

# Color vision and colorimetry

# Photoreceptor types

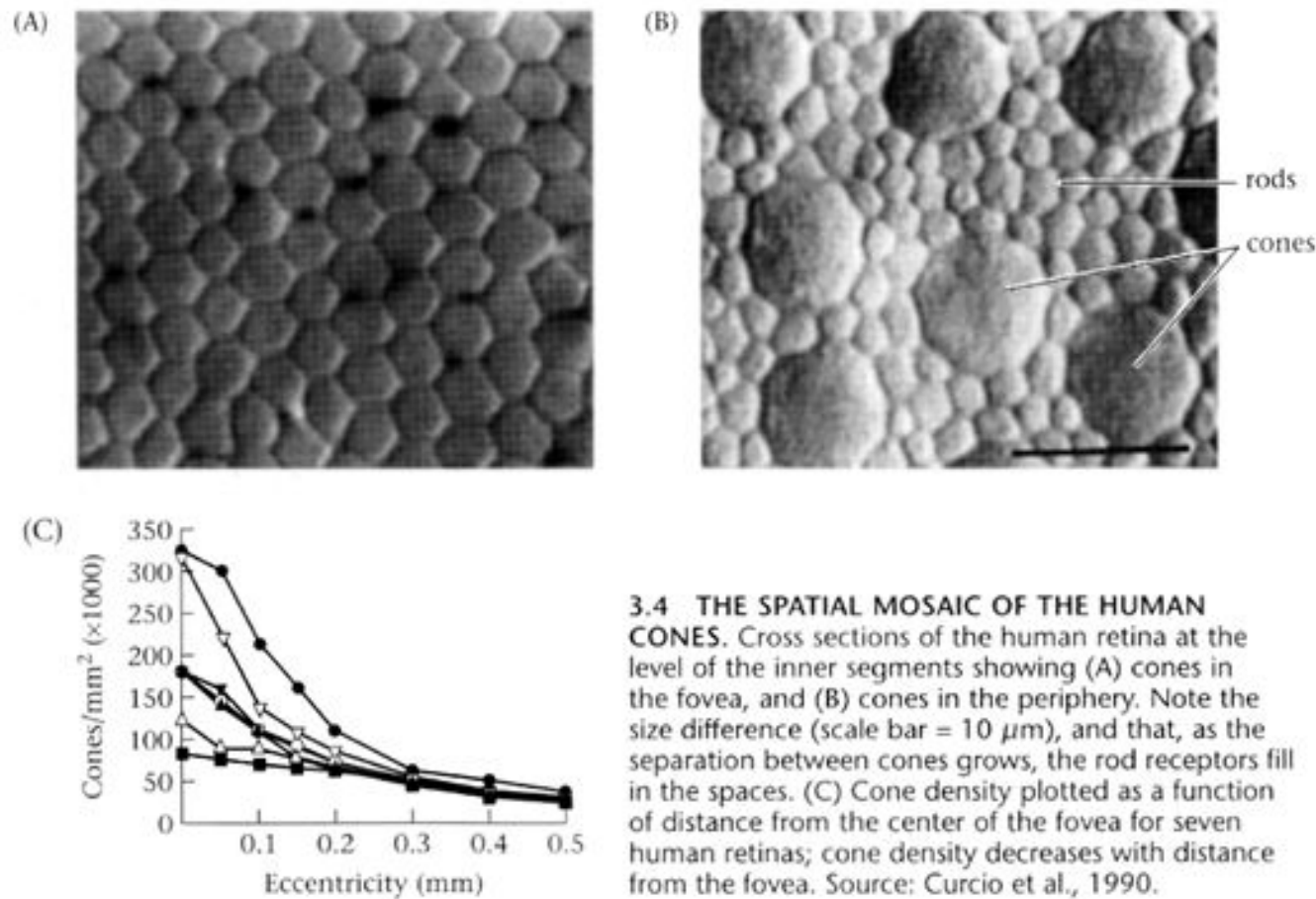
## Rods

- *Scotopic* vision (low illumination)
- Do not mediate color perception
- High density in the periphery to capture many quanta
- Low spatial resolution
- Many-to-one structure
  - The information from many rods is conveyed to a single neuron in the retina
- Very sensitive light detectors
  - Reaching high quantum efficiency could be the reason behind the integration of the signals from many receptors to a single output. The price for this is a low spatial resolution
- About 10 millions

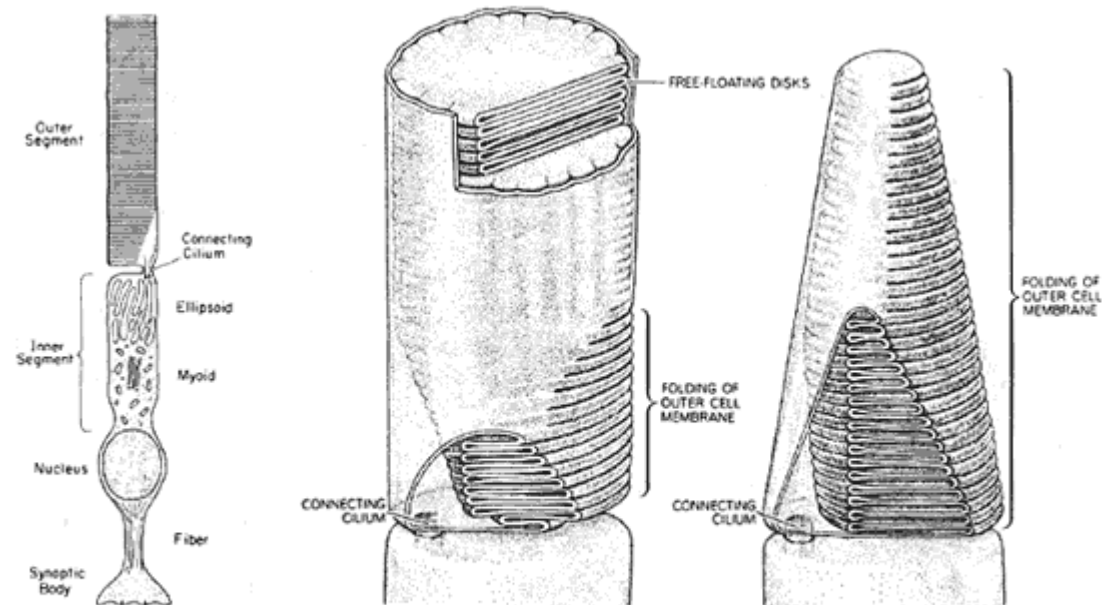
## Cones

- *Photopic* vision (high illumination)
- Mediate color perception
- High density in the fovea
- One-to-one structure
  - Do not converge into a different single neuron but are communicated along private neural channels to the cortex
- High spatial resolution
  - The lower sensitivity is compensated by the high spatial resolution, providing the eye with good acuity
- About 5 millions
  - 50000 in the central fovea

# Cones and Rods mosaic

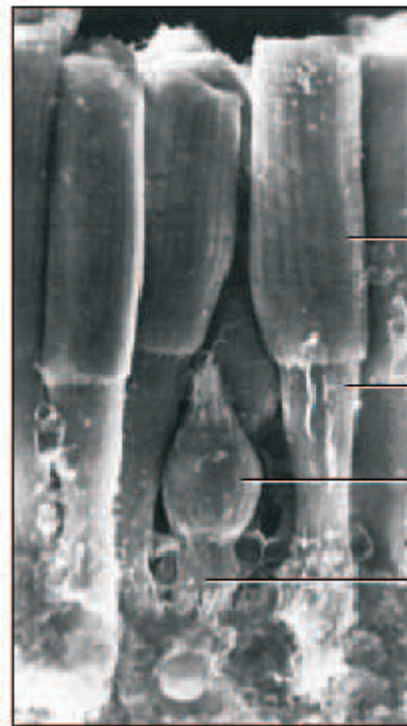


# Cones and Rods shape



At the left is a generalized conception of the important structural features of a vertebrate photoreceptor cell. At the right are shown the differences between the structure of rod (left) and cone (right) outer segments. These diagrams are from Young (1970) and Young (1971).

# Cones and rods shapes

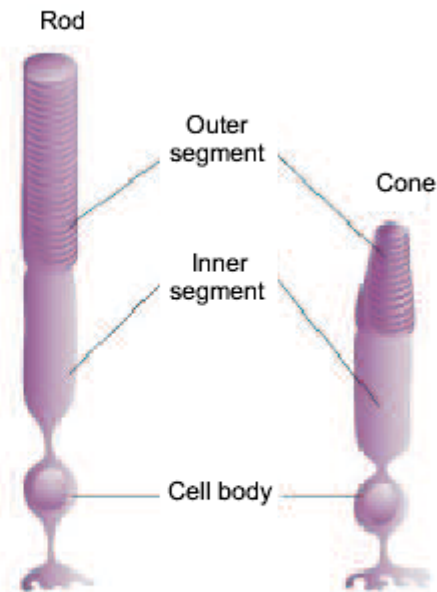


Rod outer segment

Rod inner segment

Cone outer segment

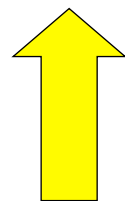
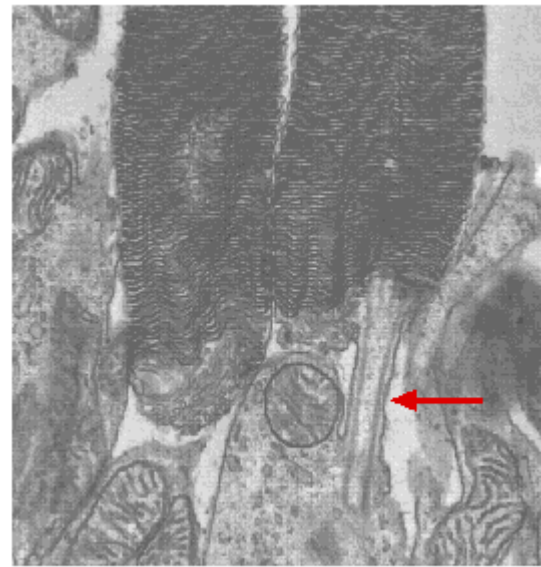
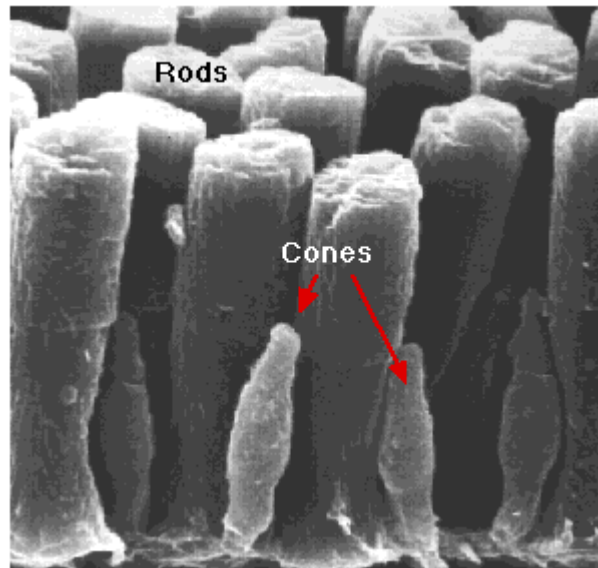
Cone inner segment



The light enters the inner segment and passes into the outer segment which contains light absorbing photopigments. Less than 10% photons are absorbed by the photopigments [Baylor, 1987].

The rods contain a photopigment called rhodopsin.

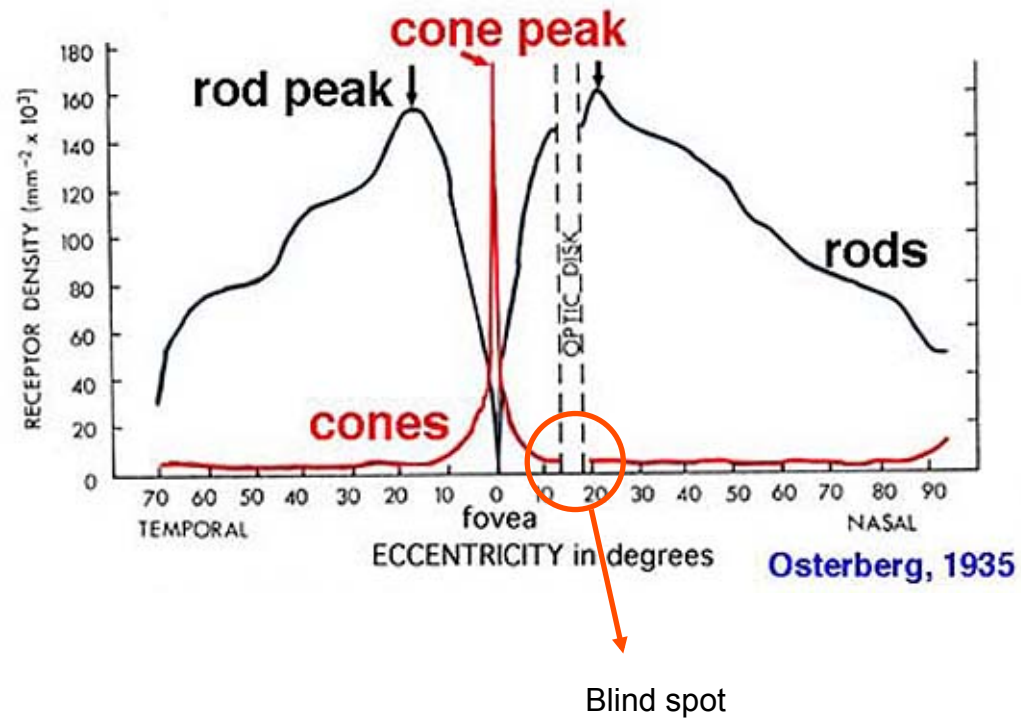
# Cone and rods



photons

# The fovea

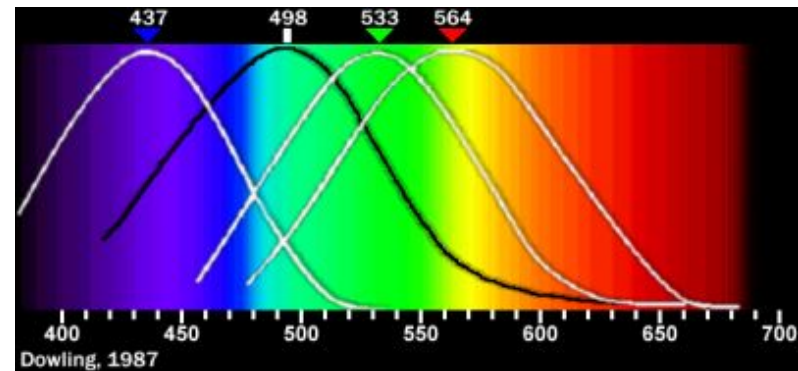
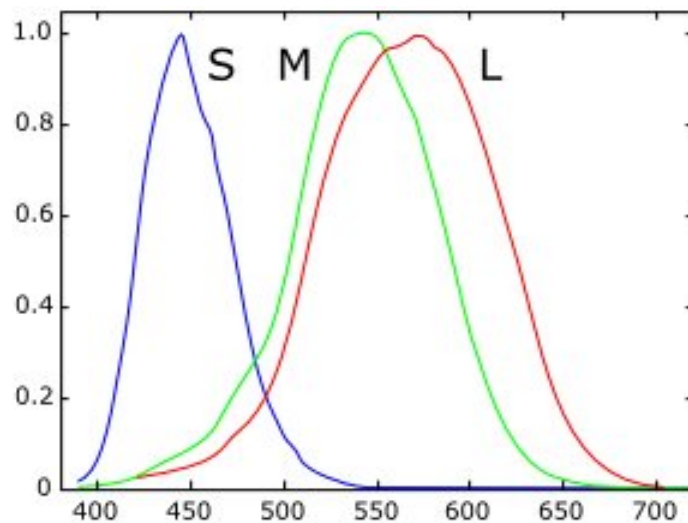
- The fovea is the region of the highest visual acuity. The central fovea contains no rods but does contain the highest concentration of cones.



# Properties of Rod and Cone Systems

| Rods                                 | Cones                                    | Comment  |
|--------------------------------------|--|--|
| More photopigment                    | Less photopigment                        |  |
| Slow response: long integration time | Fast response: short integration time    | Temporal integration   |
| High amplification                   | Less amplification                       | Single quantum detection in rods (Hecht, Schlaer & Pirenne)  |
| Saturating Response (by 6% bleached) | Non-saturating response (except S-cones) | The rods' response saturates when only a small amount of the pigment is bleached (the absorption of a photon by a pigment molecule is known as bleaching the pigment). |
| Not directionally selective          | Directionally selective                  | Stiles-Crawford effect (see later this chapter)  |
| Highly convergent retinal pathways   | Less convergent retinal pathways         | Spatial integration  |
| High sensitivity                     | Lower absolute sensitivity               |  |
| Low acuity                           | High acuity                              | Results from degree of spatial integration   |
| Achromatic: one type of pigment      | Chromatic: three types of pigment        | Color vision results from comparisons between cone responses   |

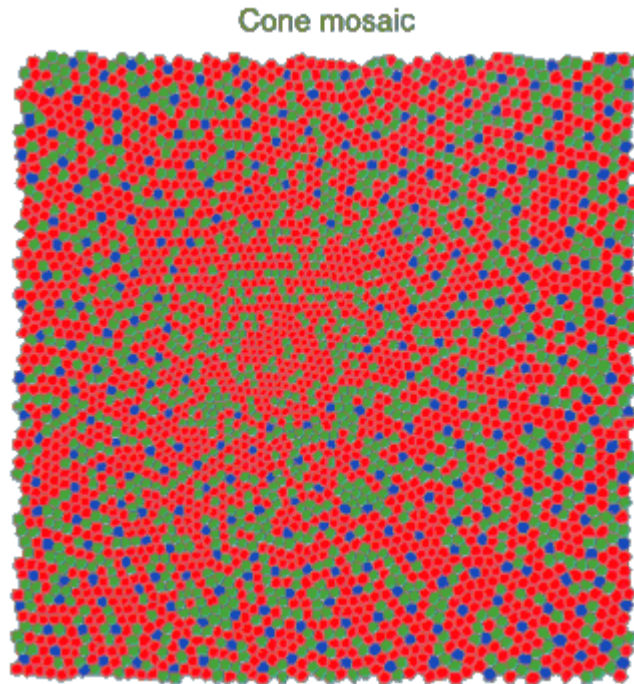
# Types of cones



The cones are classified based on their wavelength selectivity as L (long), M (medium) and S (short) wavelength sensors.

L, M and S cones have different sensitivity and spatial distributions. The S cones are far less numerous and more sensitive than the others.

# Cone mosaic



Williams (1985) measured the sampling density of the mosaic of the L- and M-cones together. His results are consistent with a sampling frequency of 60 cpd at the central fovea, consistent with a center-to-center spacing of the cones of 30 minutes of degree.

The sampling frequency then decreases when increasing the visual angle, consistently with the decrease in cone density.

This diagram was produced based on histological sections from a human eye to determine the density of the cones. The diagram represents an area of about  $1^\circ$  of *visual angle*. The number of S-cones was set to 7% based on estimates from previous studies. The L-cone:M-cone ratio was set to 1.5. This is a reasonable number considering that recent studies have shown wide ranges of cone ratios in people with normal color vision. In the central fovea an area of approximately  $0.34^\circ$  is S-cone free. The S-cones are semi-regularly distributed and the M- and L-cones are randomly distributed. Throughout the whole retina *the ratio of L- and M- cones to S-cones is about 100:1*.

