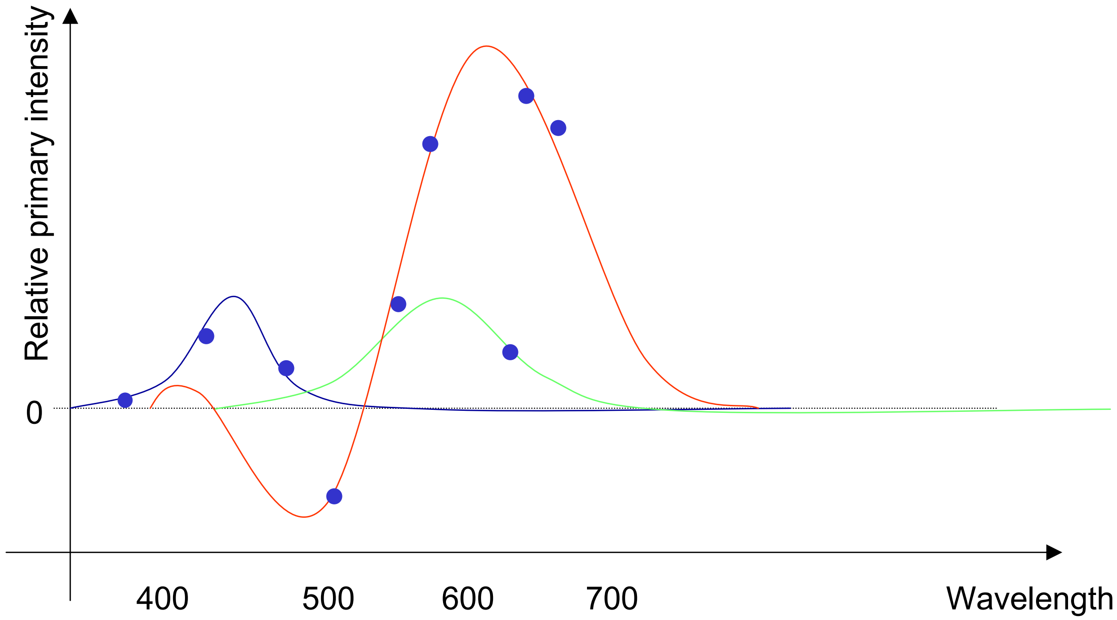
$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \text{Spectral sensitivity of L photopigments} \\ \text{Spectral sensitivity of M photopigments} \\ \text{Spectral sensitivity of S photopigments} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

Psychophyiscal measurements

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \text{CMF of primary 1} \\ \text{CMF of primary 2} \\ \text{CMF of primary 3} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

- There should be a linear transformation that maps the cone absorption curves to the system matrix of the color matching experiment
- *Linking hypothesis*

# Cone photopigments and CMFs




From the agreement between these two datasets one can conclude that the photopigment spectral responsivities provide a satisfactory biological basis to explain the photopic color matching experiments

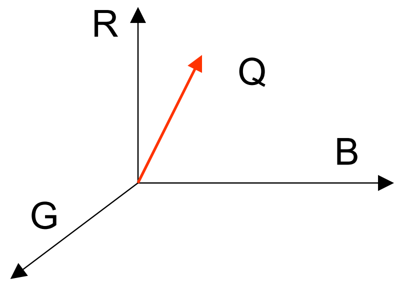
# Tristimulus values for complex stimuli

- Color stimuli are represented by vectors in a three-dimensional space, called the *tristimulus* space
  - Let Q be an arbitrary monochromatic color stimulus and **R,G and B the fixed primary stimuli** chosen for the color matching experiment
  - Each color stimulus is assumed to be produced by imaging on the retina of a surface in the external field uniformly emitting radiant power

$$\vec{Q}=R_Q\vec{R}+G_Q\vec{G}+B_Q\vec{B}$$

  - $R_Q,G_Q,B_Q$ : *tristimulus values* of Q
  - The scalar multipliers $R_Q,G_Q,B_Q$ are measured in terms of the *assigned respective units of the corresponding primaries*
    - It is customary to choose these units such that when additively mixed yield a complete color match with a specified *achromatic* stimulus, usually one with an *equal-energy spectrum* on a wavelength basis

The units of these primaries was chosen in the radiant power ratio of 72.1:1.4:1.0, which places the chromaticity coordinates of the equal energy stimulus E at the center of the $(r,g)$ chromaticity diagram
→ $R_W=G_W=B_W=1$ for the reference white

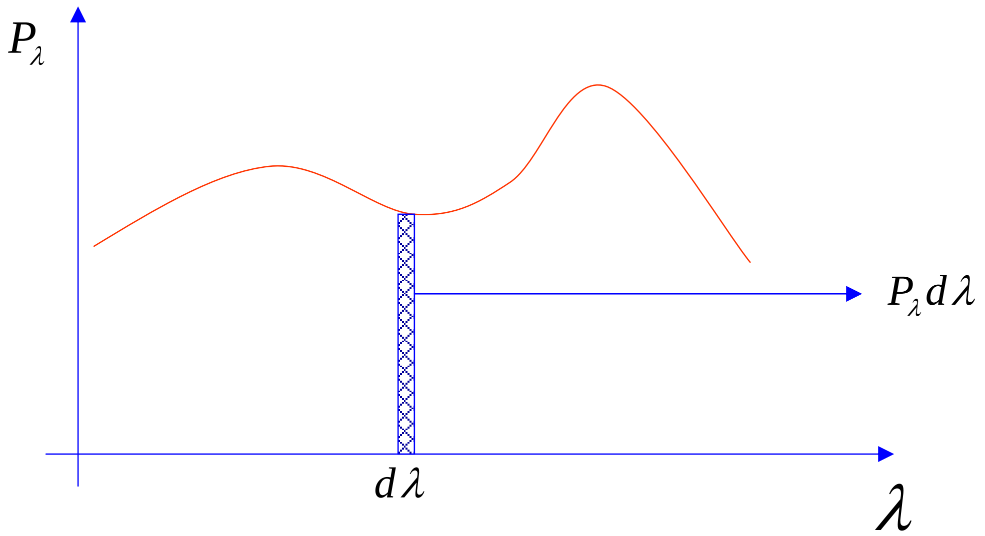


# Complex stimuli

- A given complex stimuli Q with spectral power density (SPD) $\{P_\lambda d\lambda\}_Q$ can be seen as an additive mixture of a set of monochromatic stimuli $Q_i$ with SPD $\{P_\lambda\, d\lambda\}_{Qi}$
  - For each monochromatic stimulus