# Wavelength encoding

- Scotopic matching experiment → Scotopic luminosity function  $V'(\lambda)$ 
  - Characterizes vision at low illumination conditions
  - Rod responses
  - One primary light and one test light
  - The intensity of the light beam is the parameter
- Photopic color matching experiment → Color matching functions (CMF), photopic luminosity function  $V(\lambda)$ 
  - Characterizes vision under high illumination conditions
  - Cones responses
  - Three primary lights and one test light
  - The intensities of each primary lights are the parameters

# Brightness matching

Wavelength encoding

# The photometric principle

- Basic postulate

*Whatever the visual stimulus, fixed in all respects, of one patch, and whatever the fixed relative spectral distribution of the stimulus on the second patch, a brightness match can always be achieved by varying the absolute value of the second stimulus*

- Basic laws of brightness matching

- Symmetry
  - If A matches B then B matches A
- Transitivity
  - If A matches B and B matches C then A matches C
- Proportionality
  - If A matches B then  $kA$  matches  $kB$
- Additivity
  - If A matches B and C matches D then  $(A+C)$  matches  $(B+D)$

# Brightness match

- Definition

- *Similar uniform light patches, producing visual stimuli defined by  $\{P_\lambda d\lambda\}$  and  $\{P'_\lambda d\lambda\}$ , respectively, are in brightness match for the standard photopic observer if*

$$\int_{\lambda} P_{\lambda} V(\lambda) d\lambda = \int_{\lambda} P'_{\lambda} V(\lambda) d\lambda$$

- For brightness matches, the photopic luminous flux entering the eye per unit solid angle must be the same for the two patches

# Matching experiments

- Scotopic matching experiment (brightness matching)
  - Low illumination conditions
  - Rod responses
  - One primary light and one test light
  - The *intensity* of the primary light beam is the parameter
- Measures the scotopic spectral sensitivity function  $V'(\lambda)$
- Photopic color matching experiment
  - High illumination conditions
  - Brightness matching
    - Rods
  - Color matching
    - Cones
    - Three primary lights and one test light
    - The intensities of each primary lights are the parameters
- Measures the photopic spectral sensitivity function  $V(\lambda)$
- Measures the Color Matching Functions (CMFs)

# Brightness matching

Necessary and sufficient condition for a brightness match between two stimuli of radiant power distribution  $\{P_\lambda d\lambda\}$  and  $\{P'_\lambda d\lambda\}$ , respectively

$$\int_{\lambda} P_{\lambda} \beta(\lambda) d\lambda = \int_{\lambda} P'_{\lambda} \beta(\lambda) d\lambda$$

Where  $\beta(\lambda)$  is a fixed function characterizing the brightness-matching process depending on

- the spectral radiance power distributions (relative or absolute) of the matching stimuli
- the observational conditions
  - field size, eccentricity of the field of view, state of adaptation as modified by previous or surrounding stimuli
- *Quantum efficiency* of the human visual system

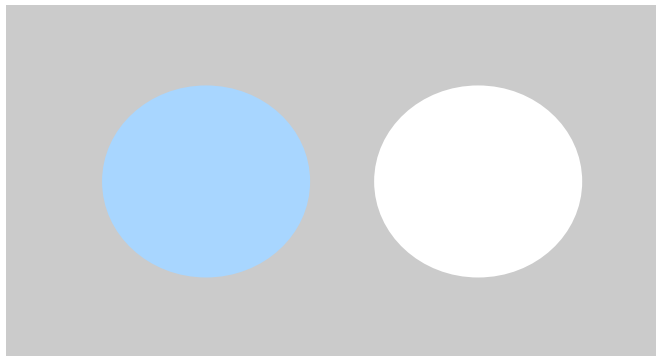
## ➤ *Ideal photometric observer*

➤ *defined by the CIE by the specification of two fixed functions*

- *Scotopic* matching  $\beta(\lambda) \rightarrow V'(\lambda)$
- *Photopic* matching  $\beta(\lambda) \rightarrow V(\lambda)$

# Scotopic brightness matching

Bipartite field



Primary light

Test light

The **primary** light has a fixed relative spectral distribution and only the *intensity* can vary

The **test** light can have any spectral distribution. It is common to use an equal-energy spectral light

Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable.

# Scotopic spectral sensitivity function

$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

**r** : system vector (transfer function)

**t** : spectral distribution of the test light

**e** : response of the observer

Assuming that the system is linear (homogeneity and superposition hold), the system vector can be measured by feeding it with  $n_\lambda$  monochromatic lights.

It is common to choose an **equal-energy spectral** light (*reference white*) as test light.

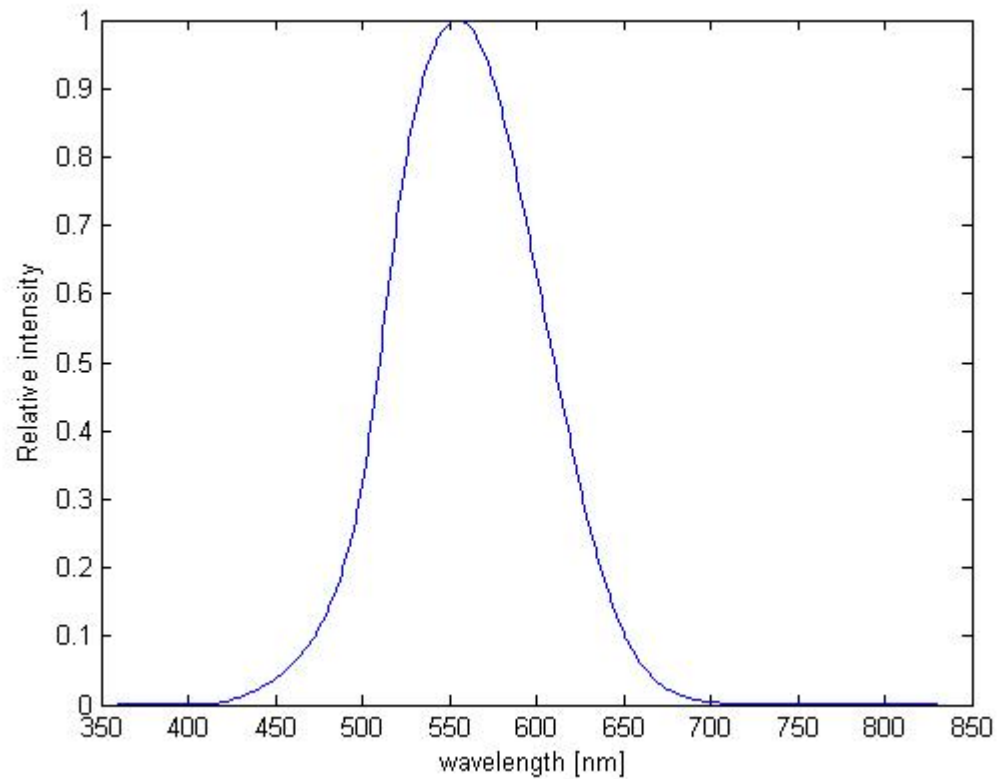
$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = r_1$$

Each monochromatic light will determine one entry of the system vector, resulting in the *Scotopic Spectral Sensitivity function*.

# Scotopic spectral sensitivity function

- In scotopic conditions, the eye is sensible only to *relative intensities* of the two lights. The spectral distribution is immaterial (in low illumination conditions, color is not “perceivable”).
  - Physiological interpretation: the rhodopsin absorption coefficient depends on the wavelength, but the response is the same for any wavelength. Once a photon is absorbed, the information about its wavelength is lost. Hence, **the appearance of the stimulus is independent of its spectrum.**
  - **The shape of  $V'(\lambda)$  reflects the dependence of the absorption coefficients from the wavelength.**
- In order to measure the spectral sensitivity at each wavelength a set of equal energy spectral (monochromatic lights) are used as test lights
- Relative intensities are recorded ( $I_{\text{REF}}/I_{\text{test}}=I_{\text{REF}}$  since  $I_{\text{test}}=\text{const.}=1$ ) and the normalized (values between zero and one)
  - Prior to normalization, due to the linearity of the system, the system vector is unique up to a scale factor.
- *$V'(\lambda)$  was adopted by CIE in 1951 in a field of 10 degrees (Crawford 1949) and eccentrically with more than 5 degrees (Wald 1945) with complete darkness adaptation.*

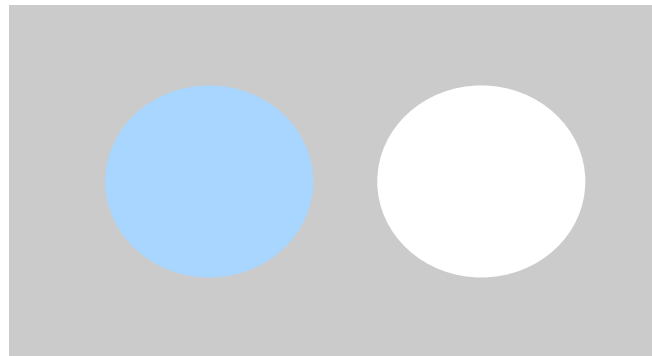
# Scotopic spectral sensitivity function $V'(\lambda)$



Data available at <http://cvision.ucsd.edu/cie.htm>

# Photopic brightness matching

Bipartite field



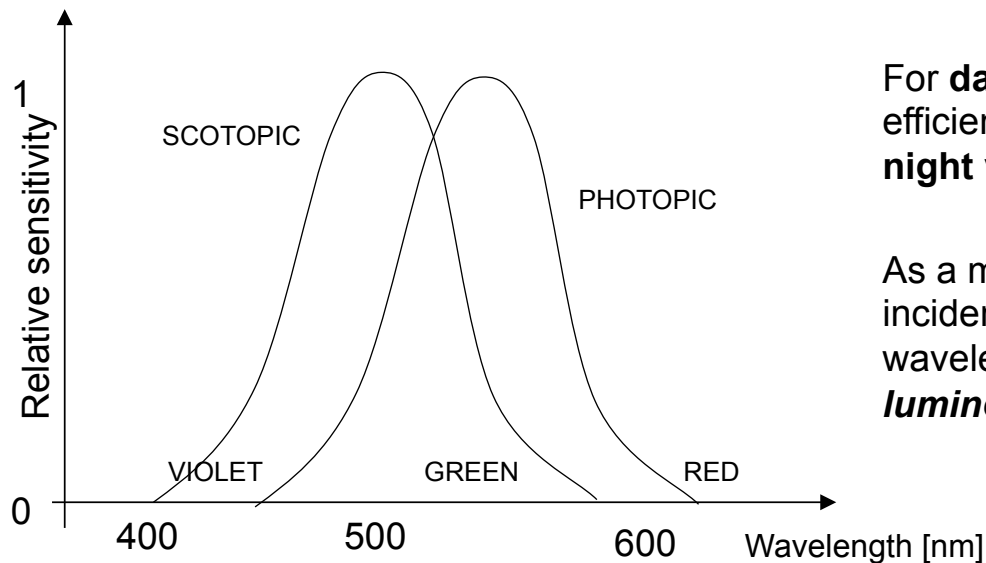
Primary light: only the  
*intensity* can vary

Test light

Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable, following a ad-hoc paradigm

# Photopic curve $V(\lambda)$

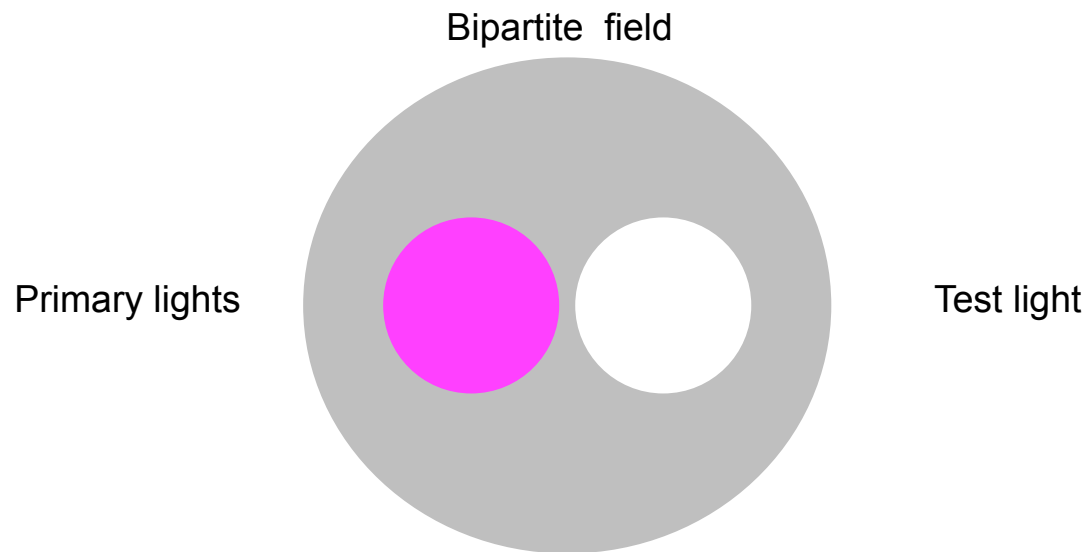
- High illumination levels
- Different paradigms
  - The direct comparison of the brightness leads to unreliable results due to the difference in color
    - Flickering method (Coblentz and Emerson, 1918)
    - Step-by-step method of heterochromatic photometry (Hyde et al. 1918)
    - CIE adopted the (Gibson and Tindall, 1923)



For **daylight** vision the maximum efficiency is at **555 nm (yellow)** while for **night** vision it shifts to **505 nm (blue)**

As a measure of the sensitivity of the eye to incident monochromatic light at each wavelength, it corresponds to a measure of the **luminous efficacy** of the eyes.

# Color matching

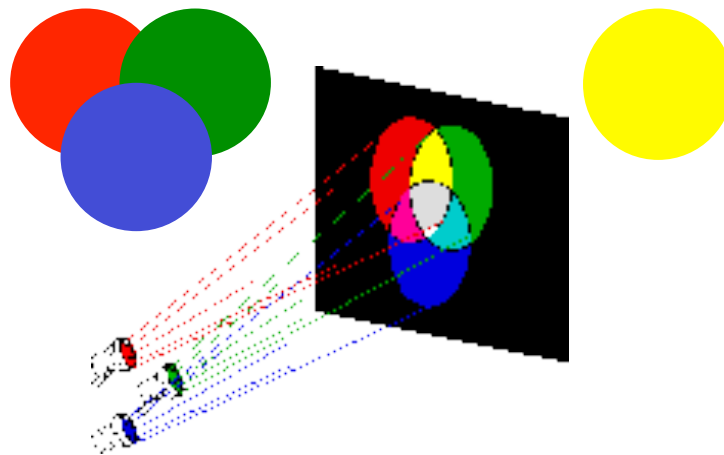
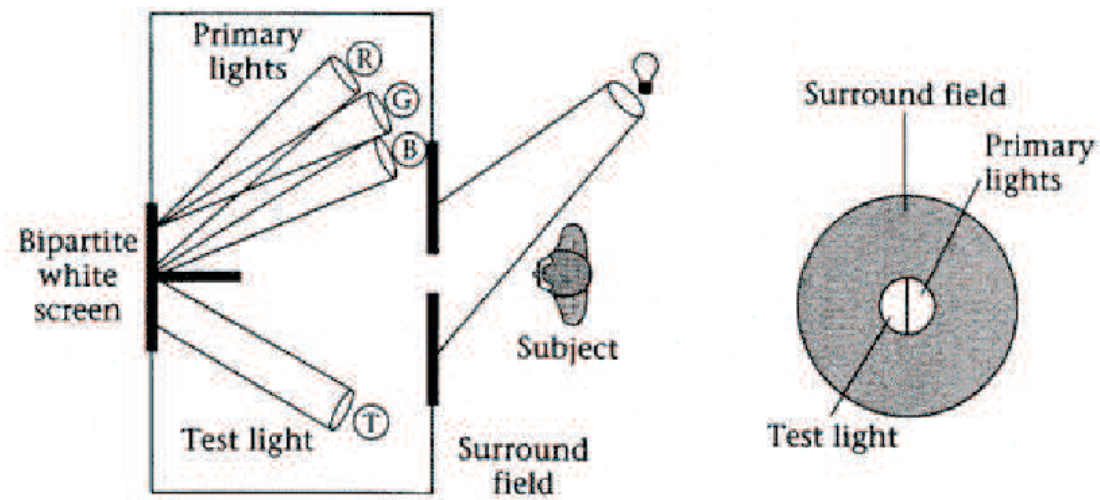


There are **three primary lights** with fixed relative spectral distribution and only the intensity can vary. These are chosen to be **monochromatic**

The test light can have any spectral distribution. It is common to choose a equal energy light and decompose it into the monochromatic components for testing the entire set of wavelengths.

**Task:** Adjust the intensities of the primary lights so that the primary and test lights appear indistinguishable

# Color matching



# Measuring the CMFs

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

**R** : system matrix (transfer function). Each line represents the *Color Matching Function* (CMF) for the corresponding primary light

**t** : spectral distribution of the test light

**e** : response of the observer

Assuming a equal-energy test light

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix} = t \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

since we are measuring relative intensities we can choose  $t=1$

# Color Matching Functions (CMFs)

Assuming that the symmetry, transitivity and homogeneity hold (*Grassmann's laws of additive color mixtures*), the system matrix can be measured by feeding it with  $n_\lambda$  monochromatic lights

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} r_1^1 \\ r_1^2 \\ r_1^3 \end{bmatrix}$$

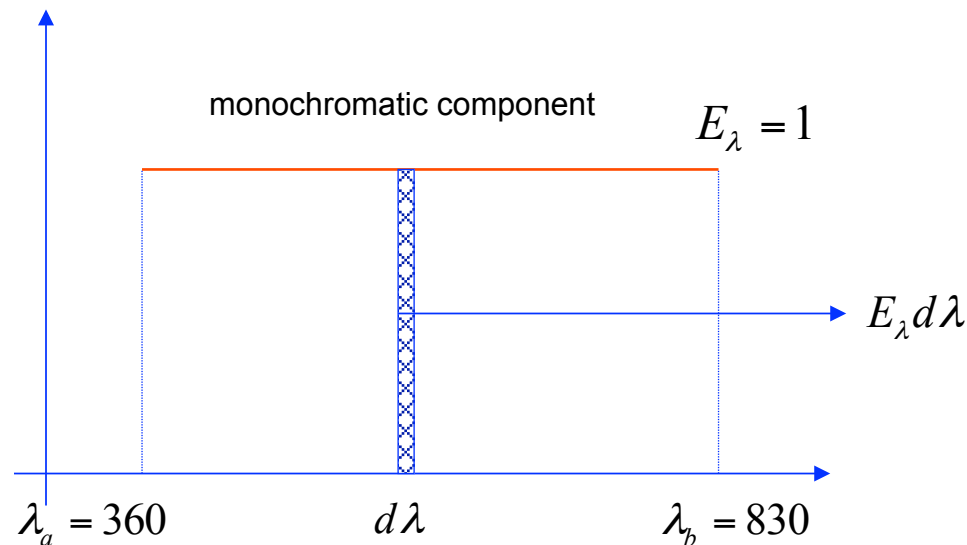
The response to each monochromatic light will determine one *column* of the system matrix, so one entry of each CMF.

It can be shown that the system matrix is not unique. Using **different sets of primaries** leads to different CMFs. Though, different sets of CMFs are related by a **linear transformation**

→ **Need to choose one set of primaries**

# Color Matching Functions

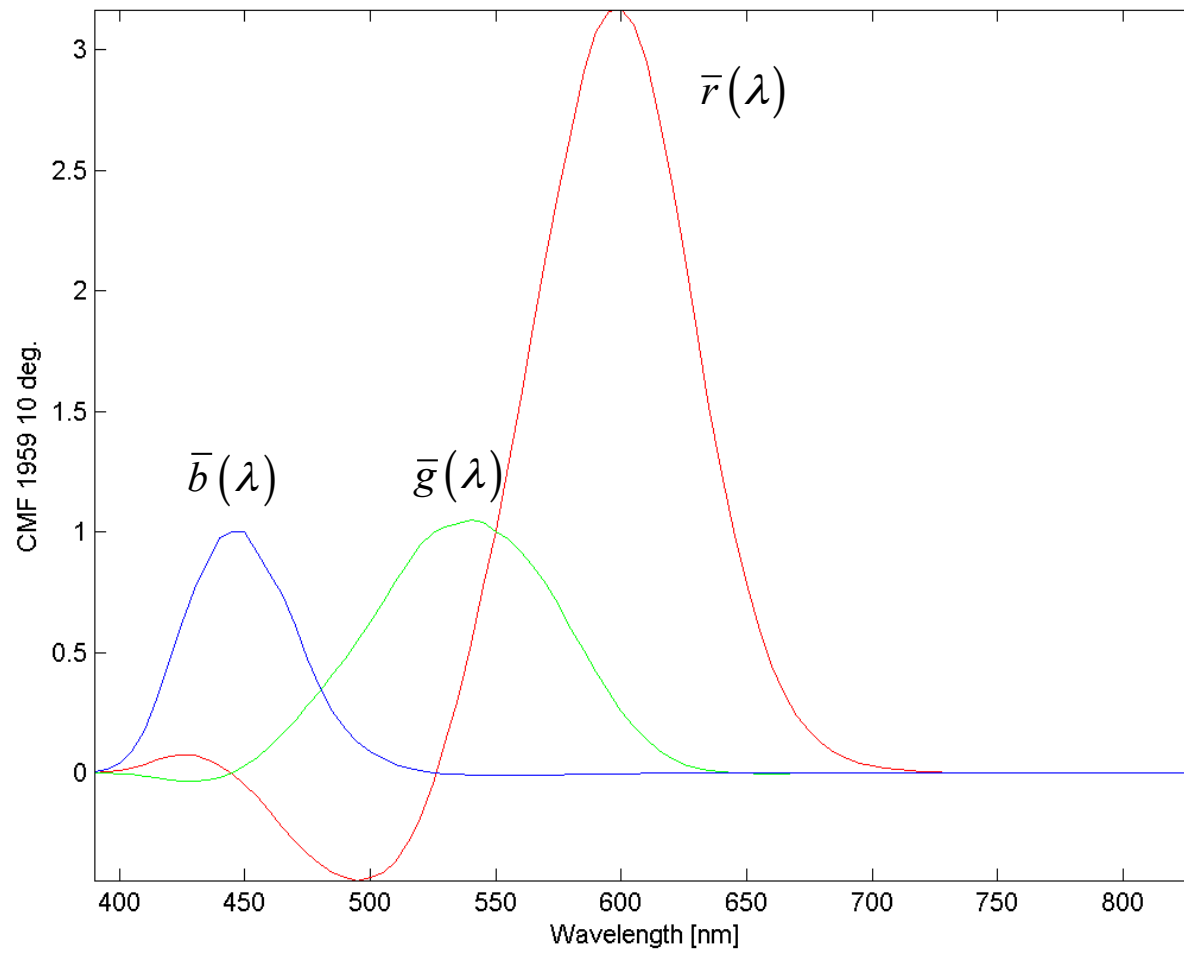
- In other words, the CMFs are the *spectral* tristimulus values of the *equal energy* stimulus E (*reference white*)



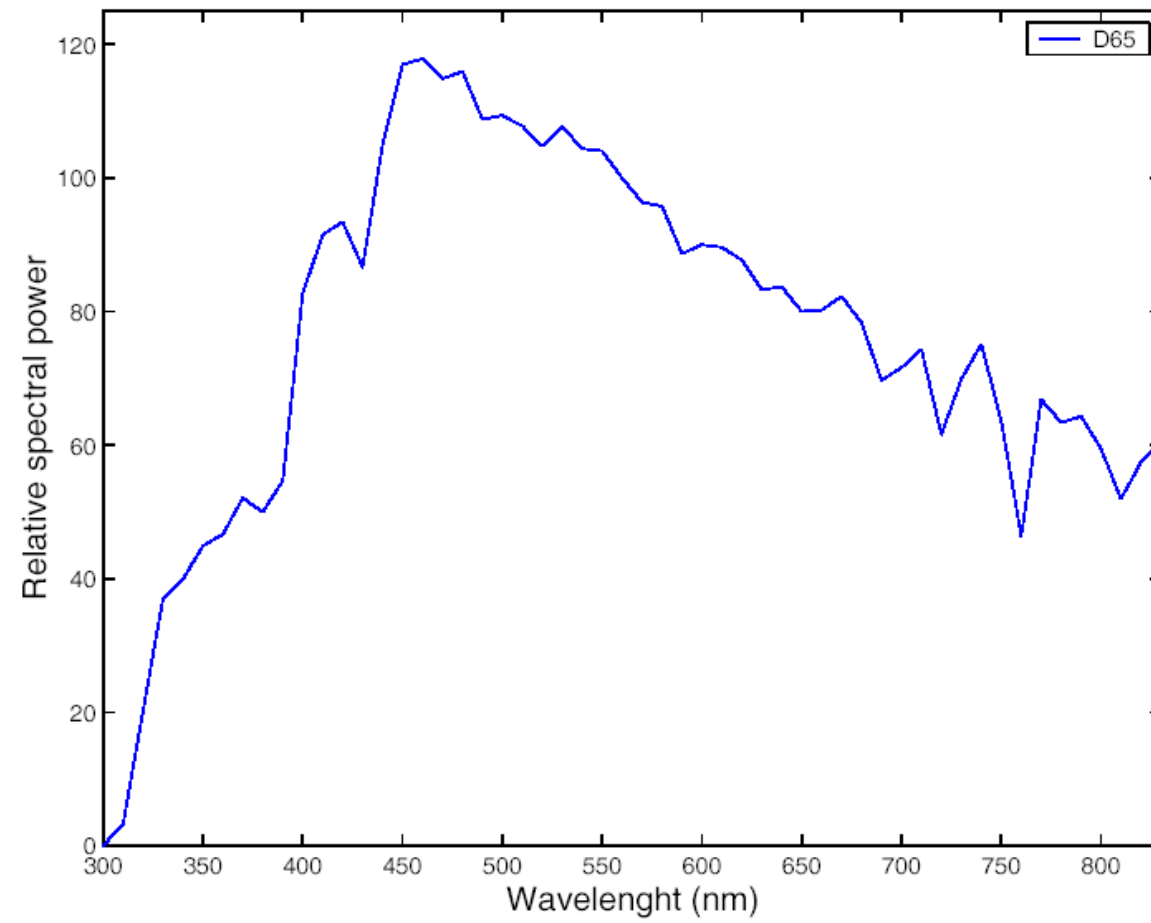
$$E_\lambda = \bar{r}(\lambda)R + \bar{g}(\lambda)G + \bar{b}(\lambda)B$$
$$\lambda_R = 700nm$$
$$\lambda_G = 546.1nm$$
$$\lambda_B = 435.8nm$$

$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$  are called color matching functions

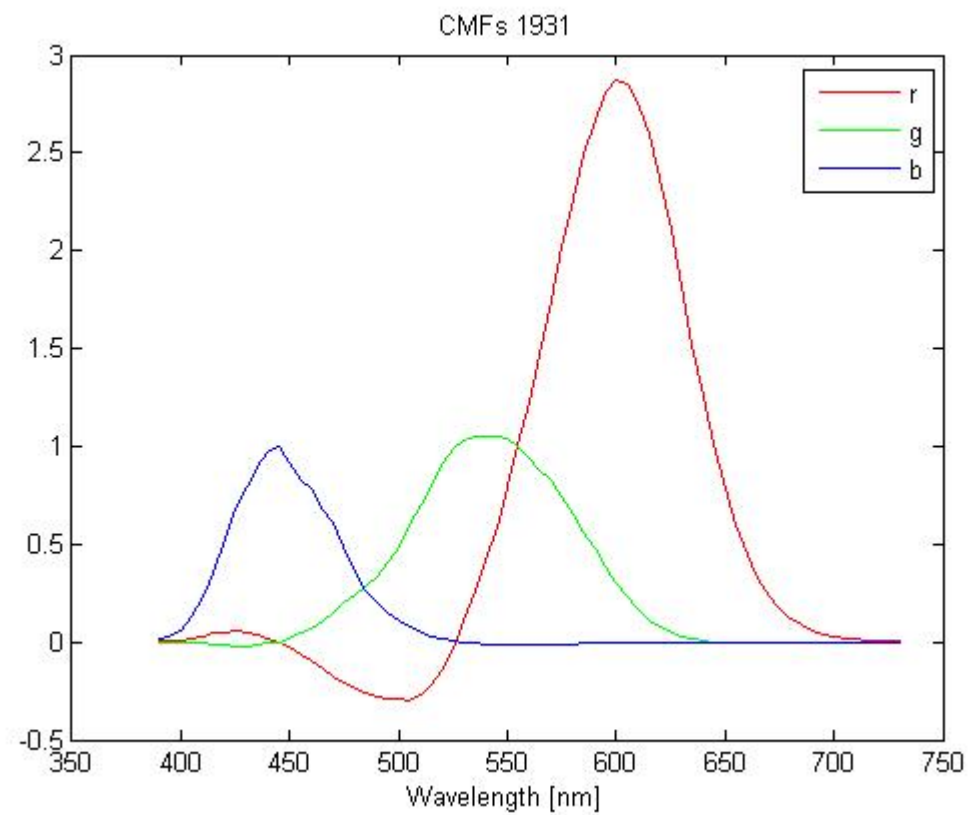
# CMFs



# D65



# CMF rgb 1931



# Stiles and Burch 10deg (1959)

Primary lights: monochromatic

$\lambda_R = 645.16 \text{ nm}$

$\lambda_G = 526.32 \text{ nm}$

$\lambda_B = 444.44 \text{ nm}$

$\bar{r}_{10}(\lambda), \bar{g}_{10}(\lambda), \bar{b}_{10}(\lambda)$  CMFs

$$t(\lambda) = R \cdot \bar{r}_{10}(\lambda) + G \cdot \bar{g}_{10}(\lambda) + B \cdot \bar{b}_{10}(\lambda)$$

$t$  : monochromatic test light

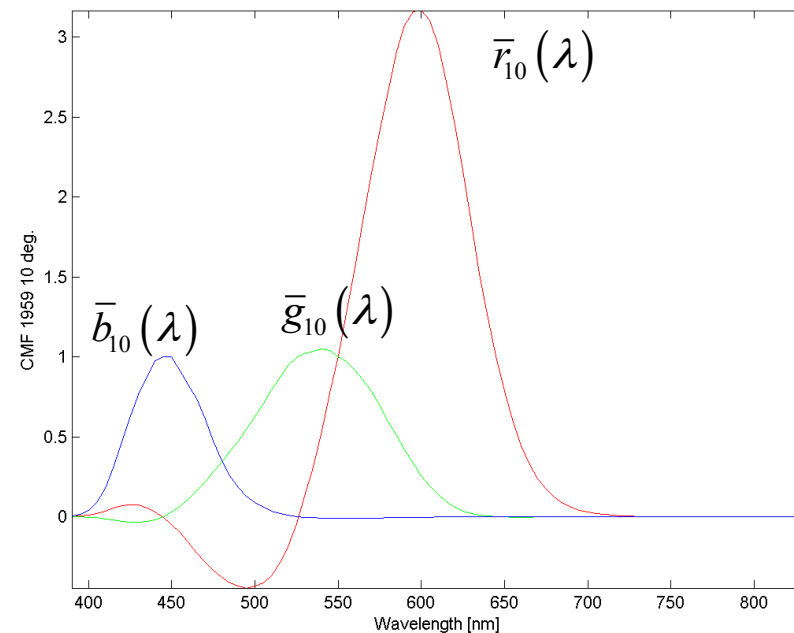
(R,G,B) : **tristimulus values** of  $t$

A **10 degrees bipartite field** was used

Negative values for the tristimulus value mean that the corresponding primary was added to the test light in order to match the color appearance.

This outlines that not every test color can be matched by an additive mixture of the three primaries.

The presence of negative values could be impractical, so another color coordinate system was chosen as the reference by the *Commission Internationale d'Eclairage (CIE)* in 1931.



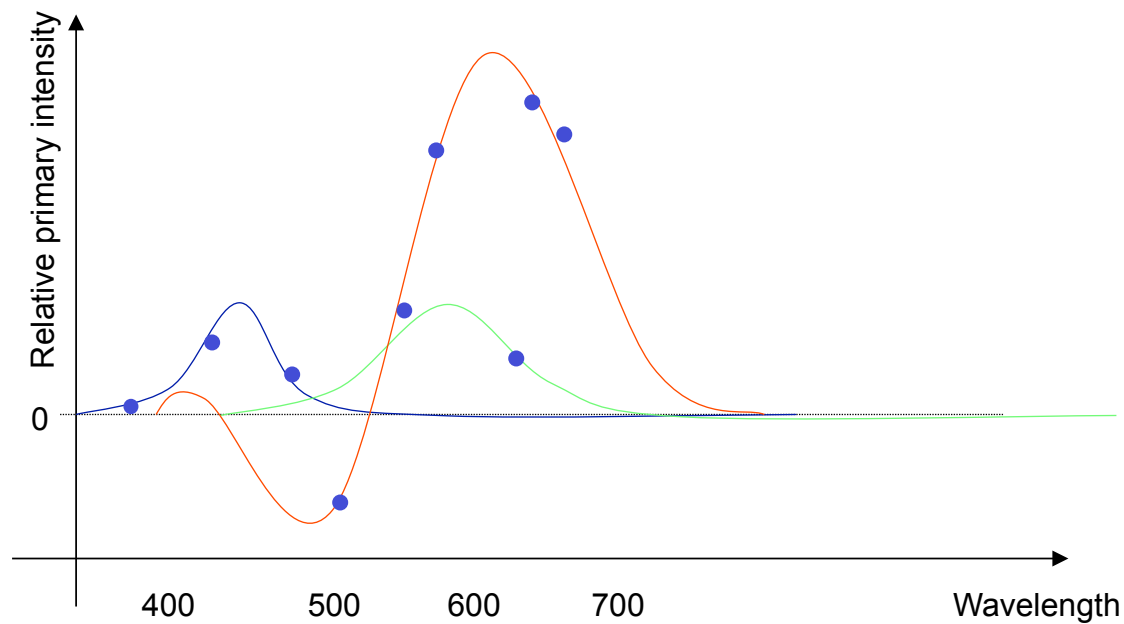
# Cone photopigments and CMF

- How well do the spectral sensitivities of the cone photopigments predict performance on the photopic color matching experiment?

| Biological measurements  | Psychophysical measurements   |
|--|---|
| $\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \text{Spectral sensitivity of L photopigments} \\ \text{Spectral sensitivity of M photopigments} \\ \text{Spectral sensitivity of S photopigments} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$ | $\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \text{CMF of primary 1} \\ \text{CMF of primary 2} \\ \text{CMF of primary 3} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$ |

- There should be a linear transformation that maps the cone absorption curves to the system matrix of the color matching experiment
- Linking hypothesis*

# Cone photopigments and CMFs



From the agreement between these two datasets one can conclude that the photopigment spectral responsivities provide a satisfactory biological basis to explain the photopic color matching experiments

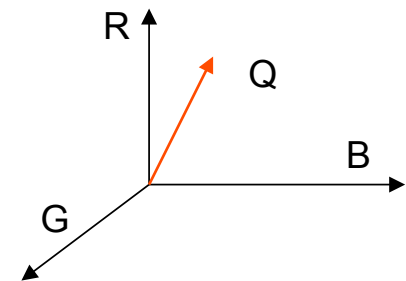
# Tristimulus values for complex stimuli

- Color stimuli are represented by vectors in a three-dimensional space, called the *tristimulus space*
  - Let  $Q$  be an arbitrary **monochromatic** color stimulus and **R,G and B** the **fixed primary stimuli** chosen for the color matching experiment

$$Q = R_Q \vec{R} + G_Q \vec{G} + B_Q \vec{B}$$

- $R_Q, G_Q, B_Q$  : *tristimulus values* of  $Q$
- The scalar multipliers  $R_Q, G_Q, B_Q$  are measured in terms of the *assigned respective units of the corresponding primaries*
- It is customary to choose these units such that when additively mixed yield a complete color match with a specified *achromatic* stimulus, usually one with an *equal-energy spectrum* on a wavelength basis

The **units** of these primaries was chosen in the radiant power ratio of **72.1:1.4:1.0**, which places the chromaticity coordinates of the equal energy stimulus  $E$  at the center of the  $(r,g)$  chromaticity diagram  
→  $R_W = G_W = B_W = 1$  for the reference white

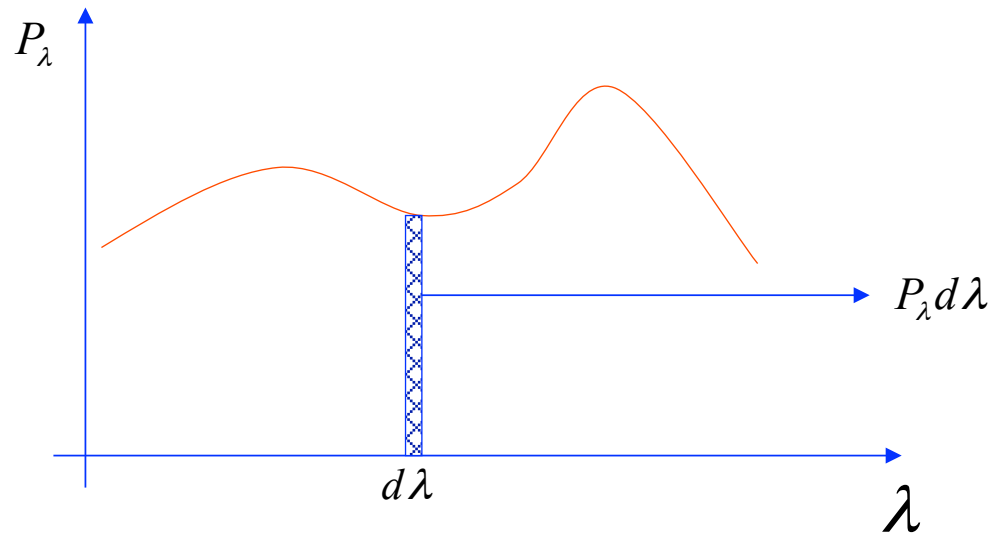


# Complex stimuli

- A given complex stimuli Q with spectral power density (SPD)  $\{P_\lambda d\lambda\}_Q$  can be seen as an *additive mixture* of a set of monochromatic stimuli  $Q_i$  with SPD  $\{P_\lambda d\lambda\}_{Q_i}$ 
  - For each monochromatic stimulus

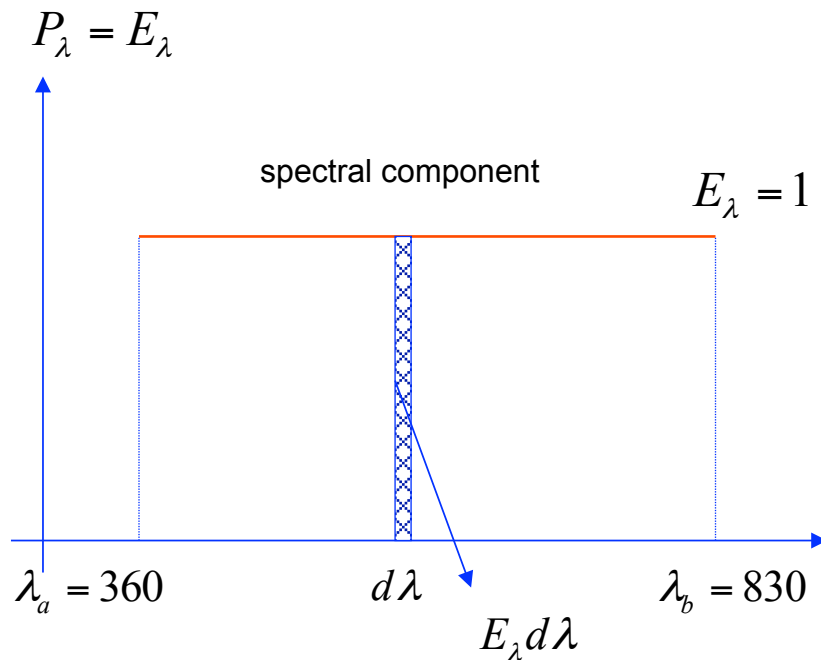
$$\vec{P}_\lambda = R_\lambda \vec{R} + G_\lambda \vec{G} + B_\lambda \vec{B}$$

$R_\lambda, G_\lambda, B_\lambda$  spectral tristimulus values



# Special case: reference white

- The *reference white* is used to express the complex spectrum in a different form



The Color Matching Functions are normalized such that the tristimulus values of the reference white are equal.