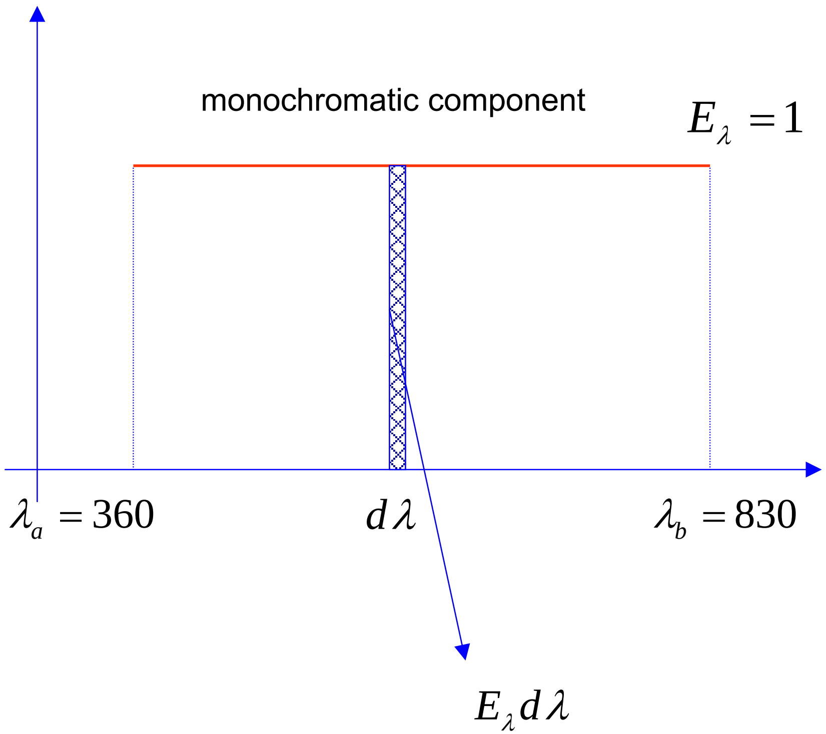
$$\vec{P}_\lambda = R_\lambda \vec{R} + G_\lambda \vec{G} + B_\lambda \vec{B}$$

$R_\lambda, G_\lambda, B_\lambda$ spectral tristimulus values



# Color Matching Functions

- The *reference white* is used to express the complex spectrum in a different form




$$\vec{E}_\lambda = \overline{r}(\lambda)\vec{R} + \overline{g}(\lambda)\vec{G} + \overline{b}(\lambda)\vec{B} \Rightarrow$$

$$E_R = \int_{-\infty}^{+\infty} \overline{r}(\lambda)\,d\lambda = 1$$

$$E_G = \int_{-\infty}^{+\infty} \overline{g}(\lambda)\,d\lambda = 1$$

$$E_B = \int_{-\infty}^{+\infty} \overline{b}(\lambda)\,d\lambda = 1$$

$$\vec{E} = 1\vec{R} + 1\vec{G} + 1\vec{B}$$

# Tristimulus values of a complex stimulus

$$Q_\lambda = \left(P_\lambda d\lambda\right) E_\lambda = \left(P_\lambda d\lambda\right) \bar{r}\left(\lambda\right) \vec{R} + \left(P_\lambda d\lambda\right) \bar{g}\left(\lambda\right) \vec{G} + \left(P_\lambda d\lambda\right) \bar{b}\left(\lambda\right) \vec{B} \rightarrow$$

$$R_Q = \int_\lambda \left(P_\lambda d\lambda\right) \bar{r}\left(\lambda\right) = \int_\lambda P_\lambda \bar{r}\left(\lambda\right) d\lambda$$

$$G_Q = \int_\lambda \left(P_\lambda d\lambda\right) \bar{g}\left(\lambda\right) = \int_\lambda P_\lambda \bar{g}\left(\lambda\right) d\lambda$$

$$B_Q = \int_\lambda \left(P_\lambda d\lambda\right) \bar{b}\left(\lambda\right) = \int_\lambda P_\lambda \bar{b}\left(\lambda\right) d\lambda$$

Metameric stimuli: different SPD, same color appearance

$$R_Q = \int P^1{}_\lambda \bar{r}(\lambda) d\lambda = \int P^2{}_\lambda \bar{r}(\lambda) d\lambda$$

$$G_Q = \int P^1{}_\lambda \bar{g}(\lambda) d\lambda = \int P^2{}_\lambda \bar{g}(\lambda) d\lambda$$

$$B_Q = \int P^1{}_\lambda \bar{b}(\lambda) d\lambda = \int P^2{}_\lambda \bar{b}(\lambda) d\lambda$$

# Chromatic coordinates

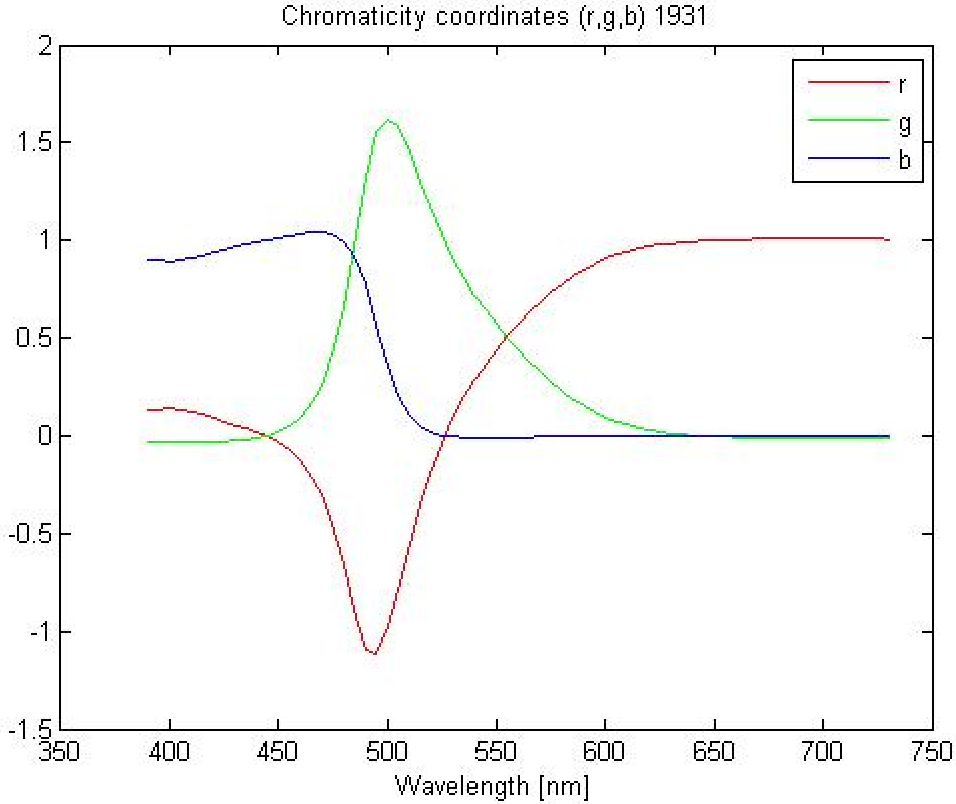
- Spectral chromaticity coordinates

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$r(\lambda) + g(\lambda) + b(\lambda) = 1$$