$$\begin{aligned}\vec{E}_\lambda &= \vec{r}(\lambda)\vec{R} + \vec{g}(\lambda)\vec{G} + \vec{b}(\lambda)\vec{B} \Rightarrow \\ \int_\lambda \vec{E}_\lambda d\lambda &= \int_\lambda (\vec{r}(\lambda)\vec{R} + \vec{g}(\lambda)\vec{G} + \vec{b}(\lambda)\vec{B}) d\lambda = \\ &= \int_\lambda (\vec{r}(\lambda)\vec{R}) d\lambda + \int_\lambda (\vec{g}(\lambda)\vec{G}) d\lambda + \int_\lambda (\vec{b}(\lambda)\vec{B}) d\lambda = \\ &= \int_\lambda \vec{r}(\lambda) d\lambda \times \vec{R} + \int_\lambda \vec{g}(\lambda) d\lambda \times \vec{G} + \int_\lambda \vec{b}(\lambda) d\lambda \times \vec{B}\end{aligned}$$

Normalization conditions

$$E_R = \int_{-\infty}^{+\infty} \vec{r}(\lambda) d\lambda = 1$$

$$E_G = \int_{-\infty}^{+\infty} \vec{g}(\lambda) d\lambda = 1$$

$$E_B = \int_{-\infty}^{+\infty} \vec{b}(\lambda) d\lambda = 1$$

Then

$$\vec{E} = 1\vec{R} + 1\vec{G} + 1\vec{B}$$

# Tristimulus values of a complex stimulus

$$Q_\lambda = (P_\lambda d\lambda) E_\lambda = (P_\lambda d\lambda) \bar{r}(\lambda) \vec{R} + (P_\lambda d\lambda) \bar{g}(\lambda) \vec{G} + (P_\lambda d\lambda) \bar{b}(\lambda) \vec{B} \rightarrow$$

$$R_Q = \int_\lambda (P_\lambda d\lambda) \bar{r}(\lambda) = \int_\lambda P_\lambda \bar{r}(\lambda) d\lambda$$

$$G_Q = \int_\lambda (P_\lambda d\lambda) \bar{g}(\lambda) = \int_\lambda P_\lambda \bar{g}(\lambda) d\lambda$$

$$B_Q = \int_\lambda (P_\lambda d\lambda) \bar{b}(\lambda) = \int_\lambda P_\lambda \bar{b}(\lambda) d\lambda$$

Metameric stimuli: different SPD, same color appearance

$$R_Q = \int P_\lambda^1 \bar{r}(\lambda) d\lambda = \int P_\lambda^2 \bar{r}(\lambda) d\lambda$$

$$G_Q = \int P_\lambda^1 \bar{g}(\lambda) d\lambda = \int P_\lambda^2 \bar{g}(\lambda) d\lambda$$

$$B_Q = \int P_\lambda^1 \bar{b}(\lambda) d\lambda = \int P_\lambda^2 \bar{b}(\lambda) d\lambda$$

# Chromatic coordinates

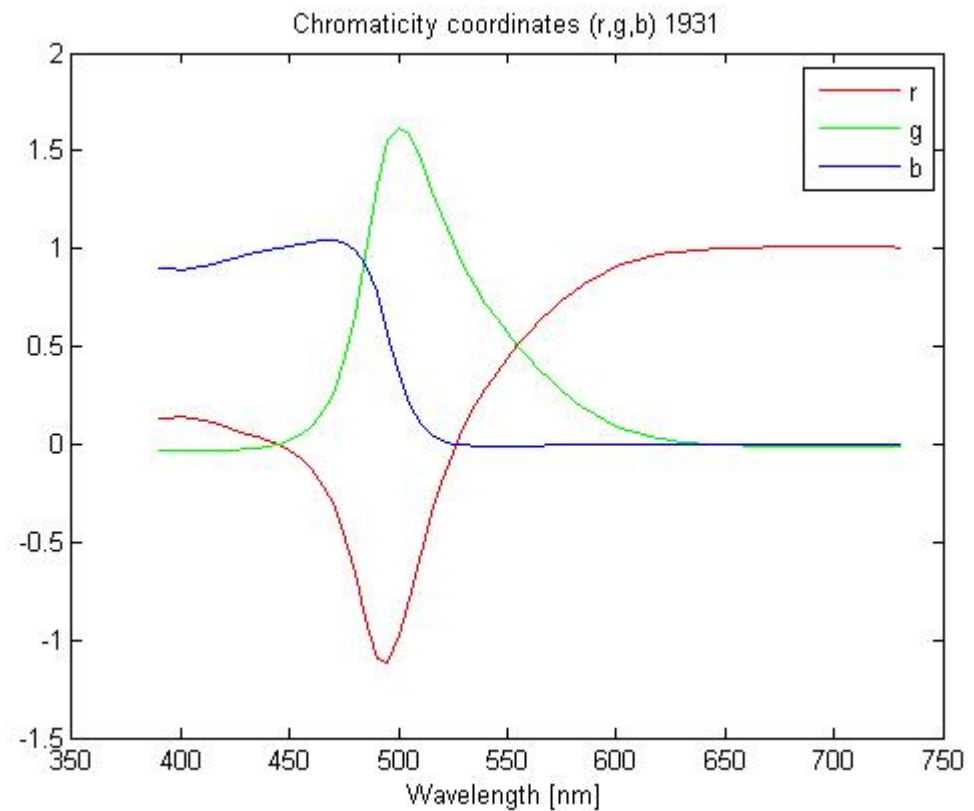
- Spectral chromaticity coordinates

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$r(\lambda) + g(\lambda) + b(\lambda) = 1$$



# (r,g) chromaticity diagram

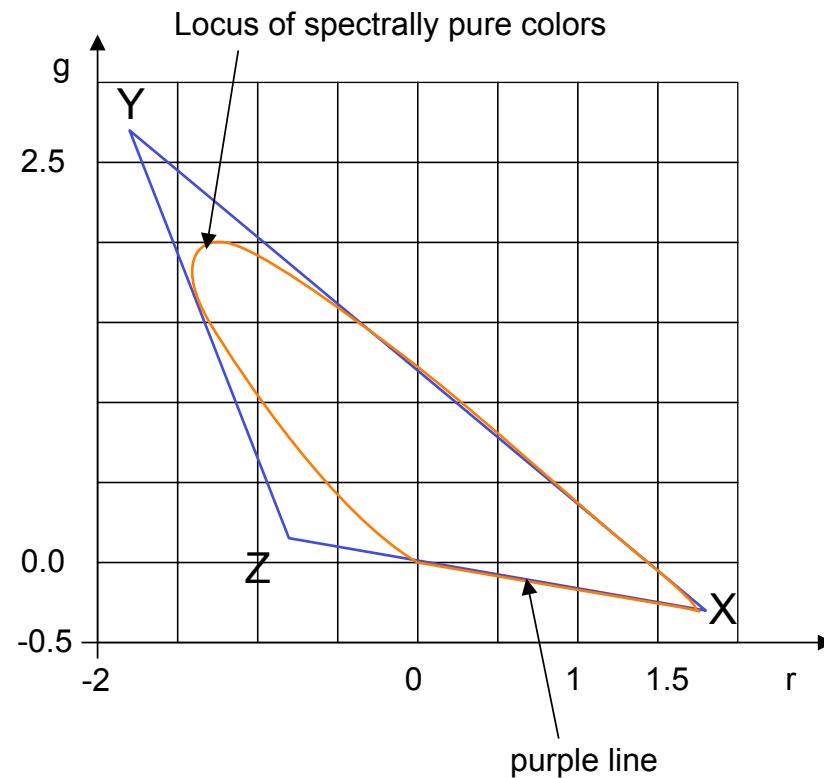
$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$r, g, b$  : chromaticity coordinates

$\bar{r}, \bar{g}, \bar{b}$  color matching functions (tristimulus values of the reference white)



# Chromaticity coordinates

*Chromaticity coordinates*

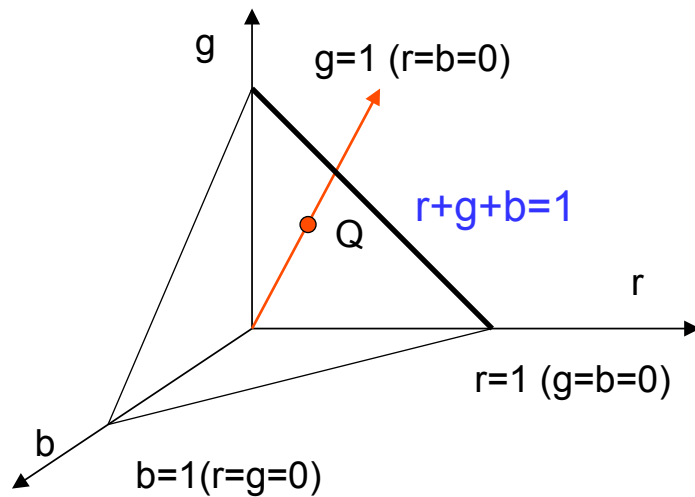
$$r = \frac{R}{R + G + B}$$

$$g = \frac{G}{R + G + B}$$

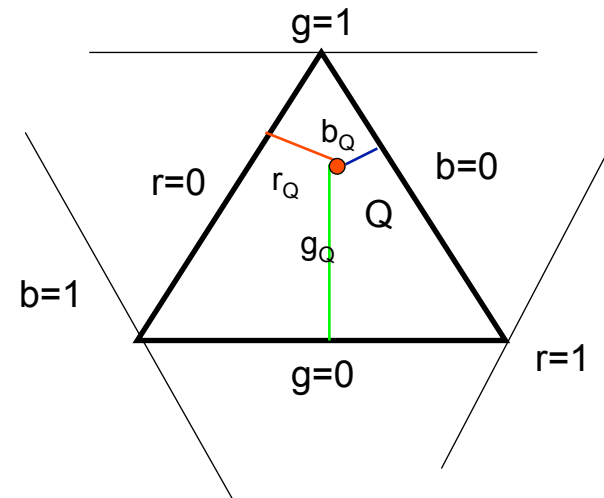
$$b = \frac{B}{R + G + B}$$

$$\Rightarrow r + g + b = 1$$

R,G,B: tristimulus value of the generic color



Maxwell color triangle



$(r,g)$  specify the *hue and saturation* of the color while the information about the luminance is lost

*A step in colorimetry...*

# CIE 1931 Standard Observer

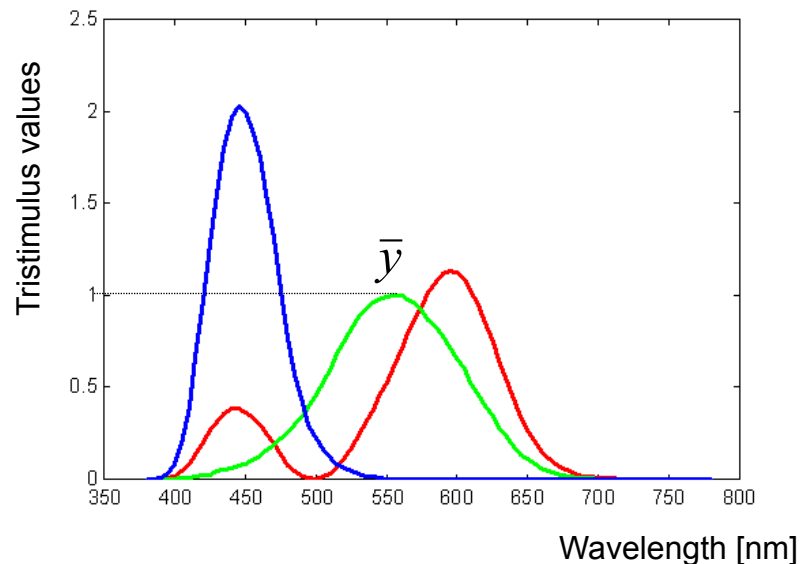
- In colorimetric practice, the main objective is to obtain results valid for the group of normal trichromats. To this end, the color matching properties of an *ideal trichromatic observer* are defined by specifying three independent functions of  $\lambda$  which are identified with the ideal observer CMFs.
- The CIE 1931 SO also embodies the additivity law for brightness ( $V(\lambda)$  photopic luminous efficiency function)
  - For an observer who makes brightness matches that *conform to the additivity law for brightness*, and who also *makes color matches* that are trichromatic in the stronger sense, it can be shown that  $V(\lambda)$  is a combination of the CMFs, provided all the pairs of metameric stimuli are also in brightness match.

For such an observer, it is possible to select from the infinitely many equivalent sets of CMFs one set for which one of the three CMFs, usually taken to be the central one  $\bar{y}$  coincides with  $V(\lambda)$ .

**In this way, the CIE 1931 SO combines both color matching and heterochromatic brightness matching properties in a single quantitative scheme.**

# CIE 1931 Standard Colorimetric Observer

- Standard system for color representation: X,Y,Z tristimulus coordinate system
- Color matching functions  $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$



## Features

- $\lambda=380$  to  $780$  nm,  $\Delta\lambda=5$ nm
- Measured at 2 degrees
- Always non negative
- $\bar{y}$  is a rough approximation of the *brightness* of monochromatic lights of equal size and duration (*Standard photopic luminosity function*  $V(\lambda)$ )
- They cannot be measured by color matching experiments
- Quite inaccurate at low wavelengths

## Improvements

- In 1959 a new set of CIE XYZ coordinates was derived based on the CMFs measured by Stiles&Burch at 10 degrees (CIE 1964 Supplementary Standard Colorimetric Observer).

# Guidelines for the derivation of CIE 1931 SO

- Projective transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

$(r_x, g_x, b_x)$  : coordinates of (1,0,0) as measured in the {r,g,b} system

....

- Need to determine the matrix A of the transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \dots & \\ & & a_{3,3} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \dots & \\ & & a_{3,3} \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1}$$

- This is accomplished by imposing some conditions

# Guidelines for the derivation of CIE 1931 SO

1. The function  $\bar{y}(\lambda)$  must be equal to the luminosity function of the eye  $V(\lambda)$

$$\bar{y}(\lambda) = V(\lambda)$$

this sets a relation among 3 coefficients

$$a_{11} + a_{12} + a_{13} = \text{const.}$$

2. The constant spectrum of white,  $E(\lambda)=1$ , should have equal tristimulus values

$$\sum_{i=1}^N \bar{x}(\lambda_i) = \sum_{i=1}^N \bar{y}(\lambda_i) = \sum_{i=1}^N \bar{z}(\lambda_i)$$

$$\sum_{i=1}^N \bar{x}(\lambda_i) = a_{11} \sum_{i=1}^N \bar{r}(\lambda_i) + a_{12} \sum_{i=1}^N \bar{g}(\lambda_i) + a_{13} \sum_{i=1}^N \bar{b}(\lambda_i)$$

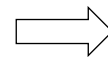
but

$$\sum_{i=1}^N \bar{r}(\lambda_i) = \sum_{i=1}^N \bar{g}(\lambda_i) = \sum_{i=1}^N \bar{b}(\lambda_i) = S$$

thus

$$\sum_{i=1}^N \bar{x}(\lambda_i) = (a_{11} + a_{12} + a_{13})S$$

.....

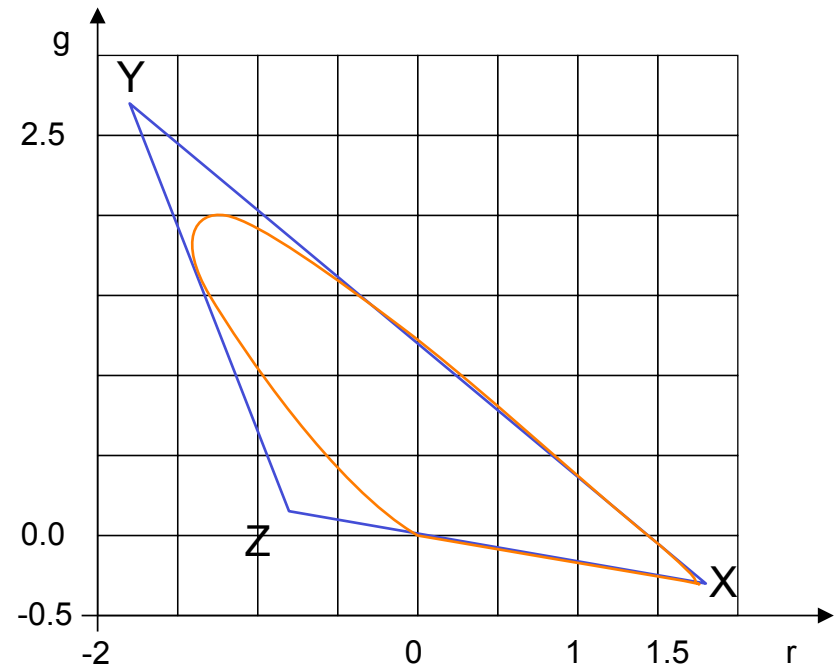


$$\begin{aligned} (a_{11} + a_{12} + a_{13}) &= \\ &= (a_{21} + a_{22} + a_{23}) = \\ &= (a_{31} + a_{32} + a_{33}) = \sigma \end{aligned}$$

[Ref: Color vision and colorimetry, D. Malacara]

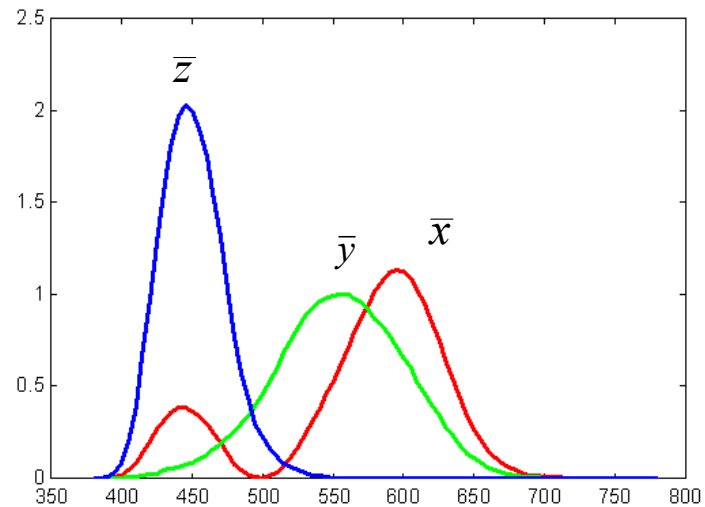
# Guidelines for the derivation of CIE 1931 SO

3. The line joining X and Y be tangent to the curve on the red side
  - In this way, a linear combination of X and Y is sufficient to describe those colors without any Z
  - This introduces other conditions on  $a_{31}$ ,  $a_{32}$ ,  $a_{33}$  and sets their values
4. No values of  $\bar{x}(\lambda)$  is negative
  - This adds a condition relating  $a_{11}$ ,  $a_{12}$  and  $a_{13}$ , whose sum must be equal to known constant  $\sigma$ . This leaves one degree of freedom that is used to set the area of the XYZ triangle at its minimum



## From rgb to xyz

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} 0.41846 & -0.1586 & -0.08283 \\ -0.09117 & 0.25243 & 0.01571 \\ 0.00092 & -0.00255 & 0.17860 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



# rgb2xyz

- Chromaticity coordinates

$$x = \frac{0.49r + 0.31g + 0.2b}{0.66697r + 1.1324g + 1.20063b}$$
$$y = \frac{0.17697r + 0.81240g + 0.01063b}{0.66697r + 1.1324g + 1.20063b}$$
$$z = \frac{0.0r + 0.01g + 0.99b}{0.66697r + 1.1324g + 1.20063b}$$

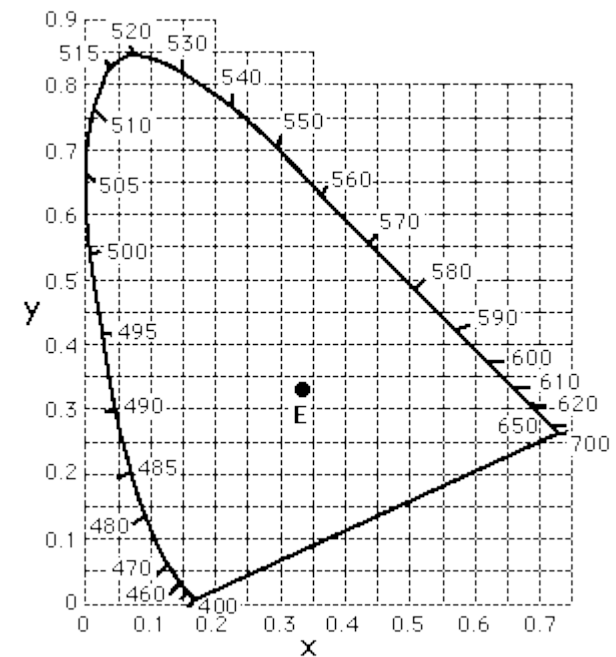
- Tristimulus values

$$X = \frac{x}{y}V \quad Y = V \quad Z = \frac{z}{y}V$$

- CMF

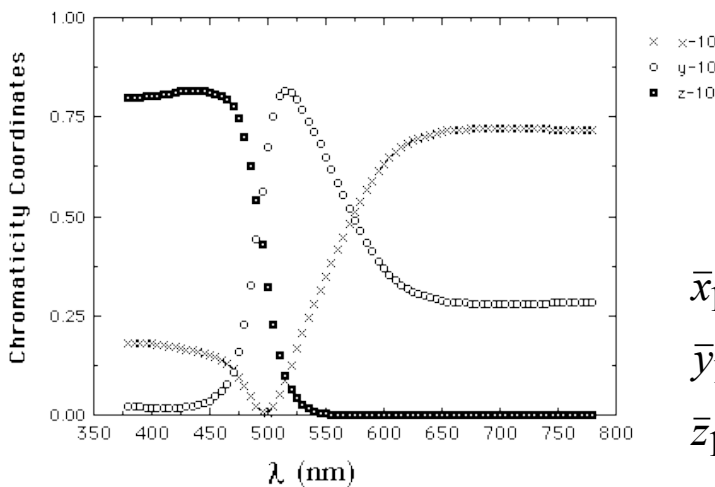
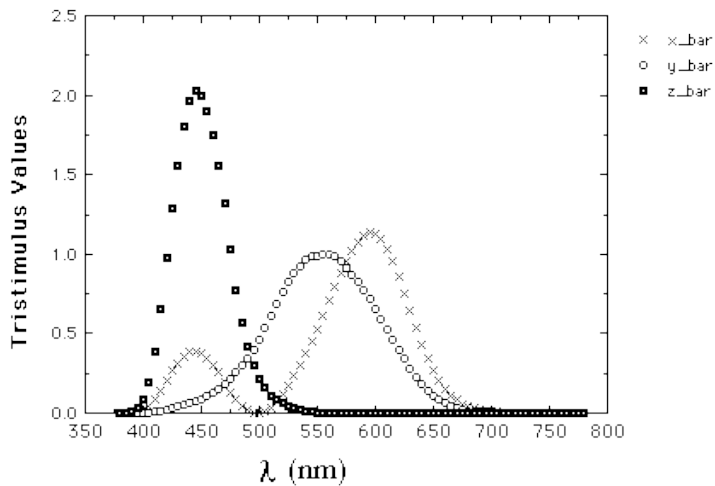
$$\bar{x}(\lambda) = \frac{x(\lambda)}{y(\lambda)}V(\lambda)$$
$$\bar{y}(\lambda) = V(\lambda)$$
$$\bar{z}(\lambda) = \frac{z(\lambda)}{y(\lambda)}V(\lambda)$$

(x,y) chromaticity diagram



$$x_E = y_E = \frac{1}{3}$$

# CIE 1964 SO



## Features

- 10 degrees field
- Extended set of wavelengths (390 to 830 nm)
- r,g,b CMFs obtained directly from the observations
  - Measures of the radiant power of each monochromatic test stimulus
- High illumination intensity
  - To minimize rods intrusion
- Data extrapolated at 1nm resolution

$$\bar{x}_{10}(\lambda) = 0.341080\bar{r}_{10}(\lambda) + 0.189145\bar{g}_{10}(\lambda) + 0.387529\bar{b}_{10}(\lambda)$$

$$\bar{y}_{10}(\lambda) = 0.139058\bar{r}_{10}(\lambda) + 0.837460\bar{g}_{10}(\lambda) + 0.073316\bar{b}_{10}(\lambda)$$

$$\bar{z}_{10}(\lambda) = 0.0\bar{r}_{10}(\lambda) + 0.039553\bar{g}_{10}(\lambda) + 2.026200\bar{b}_{10}(\lambda)$$

# CIE Chromaticity Coordinates

- (X,Y,Z) tristimulus values

$$X = \int P_{\lambda} \bar{x}(\lambda) d\lambda$$

$$Y = \int P_{\lambda} \bar{y}(\lambda) d\lambda$$

$$Z = \int P_{\lambda} \bar{z}(\lambda) d\lambda$$

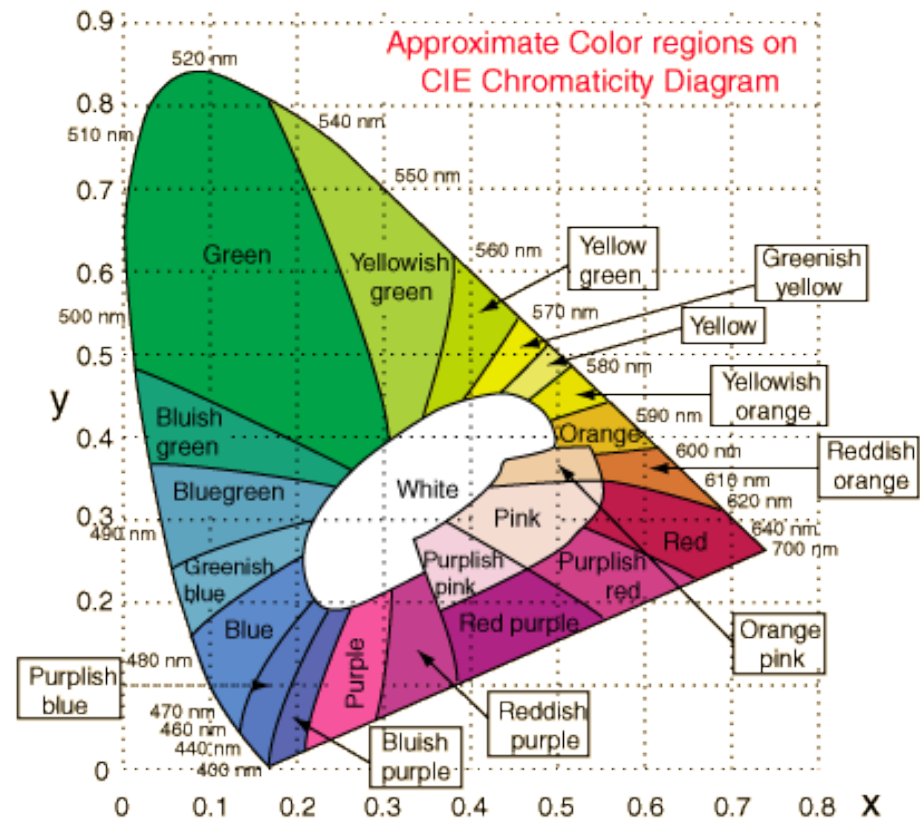
- Chromaticity coordinates

$$x(\lambda) = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

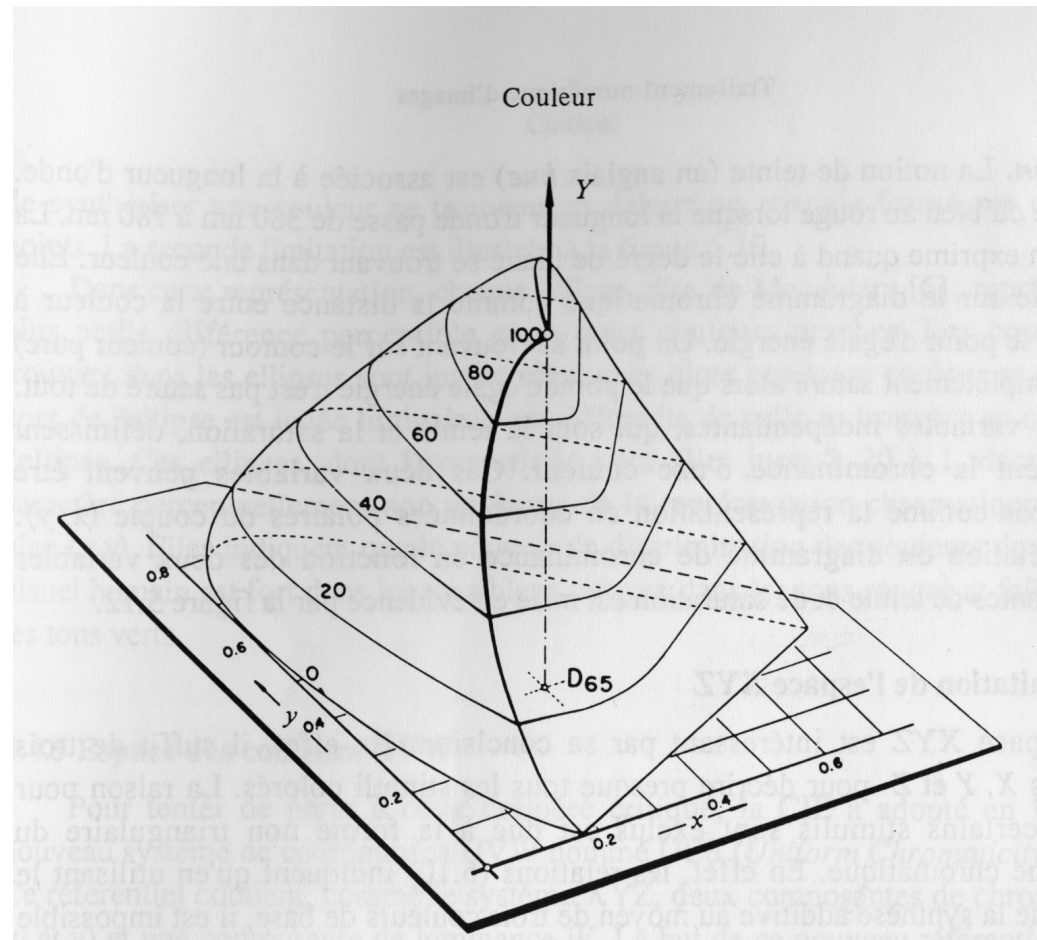
$$y(\lambda) = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

$$z(\lambda) = \frac{\bar{z}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

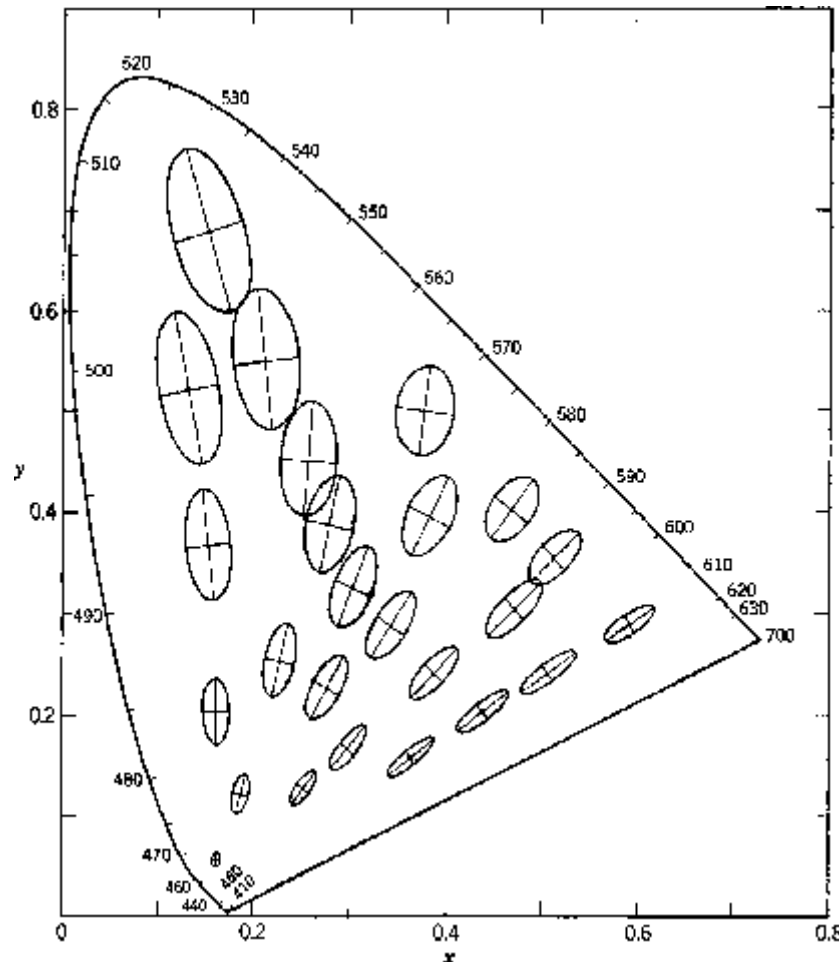
$$x(\lambda) + y(\lambda) + z(\lambda) = 1 \quad \text{x - y chromaticity diagram}$$



xyY



# Mac Adams' ellipses



The ellipses represent a **constant perceptual color stimulus**, at a constant luminance, at various positions and in various directions, in the x,y diagram.

The areas of the ellipses vary greatly.

This means that the XYZ colorspace (as the RGB color space) is *not perceptually uniform*.

To avoid this nonuniformity, CIE recommended a new CIE 1964 UCS (Uniform-Chromaticity Scale) diagram, to be used with constant luminance levels.

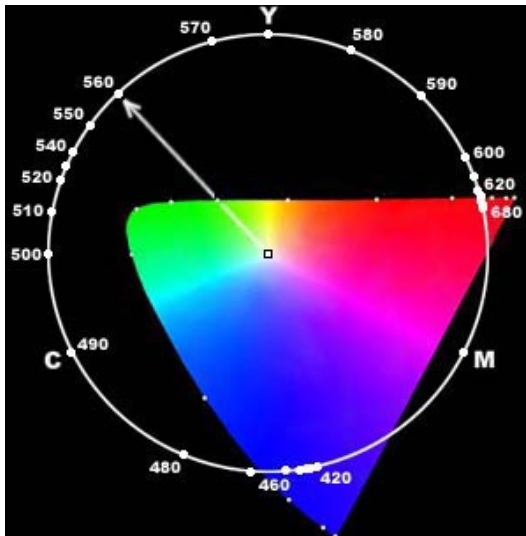
In perceptually uniform colorspace, the size of the MacAdams' ellipses are more uniform and the eccentricity is lower.

# Perceptually uniform Colorspaces

- CIE 1960 Luv colorspace
  - reversible transformation

$$u = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v = \frac{6Y}{X + 15Y + 3Z} = \frac{6x}{-2x + 12y + 3}$$



- CIE 1976 L\*u\*v\* (CIELUV)

$$u' = u$$

$$v' = 1.5v$$

$L^*$ : *perceived* lightness

$$L^* = 116 \left( \frac{Y}{Y_n} \right)^{1/3} - 16$$

$$u^* = 13L^*(u' - u'_n)$$

$$v^* = 13L^*(v' - v'_n)$$

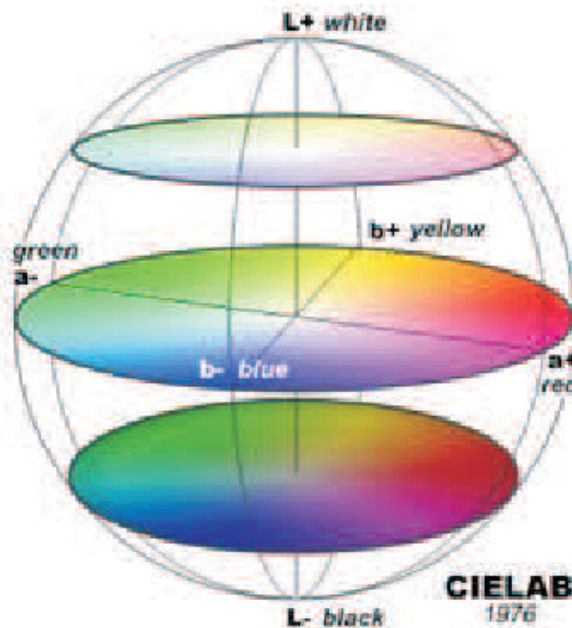
$$u' = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2 + 12y + 3}$$

$$v' = \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3}$$

$u'_n, v'_n$ : reference white

# Perceptually uniform Color models

CIE 1976 L\*a\*b\* (CIELAB)



$X_n, Y_n, Z_n$  : reference white

Tristimulus values for a nominally white object-color stimulus. Usually, it corresponds to the spectral radiance power of one of the CIE standard illuminants (as D65 or A), reflected into the observer's eye by a perfect reflecting diffuser. Under these conditions,  $X_n, Y_n, Z_n$  are the tristimulus values of the standard illuminant with  $Y_n=100$ .

For:  $\frac{Y}{Y_n}, \frac{X}{X_n}, \frac{Z}{Z_n} \geq 0.01$

$$L^* = 116(Y/Y_n)^{1/3} - 16$$

$$a^* = 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}]$$

$$b^* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}]$$

otherwise

$$L^* = 116 \left[ f\left(\frac{Y}{Y_n}\right) - \frac{16}{116} \right]$$

$$a^* = 500 \left[ f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right]$$

$$b^* = 200 \left[ f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right]$$

$$f\left(\frac{Y}{Y_n}\right) = \begin{cases} \left(\frac{Y}{Y_n}\right)^{1/3} & \text{for } \frac{Y}{Y_n} > 0.008856 \\ 7.787 \frac{Y}{Y_n} + \frac{16}{116} & \text{for } \frac{Y}{Y_n} \leq 0.008856 \end{cases}$$

Hint: the diffuse light depends on both the physical properties of the surface and the illuminant

# Perceptual correlates

- Color difference formula

$$\Delta E^*_{u,v} = \left[ (\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2 \right]^{1/2}$$

- Perceptual correlates

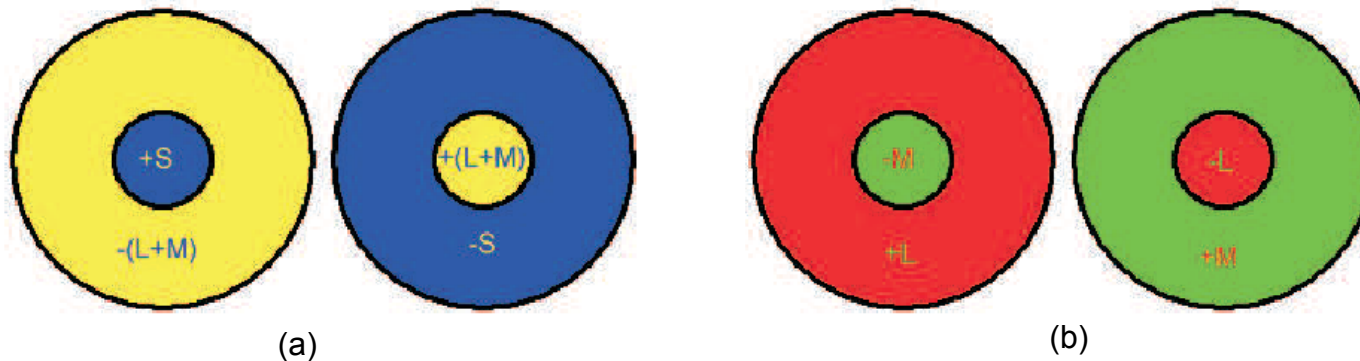
$L^*$ : lightness

$C^*_{u,v} = \left[ (u^*)^2 + (v^*)^2 \right]^{1/2}$ : chroma

$s^*_{u,v} = \frac{C^*_{u,v}}{L^*}$ : saturation

# Opponent color models

- Underlying model: *opponent channels*

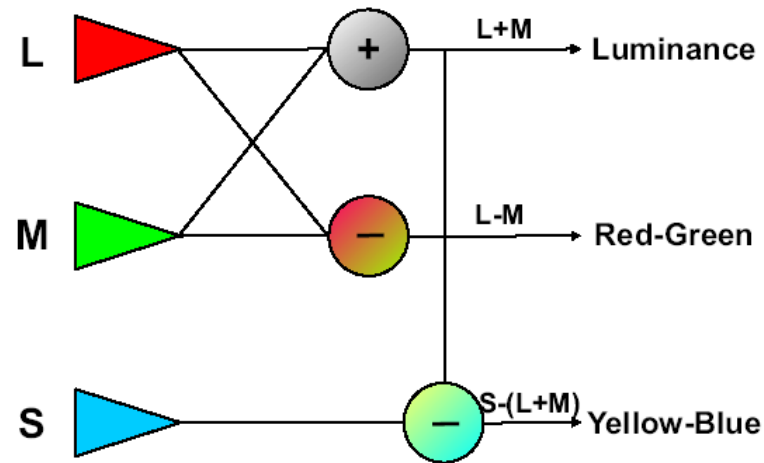


Example of typical center-surround antagonistic receptive fields: (a) on-center yellow-blue receptive fields; (b) on-center red-green receptive fields.