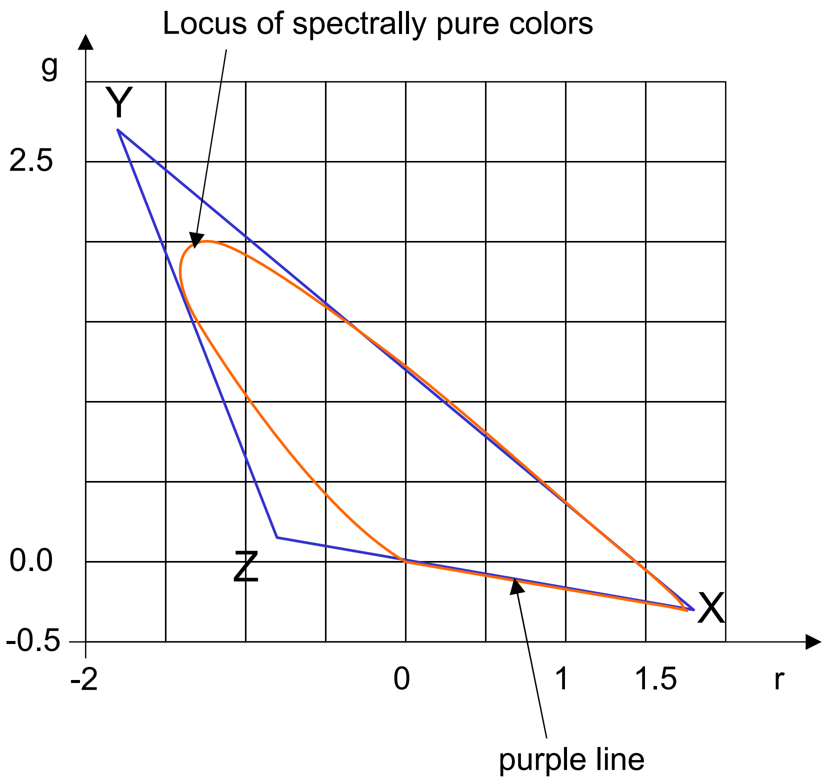


# (r,g) chromaticity diagram

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)}$$




$r,g,b$ : chromaticity coordinates

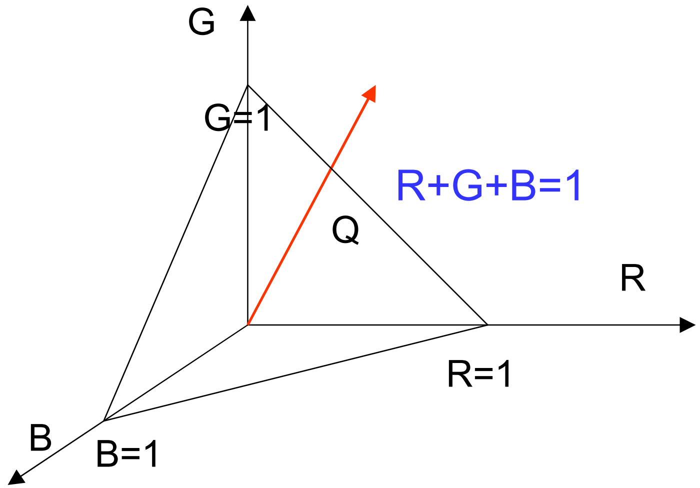
$\bar{r},\bar{g},\bar{b}$ color matching functions

# Chromaticity coordinates

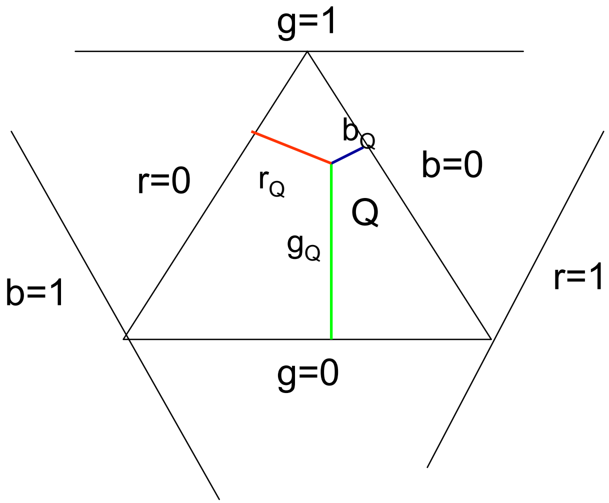
*Chromaticity coordinates*

$$r = \frac{R}{R+G+B}$$
$$g = \frac{G}{R+G+B}$$
$$b = \frac{B}{R+G+B}$$

$$\Rightarrow \quad r + g + b = 1$$




Maxwell color triangle




(r,g) specify the *hue and saturation* of the color while the information about the luminance is lost

# CIE 1931 Standard Observer

- In colorimetric practice, the main objective is to obtain results valid for the *group of normal trichromats*. To this end, the color matching properties of an *ideal trichromatic observer* are defined by specifying three independent functions of λ which are identified with the ideal observer CMFs.
- The CIE 1931 SO also embodies the additivity law for brightness (*V(λ)* photopic luminous efficiency function)

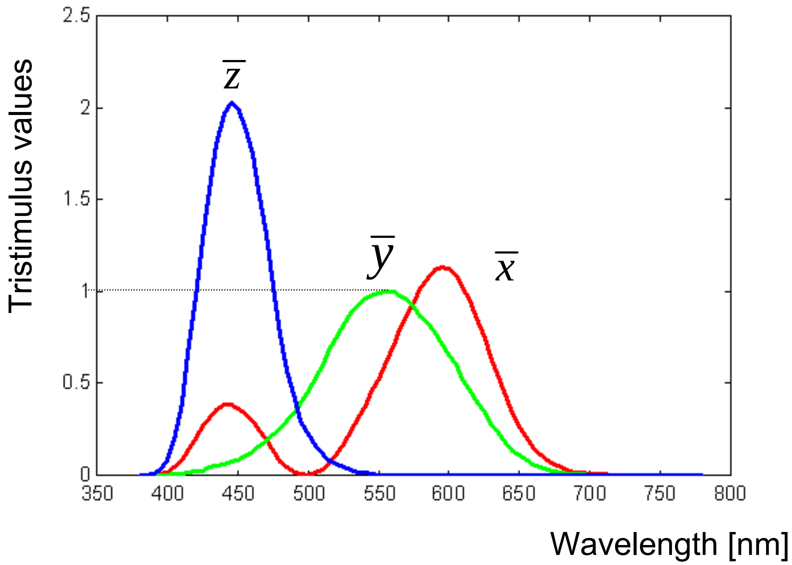
***For an observer who makes brightness matches that conform to the additivity law for brightness, and who also makes color matches that are trichromatic in the stronger sense, it can be shown that V(λ) is a combination of the CMFs, provided all the pairs of metameric stimuli are also in brightness match***

For such an observer, it is possible to select from the infinitely many equivalent sets of CMFs one set for which one of the three CMFs, usually taken to be the central one ($\bar{y}$) coincides with V(λ).