Because of the fact that the L, M and S cones have different spectral sensitivities, are in different numbers and have different spatial distributions across the retina, the respective receptive fields have quite different properties.

Experimental evidence: color after-image, non existence of colors like “greenish-red” or “yellowish-blue”

# Opponent color channels



Cone interconnections in the retina leading to opponent color channels

As a convenient simplification, the existence of three types of color receptive fields is assumed, which are called *opponent channels*.

The black-white or *achromatic* channel results from the sum of the signals coming from L and M cones ( $L + M$ ). It has the highest spatial resolution.

The *red-green* channel is mainly the result of the M cones signals being subtracted from those of the L cones ( $L - M$ ). Its spatial resolution is slightly lower than that of the achromatic channel ( $L + M - S$ ).

Finally the *yellow-blue* channel results from the addition of L and M and subtraction of S cone signals. It has the lowest spatial resolution.

# Additive Color Mixing with CIE

The result of adding two colors of light can be worked out as a weighted average of the CIE chromaticity coordinates for the two colors. The weighting factors involve the brightness parameter  $Y$ . If the coordinates of the two colors are

$(x_1, y_1)$  with brightness  $Y_1$

$(x_2, y_2)$  with brightness  $Y_2$

then the additive mixture color coordinates are

$$x_3 = \frac{Y_1}{Y_1 + Y_2} x_1 + \frac{Y_2}{Y_1 + Y_2} x_2$$
$$y_3 = \frac{Y_1}{Y_1 + Y_2} y_1 + \frac{Y_2}{Y_1 + Y_2} y_2$$

This linear procedure is valid only if the colors are relatively close to each other in value.

# Color models

# Color models

- A color model is a 3D unique representation of a color
- There are different color models and the use of one over the other is problem oriented. For instance
  - RGB color model is used in hardware applications like PC monitors, cameras and scanners
  - CMY color model is used in color printers
  - YIQ model in television broadcast
  - In color image manipulation the two models widely used are HSI and HSV.

# Color models

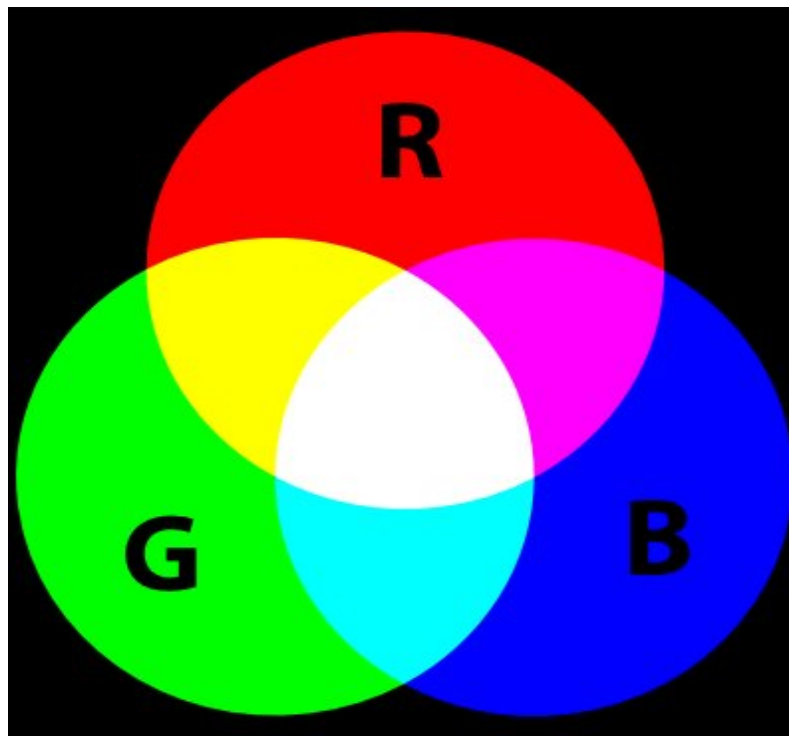
- Colorimetric color models
  - Based on the principles of trichromacy
  - Allow to predict if two colors match in appearance in given observation conditions
  - CIE XYZ
  - Perceptually uniform color models (CIELAB, CIELUV)
- User-oriented color models
  - Emphasize the intuitive color notions of brightness, hue and saturation
    - HSV (Hue, saturation, Value)
    - HSI (Hue, Saturation, Intensity)
    - HSL (Hue, Saturation, Lightness)

# Color models

- Device-oriented color models
  - The color representation depends on the device.
- Concerns both acquisition and display devices
  - Acquisition
    - The value of the color numerical descriptors depend on the spectral sensitivity of the camera sensors
  - Display
    - A color with given numerical descriptors appears different if displayed on another device or if the set-up changes
    - In RGB for instance, the R,G and B components depend on the chosen red, green and blue primaries as well as on the reference white
    - Amounts of ink expressed in CMYK or digitized video voltages expressed in RGB
  - RGB, Y' CbCr, Y' UV, CMY, CMYK
  - Towards device independence: sRGB

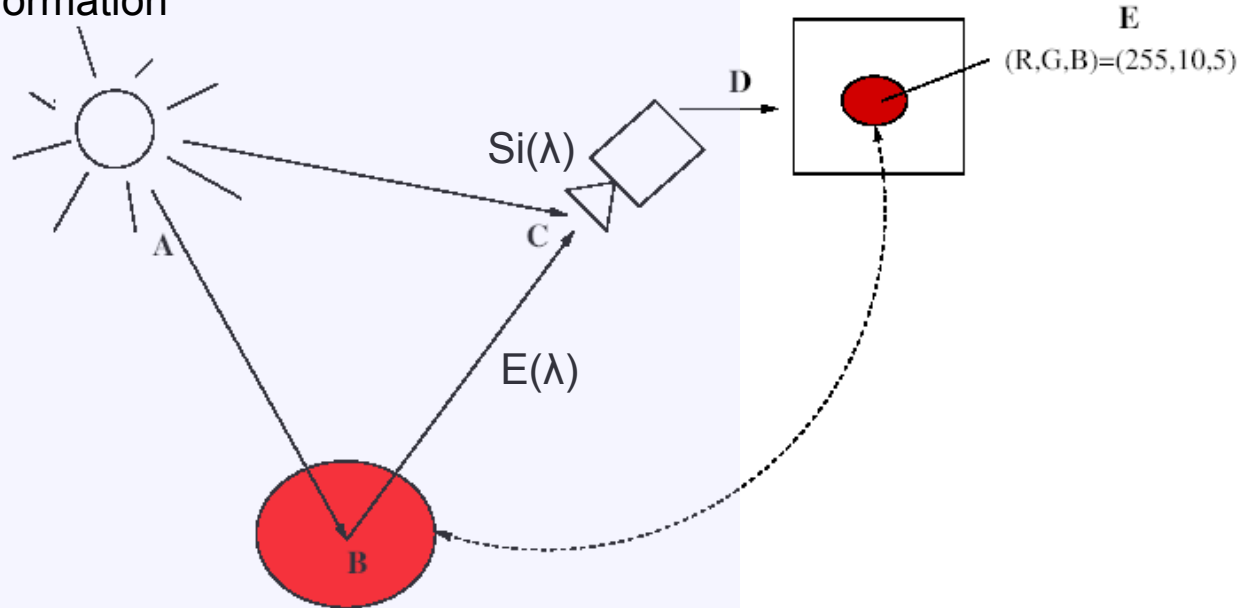
# RGB color model

- Additive color model
  - The additive reproduction process usually uses red, green and blue light to produce the other colors



# RGB color model

Image formation



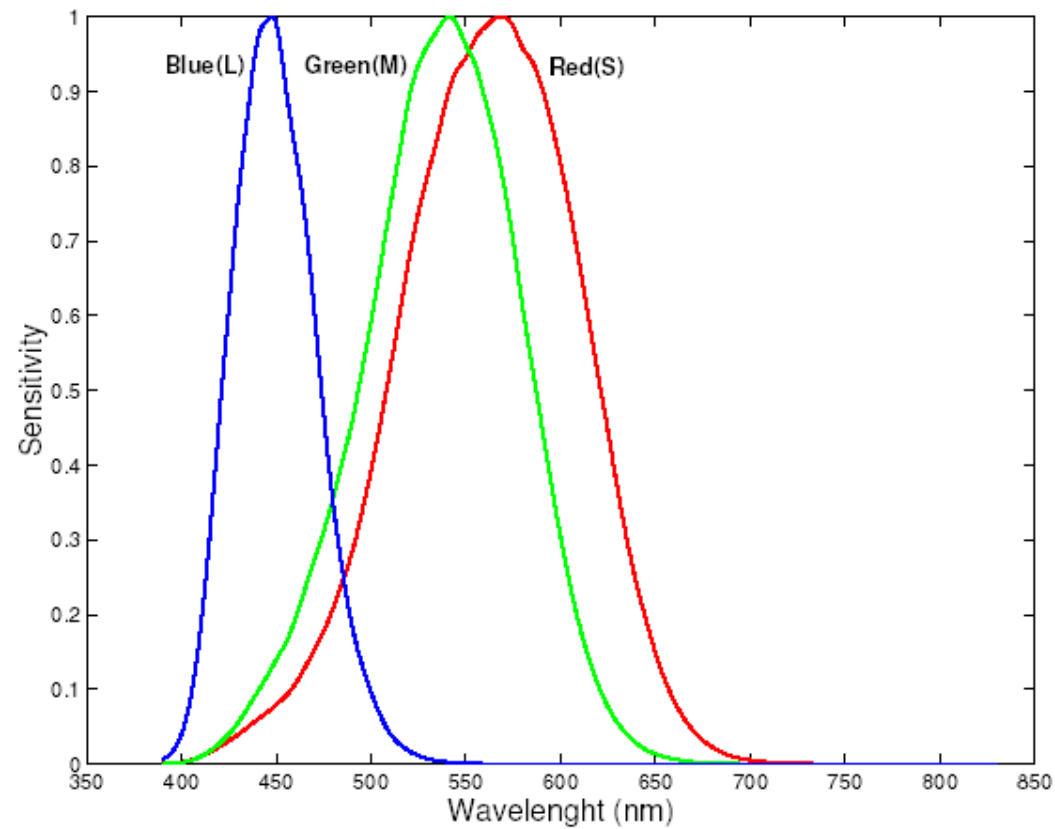
$$C_i = \int_{\lambda} E(\lambda) S_i(\lambda) d\lambda$$

$S_i(\lambda)$ : sensitivity of the  $i^{\text{th}}$  sensor

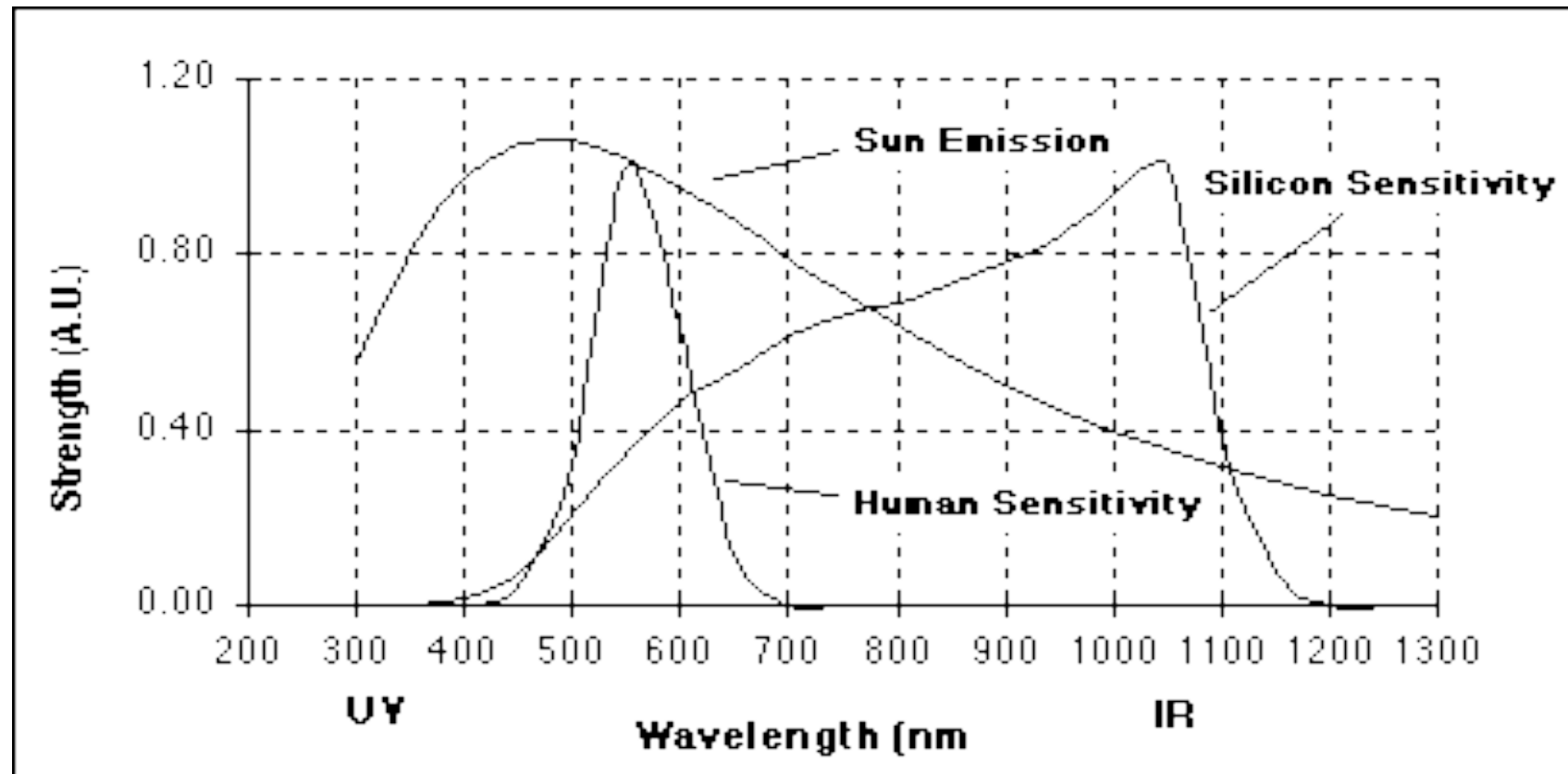
$E(\lambda)$ : Spectral Power Distribution (SPD) of the diffused light

# Spectral sensitivities

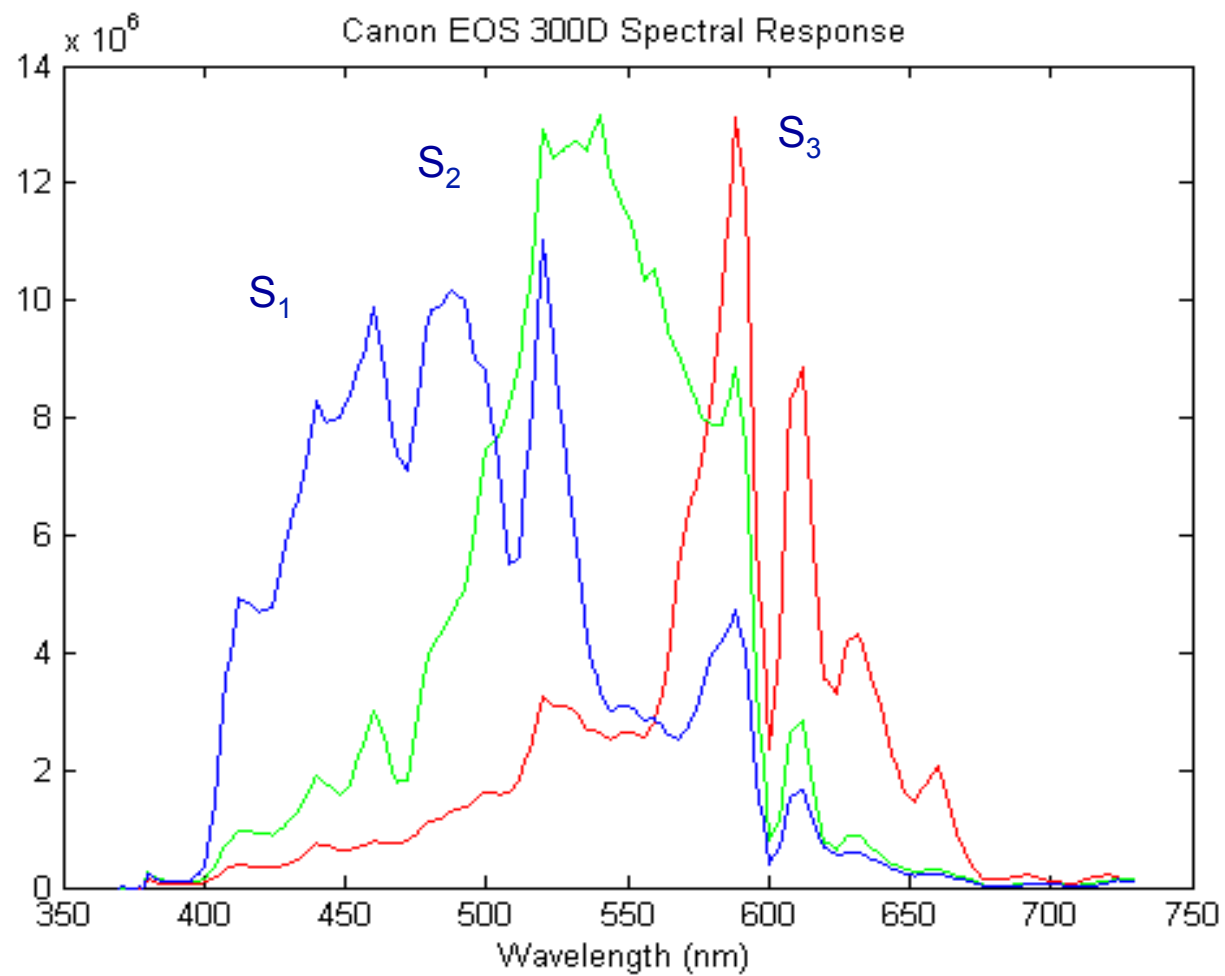
Target: spectral sensitivity of the eye



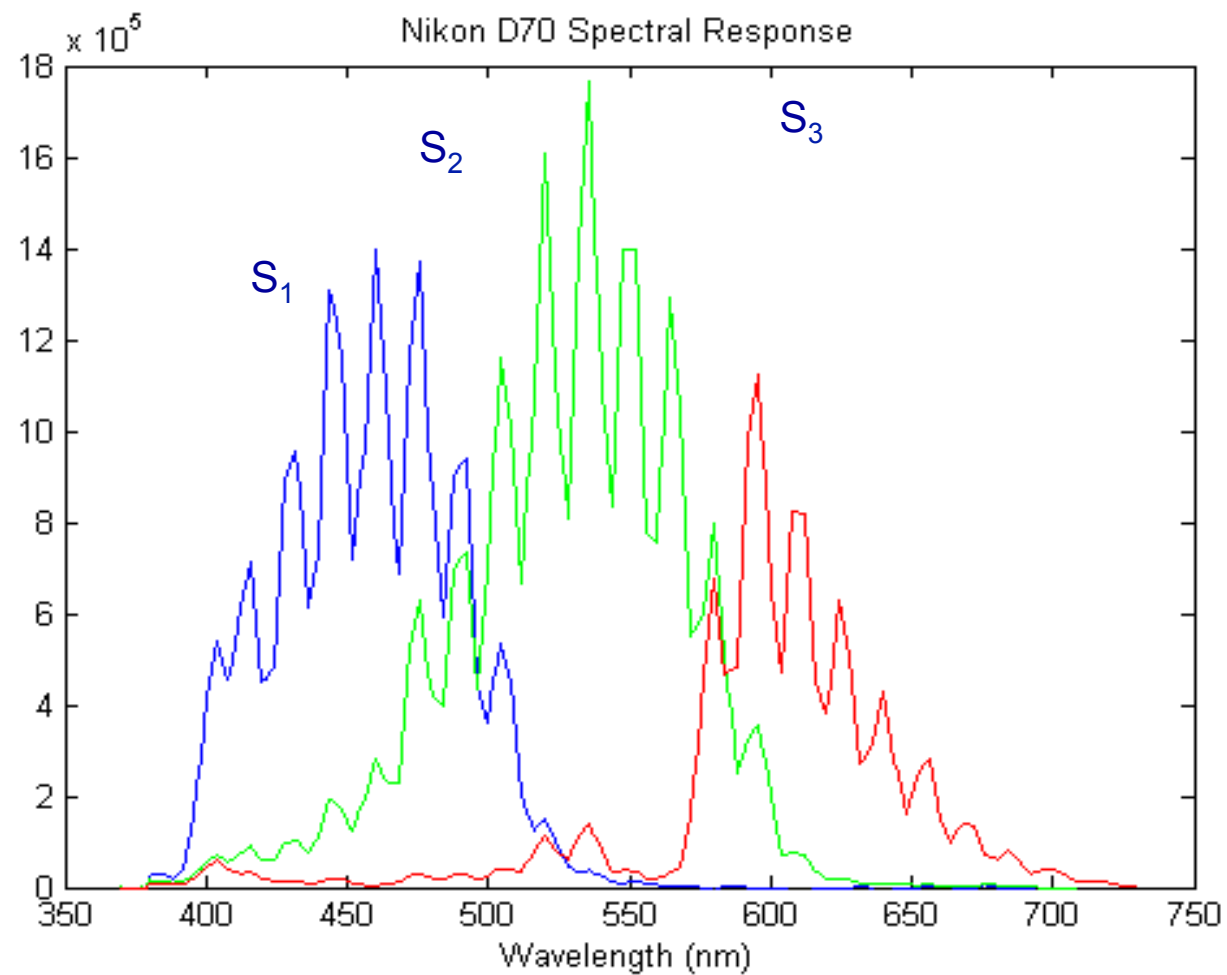
## Broad range sensitivity



# Sensor sensitivity: Ex. 1



## Spectral sensitivity: Ex. 2



# RGB model

$$C_i = \int_{\lambda} P(\lambda) S_i(\lambda) d\lambda$$

$P(\lambda)$ : PSD (Power Spectral Density of the incident light)