In this way, the CIE 1931 SO combines both color matching and heterochromatic brightness matching properties in a single quantitative scheme

# CIE 1931 Standard Colorimetric Observer

- Standard system for color representation: X,Y,Z tristimulus coordinate system
- Color matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$




http://www.cvrl.org/cmfs.htm

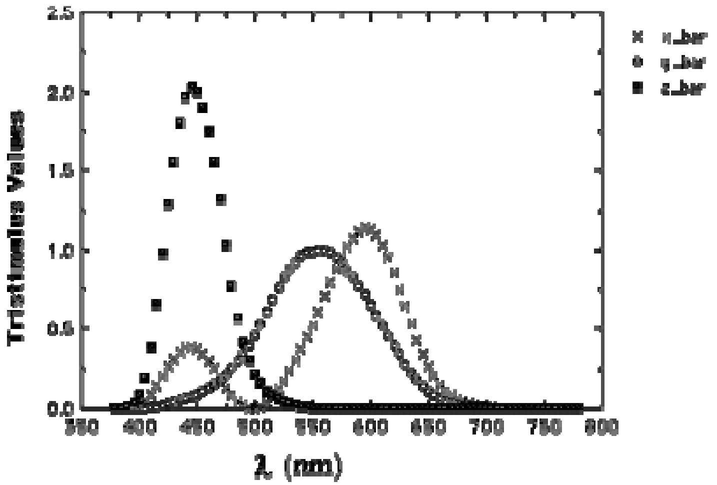
Features

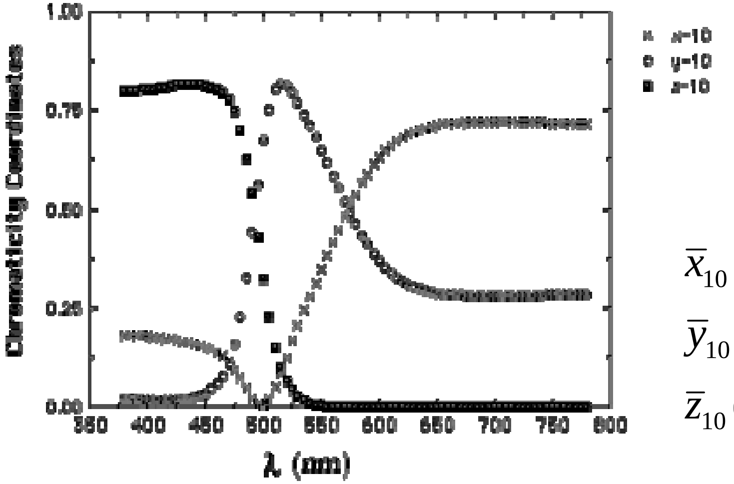
- λ=380 to 780 nm, Δλ=5nm
- Measured at 2 degrees
- Always non negative
- $\bar{y}$ is a rough approximation of the luminance of a monochromatic light of equal size and duration (*Standard photopic luminosity function V(λ)*)
- They cannot be measured by color matching experiments
- Derived such that equal amounts of X, Y, Z produce white

Improvements

- In 1959 a new set of CIE XYZ coordinates was derived based on the CMFs measured by Stiles&Burch at 10 degrees (CIE 1964 Supplementary Standard Colorimetric Observer).

# CIE 1964 SO




Features

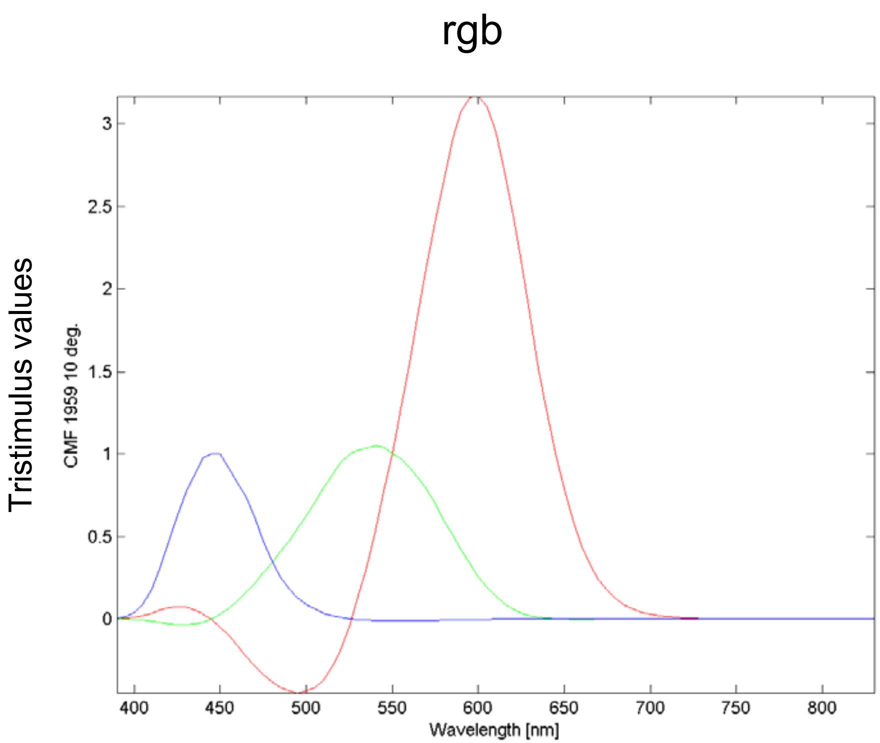
- 10 degrees field
- Extended set of wavelengths (390 to 830 nm)
- CMFs obtained directly from the observations
  - Measures of the radiant power of each monochromatic test stimulus
- High illumination intensity
  - To minimize rods intrusion
- Data extrapolated at 1nm resolution

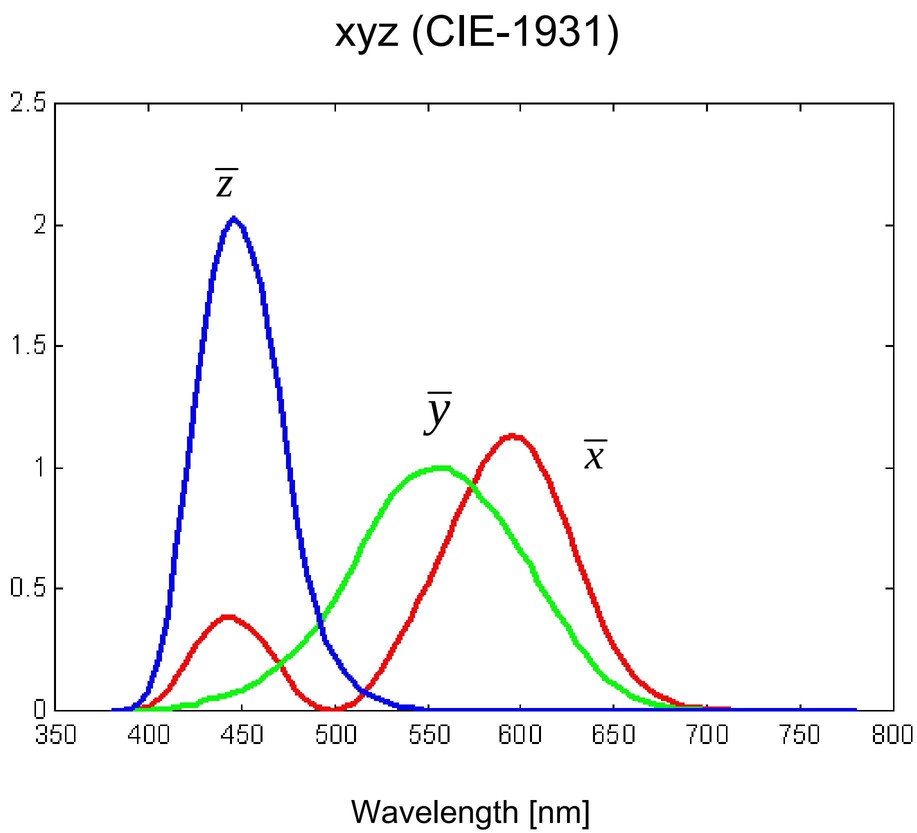
$$\bar{x}_{10}(\lambda) = 0.341080 \times \bar{r}_{10}(\lambda) + 0.189145 \times \bar{g}_{10}(\lambda) + 0.387529 \times \bar{b}_{10}(\lambda)$$

$$\bar{y}_{10}(\lambda) = 0.139058 \times \bar{r}_{10}(\lambda) + 0.837460 \times \bar{g}_{10}(\lambda) + 0.073316 \times \bar{b}_{10}(\lambda)$$

$$\bar{z}_{10}(\lambda) = 0.0 \times \bar{r}_{10}(\lambda) + 0.039553 \times \bar{g}_{10}(\lambda) + 2.026200 \times \bar{b}_{10}(\lambda)$$

# From rgb to xyz



# Guidelines for the derivation of CIE 1931 SO

- Projective transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

$(r_x,g_x,b_x)$ : coordinates of (1,0,0) as measured in the {r,g,b} system

....

  - Need to determine the matrix A of the transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \dots & \\ & & a_{3,3} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

  - This is accomplished by imposing some conditions

# Guidelines for the derivation of CIE 1931 SO

- The function $\bar{y}(\lambda)$ should be equal to the luminosity function of the eye V(λ)
- The constant spectrum of white, Eλ should have equal tristimulus value

$$\sum_{i=1}^{N} \bar{x}(\lambda_i) = \sum_{i=1}^{N} \bar{y}(\lambda_i) = \sum_{i=1}^{N} \bar{z}(\lambda_i) = 1$$

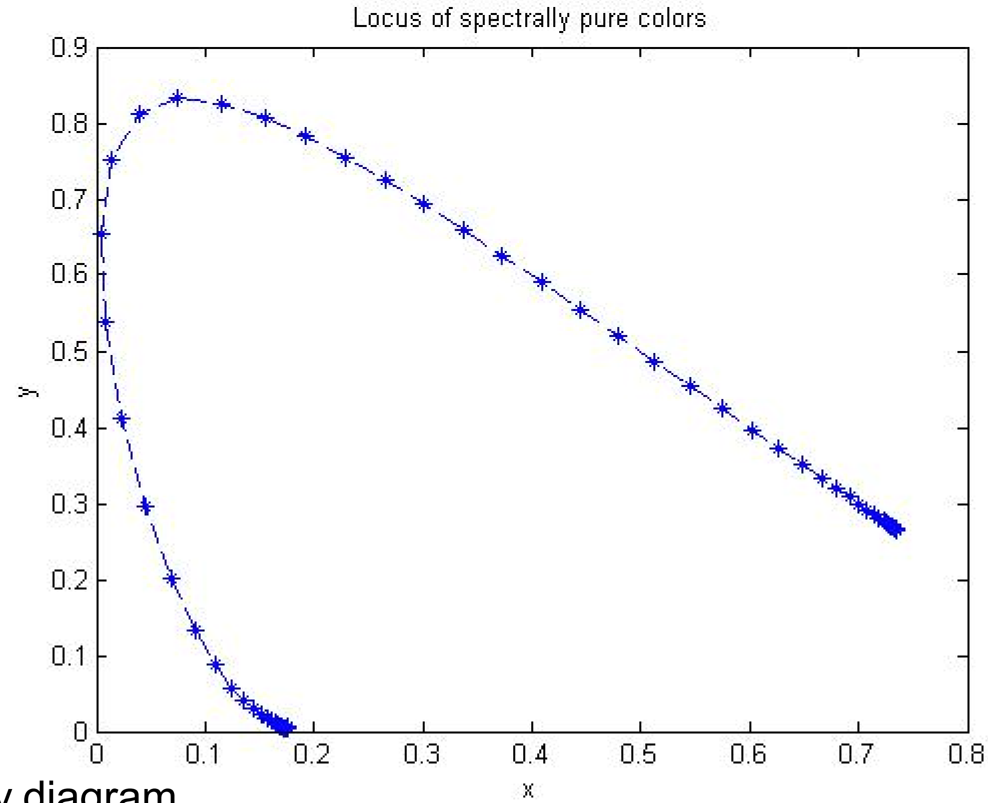
- No x value is negative
- Chromaticity coordinates

$$x(\lambda) = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}$$

$$y(\lambda) = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}$$

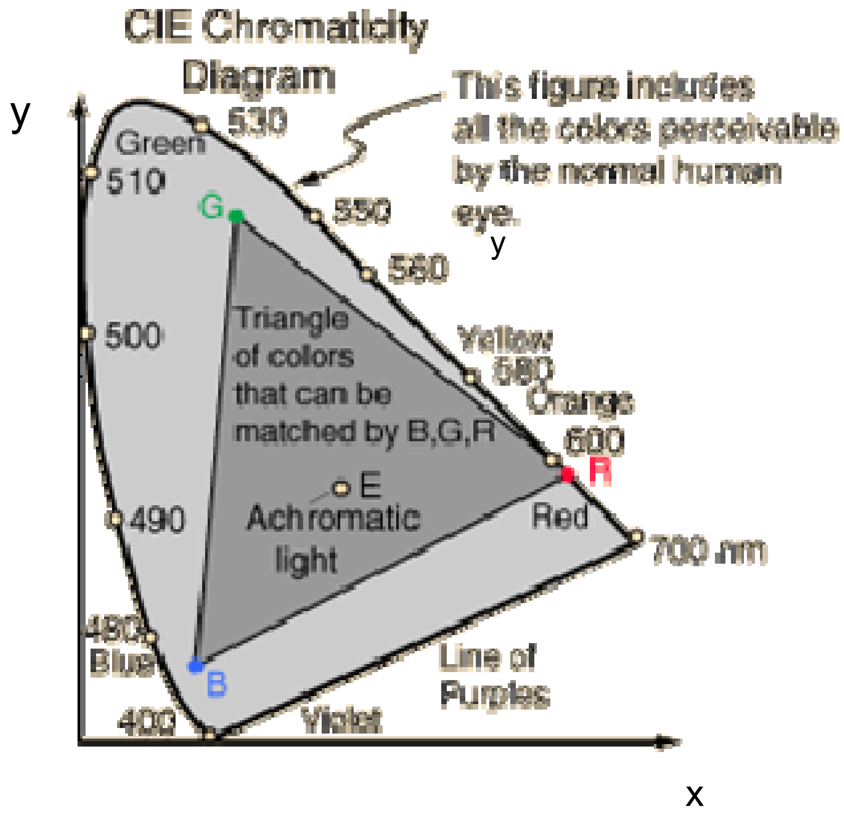
$$z(\lambda) = \frac{\bar{z}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}$$

$$x(\lambda)+y(\lambda)+z(\lambda) = 1$$




x - y chromaticity diagram

# Guidelines for the derivation of CIE 1931 SO



# rgb2xyz

- Chromaticity coordinates

$$x = \frac{0.49r + 0.31g + 0.2b}{0.66697r + 1.1324g + 1.20063b}$$
$$y = \frac{0.17697r + 0.81240g + 0.01063b}{0.66697r + 1.1324g + 1.20063b}$$
$$z = \frac{0.0r + 0.01g + 0.99b}{0.66697r + 1.1324g + 1.20063b}$$