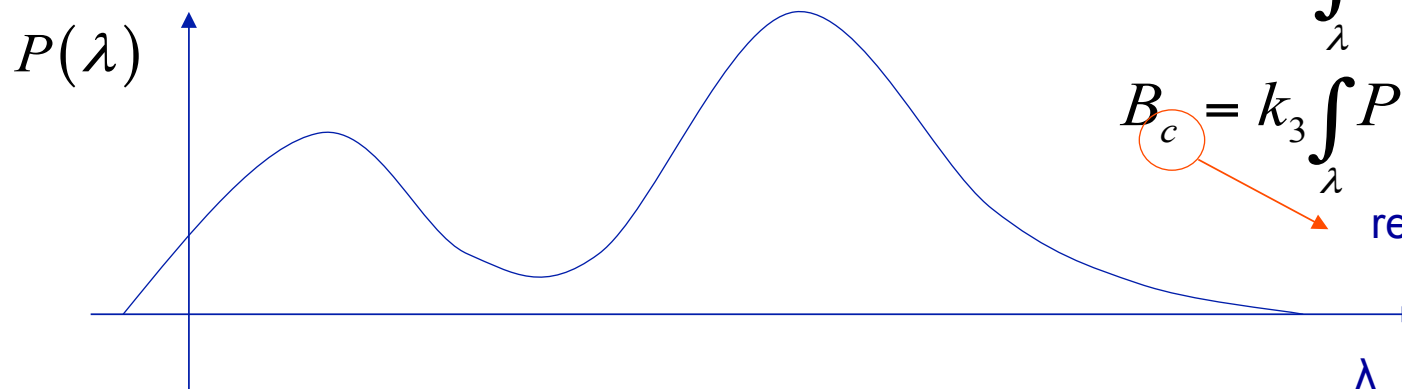
$S_i(\lambda)$ : spectral sensitivity of the "red", "green" and "blue" sensors

$$R_c = k_1 \int_{\lambda} P(\lambda) S_1(\lambda) d\lambda$$

$$G_c = k_2 \int_{\lambda} P(\lambda) S_2(\lambda) d\lambda$$

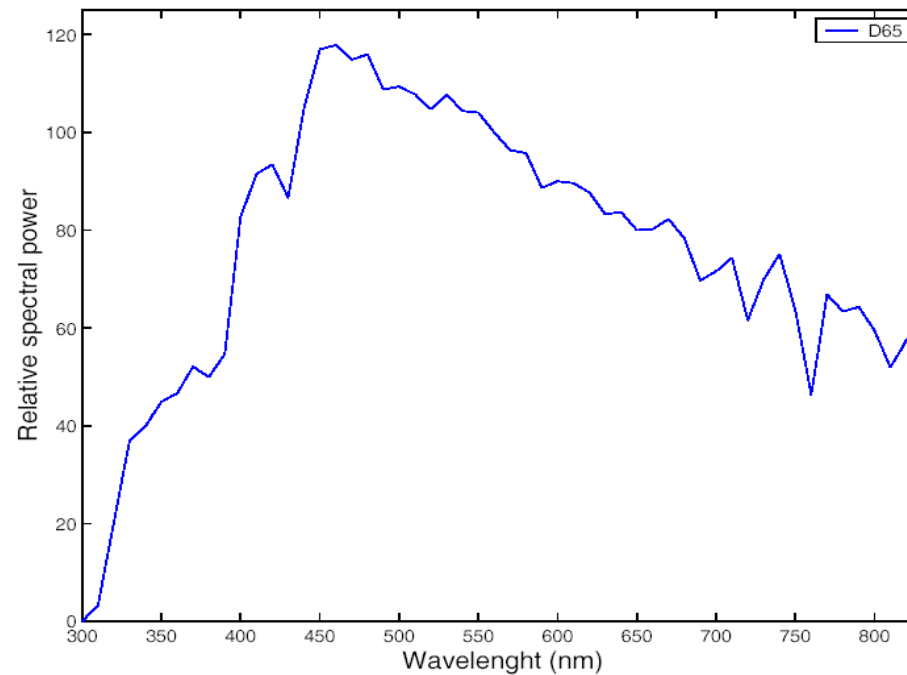
$$B_c = k_3 \int_{\lambda} P(\lambda) S_3(\lambda) d\lambda$$

relative to the camera



# Reference white

- The reference white is the light source that is chosen to approximate the white light
  - D65, D50



## Reference white

- The reference white,  $E(\lambda)$ , will be given the maximum tristimulus values in all channels ( $R=G=B=255$ )
- The numerical values of the R,G,B coordinates of a generic PSD  $P(\lambda)$  will depend on the choice of  $E(\lambda)$

$$R_{Ec} = k_1 \int_{\lambda} E(\lambda) S_1(\lambda) d\lambda = 255$$

$$G_{Ec} = k_2 \int_{\lambda} E(\lambda) S_2(\lambda) d\lambda = 255 \rightarrow k_1, k_2, k_3$$

$$B_{Ec} = k_3 \int_{\lambda} E(\lambda) S_3(\lambda) d\lambda = 255$$

# RGB tristimulus values

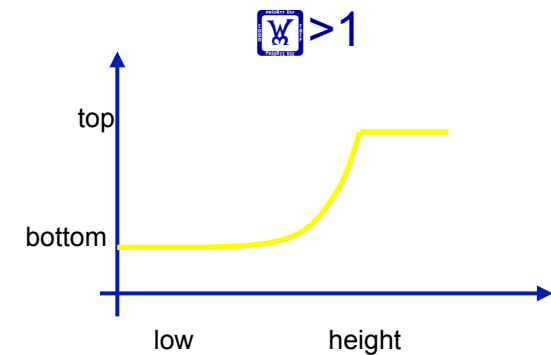
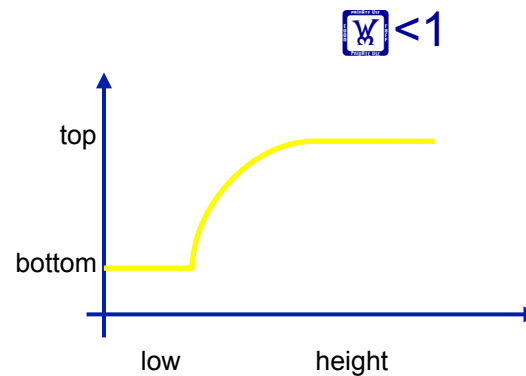
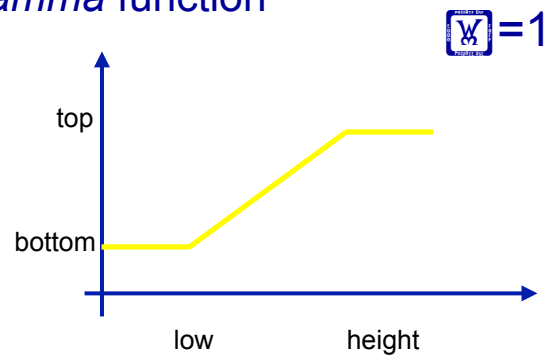
- The R,G,B coordinates does not have an absolute meaning, as their value depend on
  - The spectral sensitivity of the sensors that are used in the capture device
  - The reference white
- Thus, R,G,B values of the same physical stimulus (image) acquired with different cameras are different, in general
- Gamut: set of colors that is “manageable” by the device
  - Acquisition devices: set of colors that are represented by the device
  - → gamut mapping

# RGB model

- Similar considerations apply to rendering devices: the rendering of a color with given tristimulus coordinates (R,G,B) will depend on
  - The spectral responses of the emitters
    - phosphors for a CRT
    - color filters in a LCD
  - The calibration of the device
    - As for the acquisition devices, the color corresponding to the rendered white must be set
    - To define the entire gamut for a monitor, you only need mark the points on the diagram that represent the colors the monitor actually produces. You can measure these colors with either a colorimeter or a photospectrometer along with software that ensures the monitor is showing 100 percent red for the red measurement, 100 percent green for the green measurement, and 100 percent blue for the blue measurement.
  - The linearity of the monitor transfer function (gamma)

# Gamma function

Gamma function

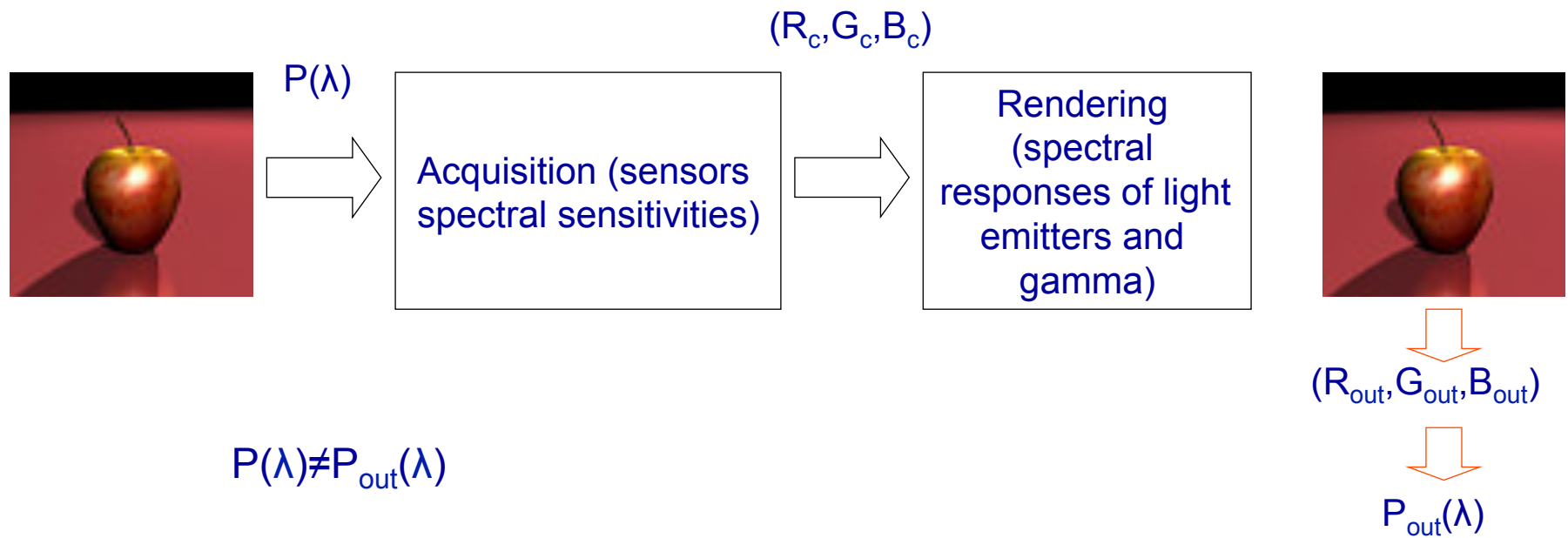
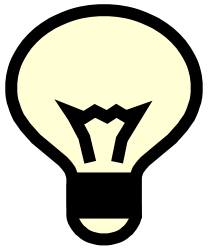


- Typical CRT monitors: gamma=2.2
- The non-linearity of the monitor can be compensated by non-uniform scaling of the RGB coordinates at input (*RGB linearization*)
- This led to the definition of the sRGB color model

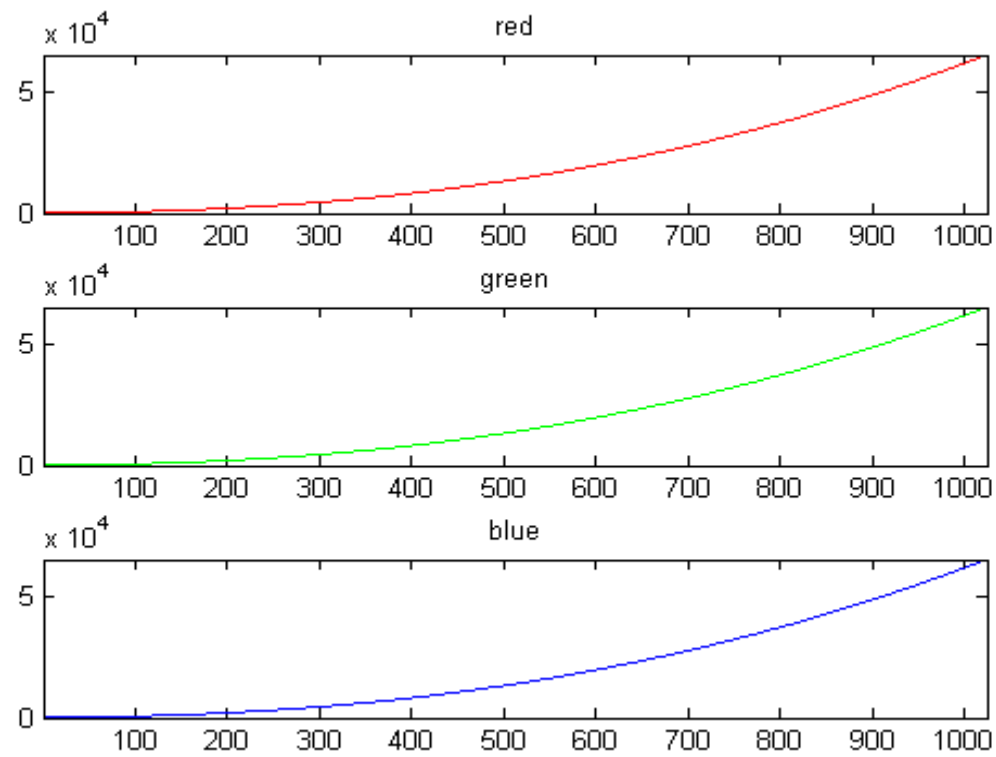
# RGB model: rendering ex.

- The RGB values depend on the phosphores coordinates
  - Different for the different reproduction media (CRT, television displays)
  - Example:
    - Red phosphore:  $x=0.68$ ,  $y=0.32$
    - Green phosphore:  $x=0.28$ ,  $y=0.60$
    - Blue phosphore:  $x=0.15$ ,  $y=0.07$
  - Given the  $x,y$  coordinates of the phosphores, the reference white point and the illuminant (D65), the RGB coordinates can be calculated
  - Calibration
    - the  $R=G=B=100$  points must match in appearance with the white color as observed by 10 deg observer under the D65 illuminant
    - The brightness of the three phosphores is non linear with the RGB values. A suitable correction factor must be applied (Gamma correction)

# RGB model

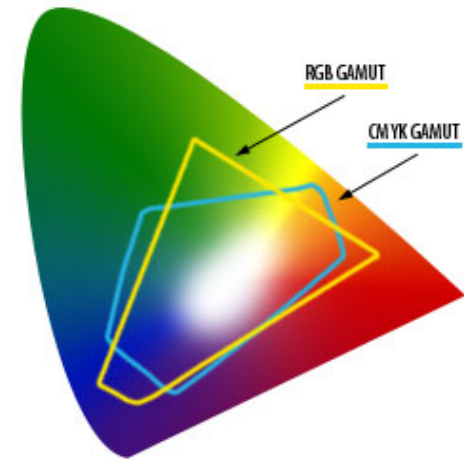


# sRGB

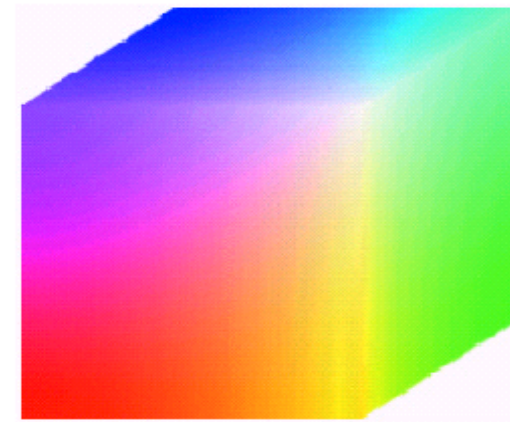
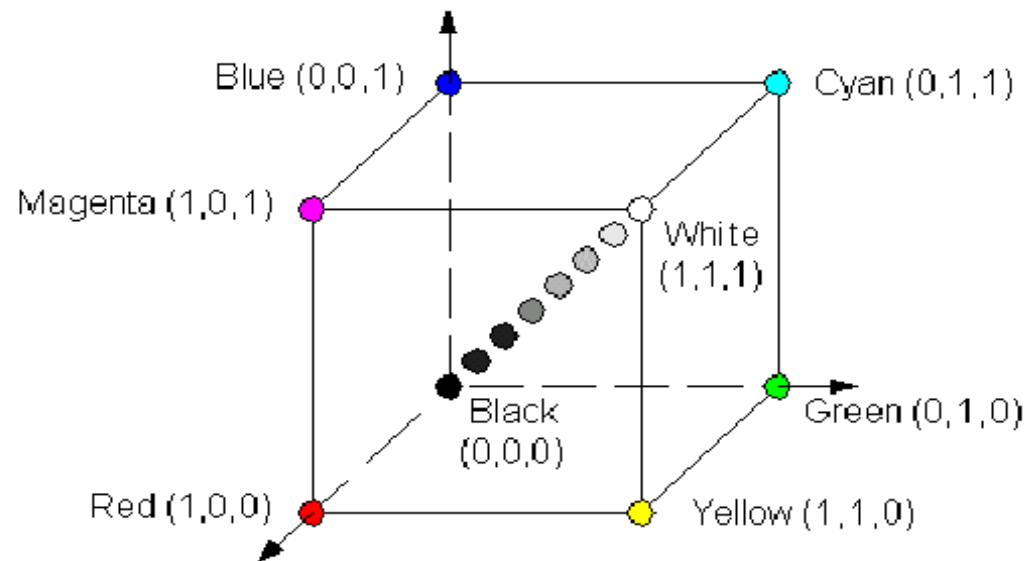