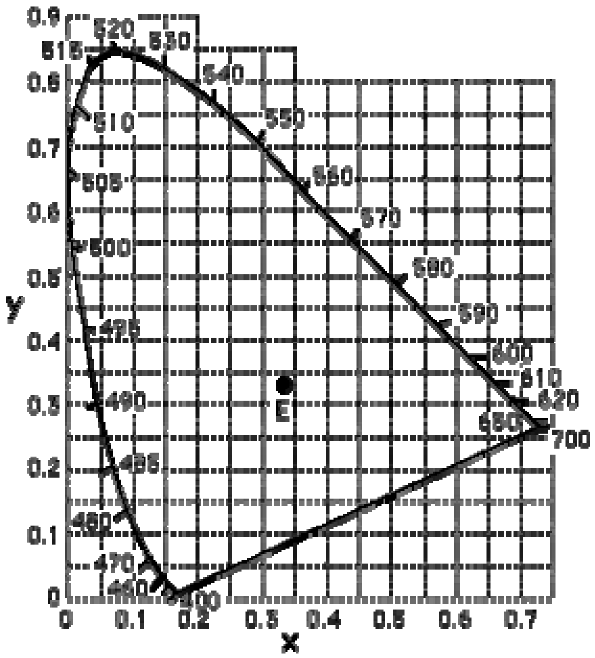
- Tristimulus values

$$X = \frac{x}{y}V \quad Y = V \quad Z = \frac{z}{y}V$$

- CMF

$$\bar{x}(\lambda) = \frac{x(\lambda)}{y(\lambda)}V(\lambda)$$
$$\bar{y}(\lambda) = V(\lambda)$$
$$\bar{z}(\lambda) = \frac{z(\lambda)}{y(\lambda)}V(\lambda)$$

(x,y) chromaticity diagram




$$x_E = y_E = \frac{1}{3}$$

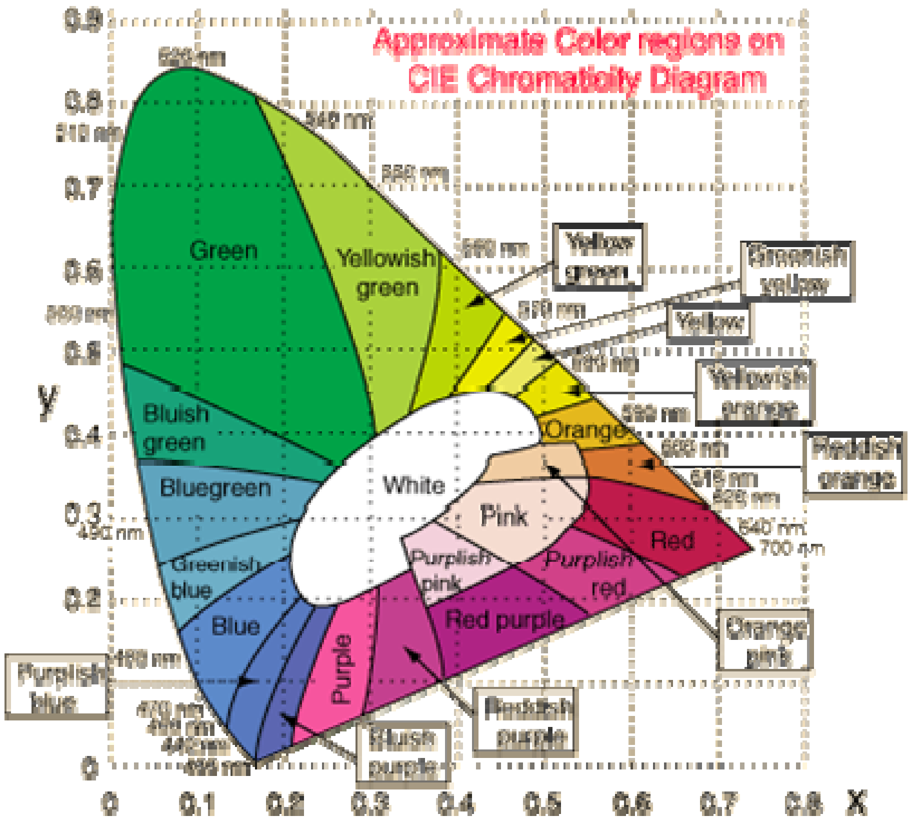
# CIE Chromaticity Coordinates

- (X,Y,Z) tristimulus values

$$X = \int P_\lambda \bar{x}(\lambda) d\lambda$$
$$Y = \int P_\lambda \bar{y}(\lambda) d\lambda$$
$$Z = \int P_\lambda \bar{z}(\lambda) d\lambda$$

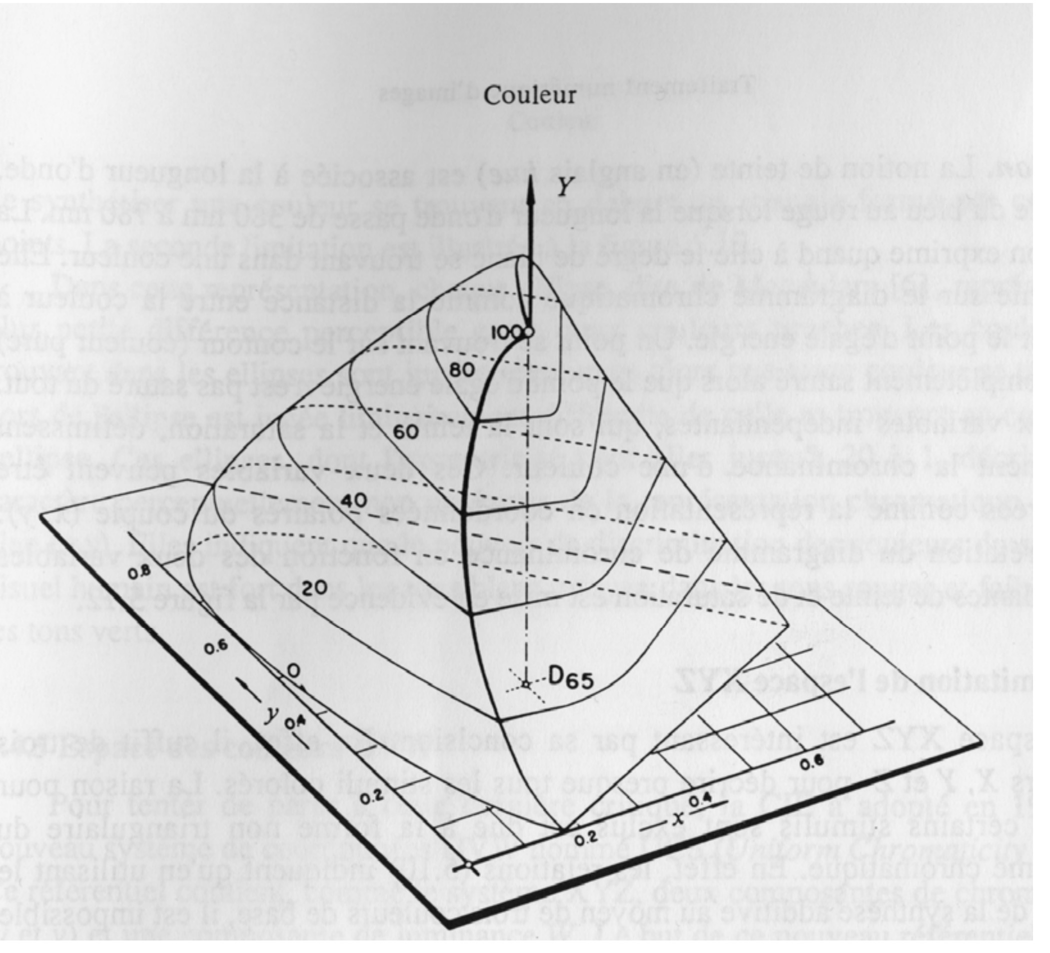
- Chromaticity coordinates

$$x(\lambda) = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}$$
$$y(\lambda) = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}$$
$$z(\lambda) = \frac{\bar{z}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}$$




$x(\lambda)+y(\lambda)+z(\lambda)=1$ x - y chromaticity diagram

# xyY

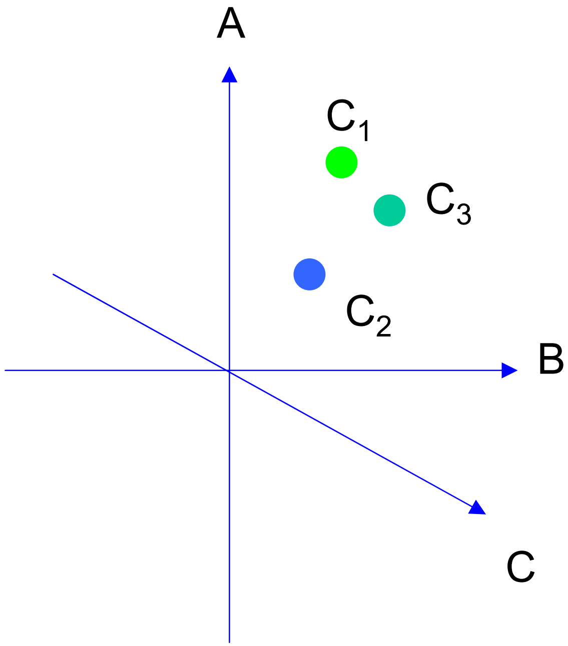


# Color models

# Color models

- Colorimetric color models
  - Based on the principles of trichromacy
  - Allow to predict if two colors match in appearance in given observation conditions
  - CIE XYZ
  - ***Perceptually uniform color models*** (CIELAB, CIELUV)
- User-oriented color models
  - Emphasize the intuitive color notions of brightness, hue and saturation
    - HSV (Hue, saturation, Value), HSI (Hue, Saturation, Intensity), HSL
- Device-oriented color models
  - The color representation depends on the device. It appears different if displayed on another device or if the set-up changes,
    - In RGB for instance, the R,G and B components depend on the chosen red, green and blue primaries as well as on the reference white
    - Amounts of ink expressed in CMYK or digitized video voltages expressed in RGB
  - RGB, sRGB, Y'CbCr,Y'UV, CMY, CMYK

# Perceptually uniform color models




Perceptual distance:

- Scaling the perceptual similarity among color samples
  - C1 is most similar to C3 than it is to C2

Measurable distance

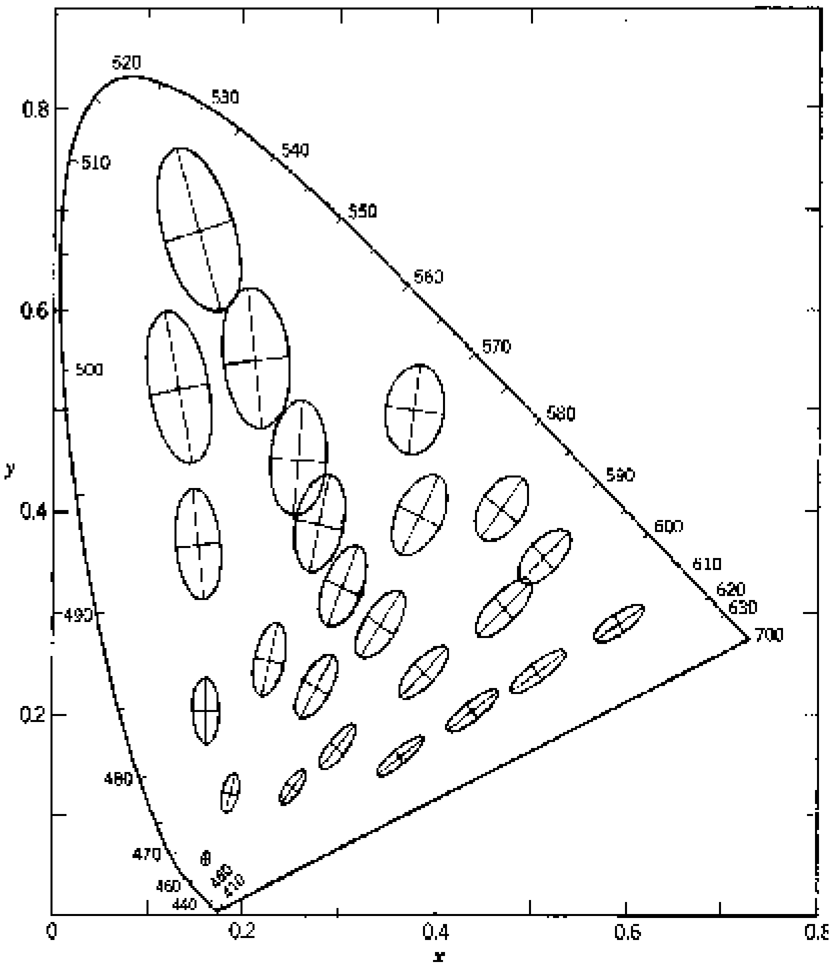
- Metric in the color space
  - Euclidean distance among the color samples

Does the perceptual distance match with the measurable distance among colors?

$$d\left(C_1 C_3\right) \overset{?}{\leq} d\left(C_1 C_2\right)$$

**Color models whose metric is representative of the perceptual distance are *perceptually uniform***

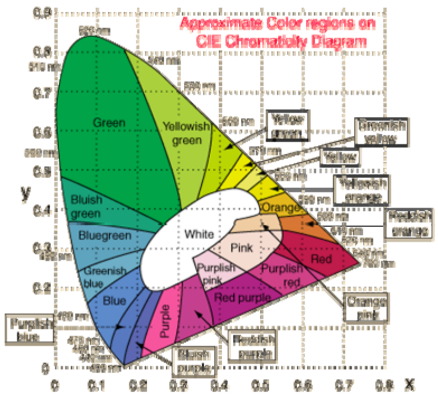
# Mac Adams’ ellipses




- The ellipses represent a **constant perceptual color stimulus**, at a constant luminance, at various positions and in various directions, in the x,y diagram.
- The areas of the ellipses vary greatly ↔ XYZ (as RGB) is not perceptually uniform.

→ CIE recommended a new CIE 1964 UCS (Uniform-Chromaticity Scale) diagram, to be used with **constant luminance levels**.

→ The size of the MacAdams’ ellipses are more uniform and the eccentricity is lower.



# Uniform color scales

Attributes: hue, saturation (chroma), brightness (lightness)

Scaling methods: procedures that attempt to give *quantitative* descriptions to *perceptual* attributes

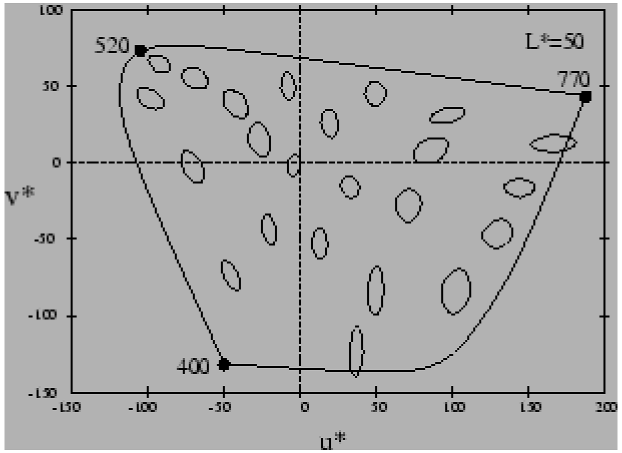
- Brightness
  - The attribute of a visual sensation according to which a visual stimulus appears to be ***more or less "intense"***, or to emit more or less light
  - Ranges from "bright" to "dim"
- Lightness
  - The attribute of a visual sensation according to which a visual stimulus appears to be more or less "intense", or to emit more or less light ***in proportion to that emitted by a similarly illuminated area perceived as "white"***
  - *Relative* brightness
  - Ranges from "light" to "dark"
- Colorfulness
  - The attribute of a visual sensation according to which a visual stimulus appears to be more or less **"chromatic"**
- Chroma
  - The attribute of a visual sensation which permits a judgment to be made of the degree to which a chromatic stimulus differs from an "achromatic" stimulus *of the same brightness*
- Saturation
  - The attribute of a visual sensation which permits a judgment to be made of the degree to which a chromatic stimulus differs from an "achromatic" stimulus *regardless of their brightness*
- Chroma and saturation are often considered as equivalent

# Perceptually uniform Color models: Luv

- CIE 1960 Luv colorspace
  - reversible transformation

$$u = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v = \frac{6Y}{X + 15Y + 3Z} = \frac{6x}{-2x + 12y + 3}$$




- CIE 1976 L*u*v* (CIELUV)

$$u' = u$$
$$v' = 1.5v$$

L*: *perceived* lightness

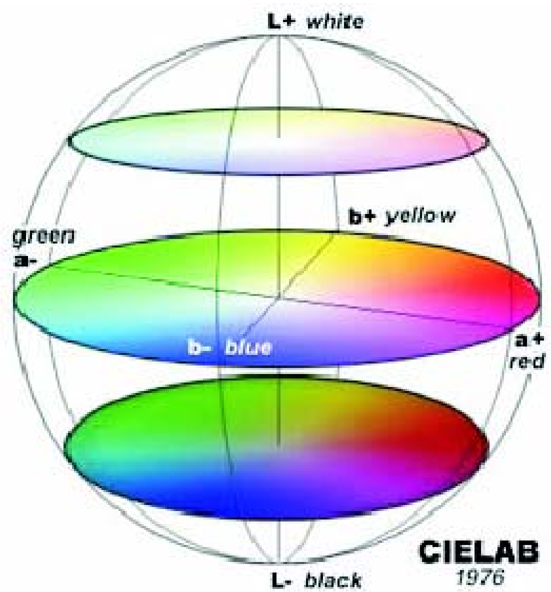
$$\begin{aligned} L^* &= 116 \left(\frac{Y}{Y_n}\right)^{1/3} - 16 \\ u^* &= 13L^*(u' - u'_n) \\ v^* &= 13L^*(v' - v'_n) \end{aligned}$$

$$\begin{aligned} u' &= \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2 + 12y + 3} \\ v' &= \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3} \end{aligned}$$

$u'_n, v'_n$ : reference white

# Perceptually uniform Color models: Lab

CIE 1976 L*a*b* (CIELAB)




For: $$\frac{Y}{Y_n}, \frac{X}{X_n}, \frac{Z}{Z_n} \geq 0.01$$

$$\begin{aligned} L^* &= 116(Y/Y_n)^{1/3} - 16 \\ a^* &= 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}] \\ b^* &= 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}] \end{aligned}$$

otherwise

$$\begin{aligned} L^* &= 116\left[f\left(\frac{Y}{Y_n}\right) - \frac{16}{116}\right] \\ a^* &= 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \\ b^* &= 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right] \end{aligned}$$

$$f\left(\frac{Y}{Y_n}\right) = \begin{cases} \left(\frac{Y}{Y_n}\right)^{1/3} & \textit{for } \frac{Y}{Y_n} > 0.008856 \\ 7.787\frac{Y}{Y_n} + \frac{16}{116} & \textit{for } \frac{Y}{Y_n} \leq 0.008856 \end{cases}$$

$X_n$, $Y_n$, $Z_n$ : *reference white*

Tristimulus values for a nominally white object-color stimulus. Usually, it corresponds to the spectral radiance power of one of the CIE standard illuminants (as D65 or A), reflected into the observer's eye by a perfect reflecting diffuser. Under these conditions, $X_n$, $Y_n$, $Z_n$ are the tristimulus values of the standard illuminant with $Y_n$=100.

Hint: the diffuse light (✹ color) depends on both the physical properties of the surface and the illuminant

# Perceptual correlates

- Color difference formula

$$\Delta E^*_{u,v} = \left[ \left( \Delta L^* \right)^2 + \left( \Delta u^* \right)^2 + \left( \Delta v^* \right)^2 \right]^{1/2}$$

- Perceptual correlates

$L^*$: lightness

$C^*_{u,v} = \left[ (u^*)^2 + (v^*)^2 \right]^{1/2}$ : chroma

$s^*_{u,v} = \dfrac{C^*_{u,v}}{L^*}$: saturation

# Summary

- References
  - B. Wandell, “Foundations of visions”
  - Wyszecki&Stiles, “Color science, concepts, methods, quantitative data and formulae”, Wiley Classic Library
  - D. Malacara, “Color vision and colorimertry, theory and applications”, SPIE Press