# sRGB

- RGB specification that is based on the average performance of personal computer displays. This solution is supported by the following observations:
  - Most computer monitors are similar in their key color characteristics - the phosphor chromaticities (primaries) and transfer function → RGB primaries,  $\gamma$  value
  - Reference viewing environments are defined for standard RGB
    - Luminance level 80 cd/m<sup>2</sup>
    - Illuminant White  $x = 0.3127$ ,  $y = 0.3291$  (D65)
    - Image surround 20% reflectance
    - Encoding Ambient Illuminance Level 64 lux
    - Encoding Ambient White Point  $x = 0.3457$ ,  $y = 0.3585$  (D50)
    - Encoding Viewing Flare 1.0%
    - Typical Ambient Illuminance Level 200 lux
    - Typical Ambient White Point  $x = 0.3457$ ,  $y = 0.3585$  (D50)
    - Typical Viewing Flare 5.0%

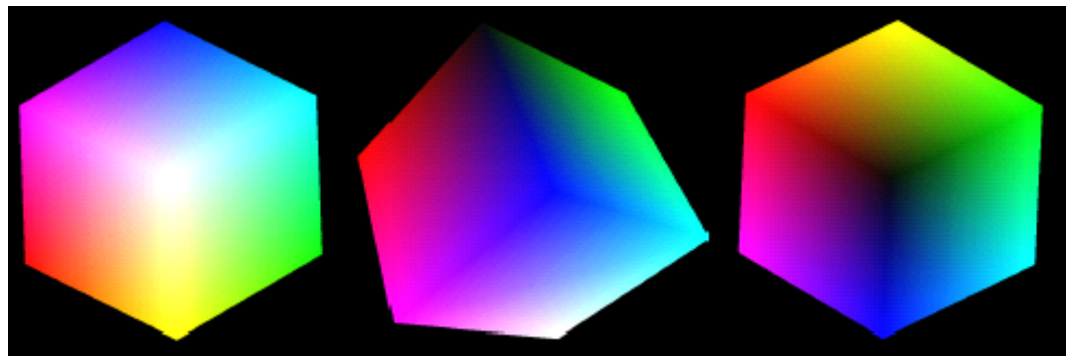


# RGB model

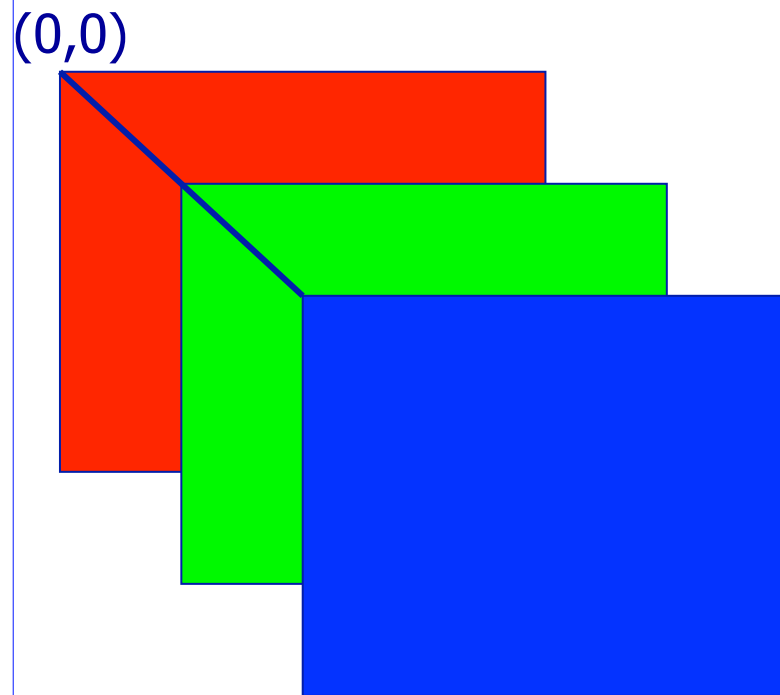


# RGB model

- Normalized values in  $[0,1]$  (chromaticity coordinates) may be convenient for some applications
- For a given device, the set of manageable colors lies inside the RGB cube
  - The R,G,B values must be represented as CIE coordinates



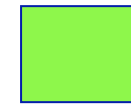
# RGB model



A single pixel consists of three components.

|     |     |    |
|-----|-----|----|
| 128 | 251 | 60 |
|-----|-----|----|

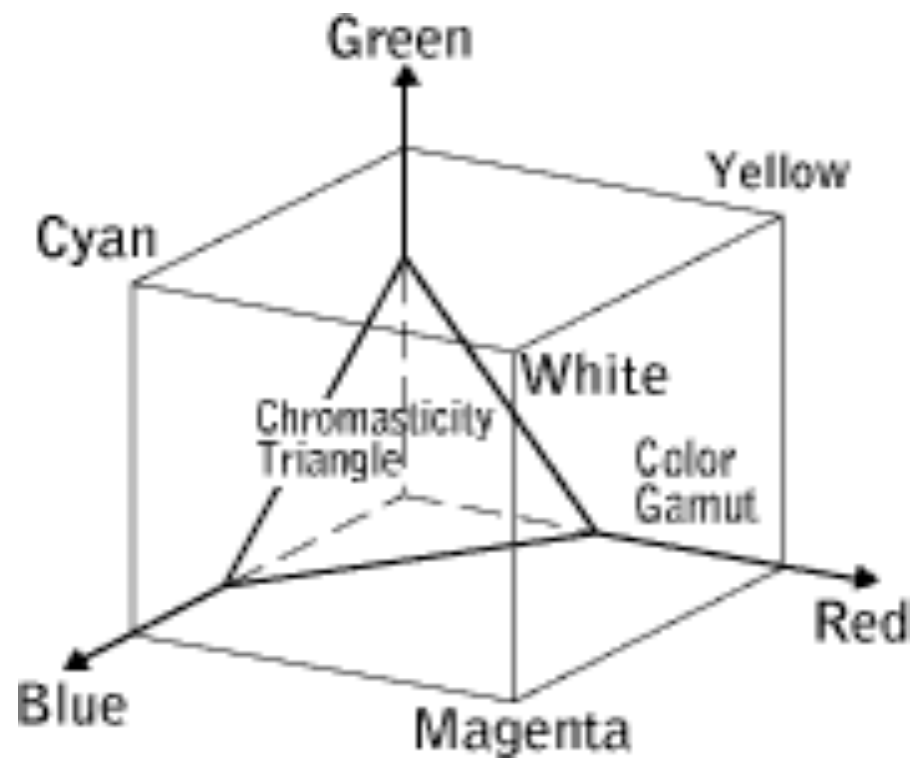
=



Final pixel in the image

If R,G, and B are represented with 8 bits (24-bit RGB image), the total number of colors is  $256^3=16,777,216$

# RGB Color Space



## Example RGB



Original Image



R-Component



G-Component



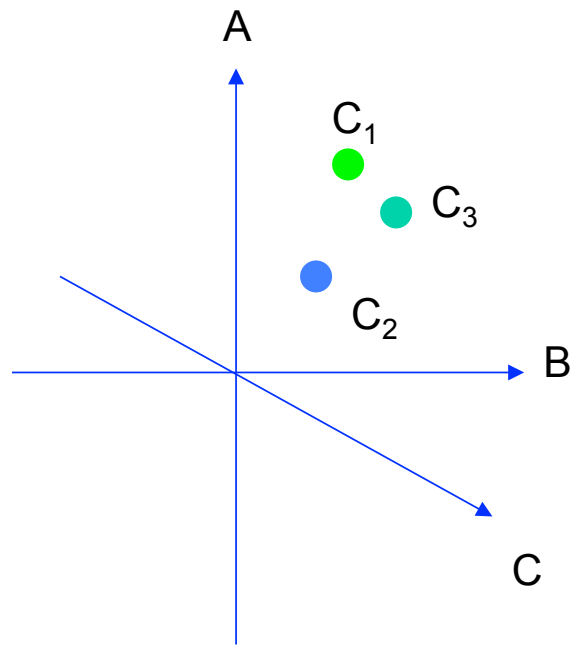
B-Component

# Uniform color scales

Attributes: hue, saturation (chroma), brightness (lightness)

- Brightness
  - The attribute of a visual sensation according to which a visual stimulus appears to be more or less “intense”, or to emit more or less light
  - Ranges from “bright” to “dim”
- Lightness
  - The attribute of a visual sensation according to which a visual stimulus appears to be more or less “intense”, or to emit more or less light in proportion to that emitted by a similarly illuminated area perceived as “white”
  - Relative brightness
  - Ranges from “light” to “dark”
- Colorfulness
  - The attribute of a visual sensation according to which a visual stimulus appears to be more or less “chromatic”
- Chroma
  - The attribute of a visual sensation which permits a judgment to be made of the degree to which a chromatic stimulus differs from an “achromatic” stimulus of the same brightness
- Saturation
  - The attribute of a visual sensation which permits a judgment to be made of the degree to which a chromatic stimulus differs from an “achromatic” stimulus regardless of their brightness
- Chroma and saturation are often considered as equivalent

# Perceptually uniform color models



Perceptual distance:

- Scaling the perceptual similarity among color samples
  - C<sub>1</sub> is most similar to C<sub>3</sub> than it is to C<sub>2</sub>

Measurable distance

- Metric in the color space
  - Euclidean distance among the color samples

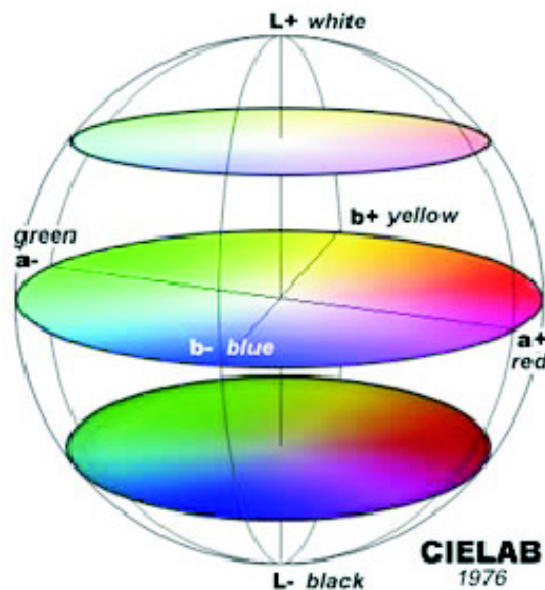
Does the perceptual distance match with the measurable distance among colors?

$$d(C_1C_3) \stackrel{?}{\leq} d(C_1C_2)$$

**Color models whose metric is representative of the perceptual distance are *perceptually uniform***

# Perceptually uniform Color models: Lab

CIE 1976 L\*a\*b\* (CIELAB)



$X_n, Y_n, Z_n$  : reference white

Tristimulus values for a nominally white object-color stimulus. Usually, it corresponds to the spectral radiance power of one of the CIE standard illuminants (as D65 or A), reflected into the observer's eye by a perfect reflecting diffuser. Under these conditions,  $X_n, Y_n, Z_n$  are the tristimulus values of the standard illuminant with  $Y_n=100$ .

For:  $\frac{Y}{Y_n}, \frac{X}{X_n}, \frac{Z}{Z_n} \geq 0.01$

$$L^* = 116(Y/Y_n)^{1/3} - 16$$

$$a^* = 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}]$$

$$b^* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}]$$

otherwise

$$L^* = 116 \left[ f\left(\frac{Y}{Y_n}\right) - \frac{16}{116} \right]$$

$$a^* = 500 \left[ f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right]$$

$$b^* = 200 \left[ f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right]$$

$$f\left(\frac{Y}{Y_n}\right) = \begin{cases} \left(\frac{Y}{Y_n}\right)^{1/3} & \text{for } \frac{Y}{Y_n} > 0.008856 \\ 7.787 \frac{Y}{Y_n} + \frac{16}{116} & \text{for } \frac{Y}{Y_n} \leq 0.008856 \end{cases}$$

Hint: the diffuse light (☒ color) depends on both the physical properties of the surface and the illuminant

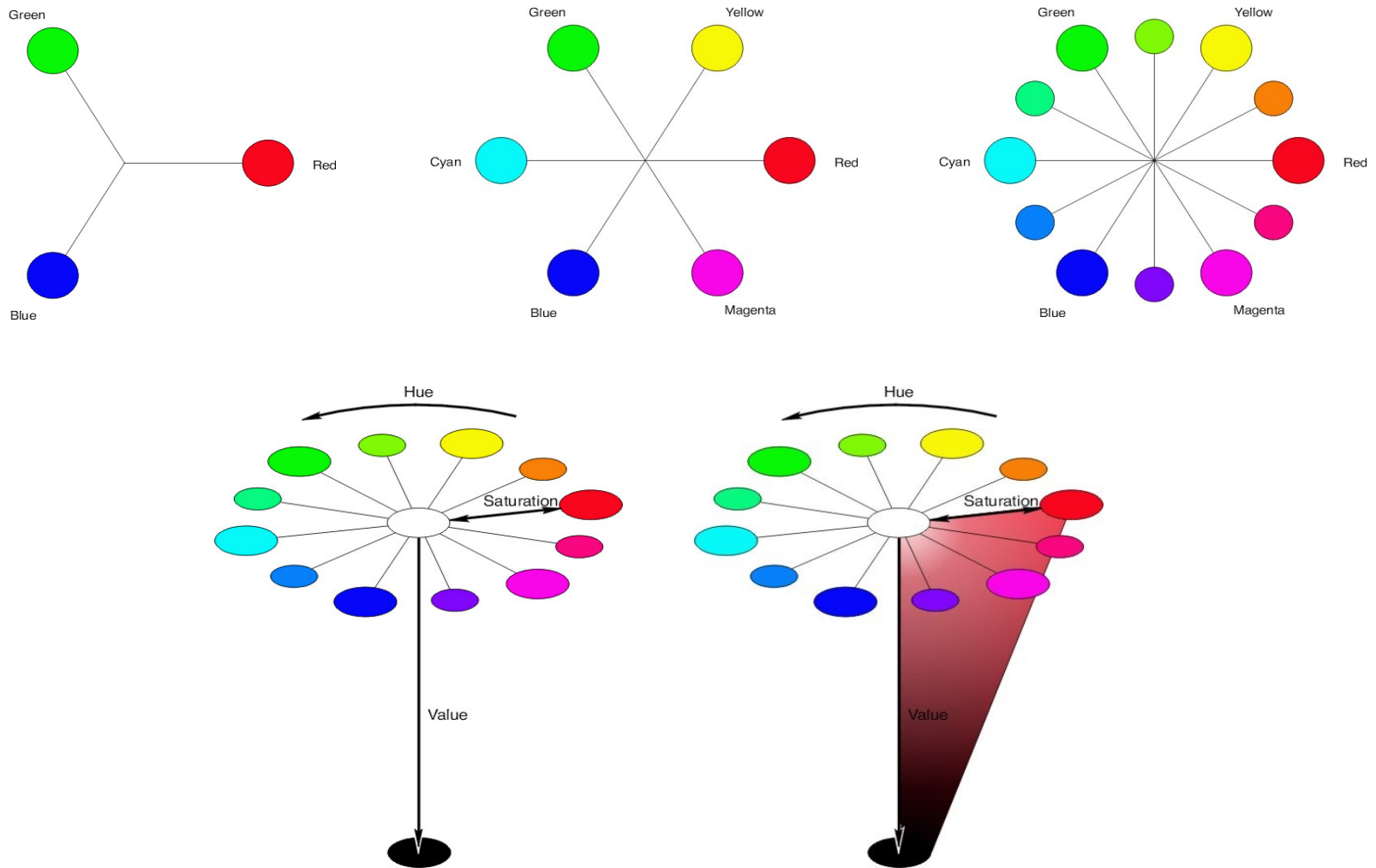
# User-oriented CM

- HSV (Hue, Saturation, and Value). Sometimes variations include HSB (Brightness), HSL (Lightness/Luminosity), HIS (Intensity)
  - The hue of a color places it on the color wheel where the color spectrum (rainbow) is evenly spaced
  - The saturation or chroma of a hue defines its intensity
    - Decreasing the saturation via a contrast control adds gray.
  - The value of a hue defines how bright or dark a color is
- They all are effectively the RGB space twisted so that the neutral diagonal becomes the lightness axis, the saturation the distance from the central lightness axis and the hue the position around the center.
- The only difference between these models is the measurement of saturation, or the strength of the colour

# User-oriented CM

- Drawbacks
  - Singularities in the transform (such as undefined hue for achromatic points)
  - Sensitivity to small deviations of RGB values near the singularities
  - Numerical instability when operating on hue due to its angular nature

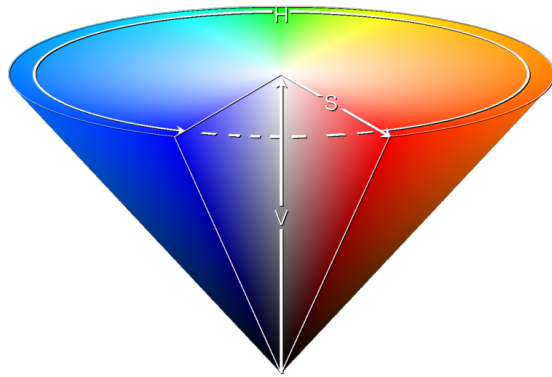
# User-oriented CM: HSV



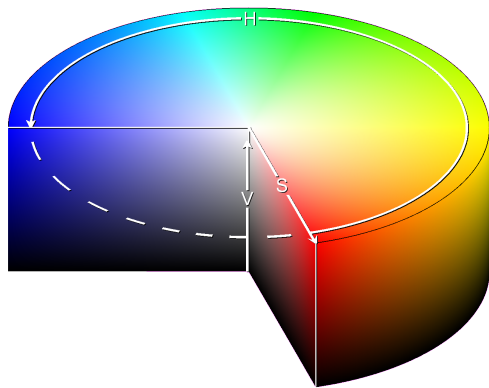
# HSV, HSL

Hue, Saturation, Value (Brightness)

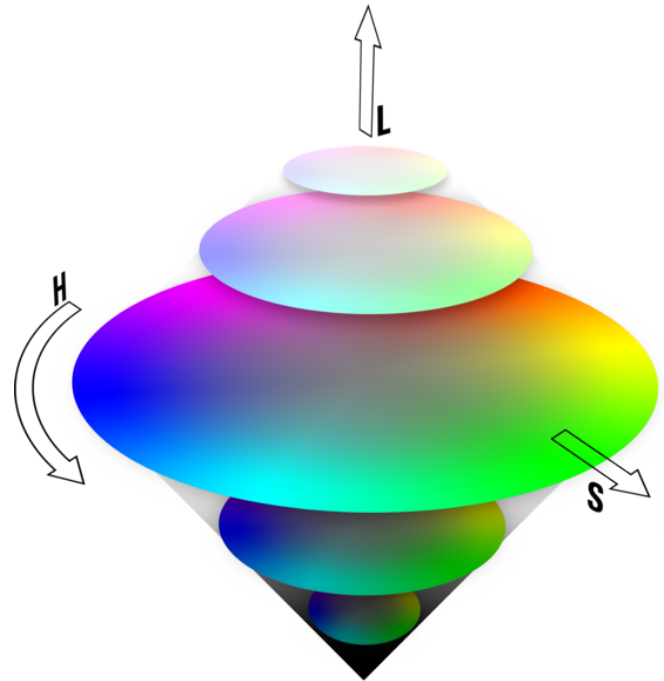
HSV cone



HSV cylinder



Hue, Saturation, Lightness

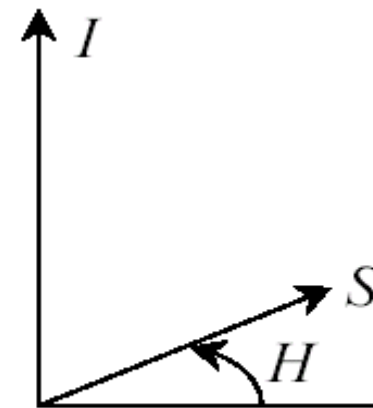


# HSI (HSV, HSL) Color Space

- Recall:
  - **Hue** is color attribute that describes a pure color
  - **Saturation** gives the measure to which degree the pure color is diluted by white light.
- 1. Intensity (Value or Lightness) component  $I$  (V,L), is decoupled from the chromaticity information!
- 2. Hue and saturation can be accessed independently from illumination

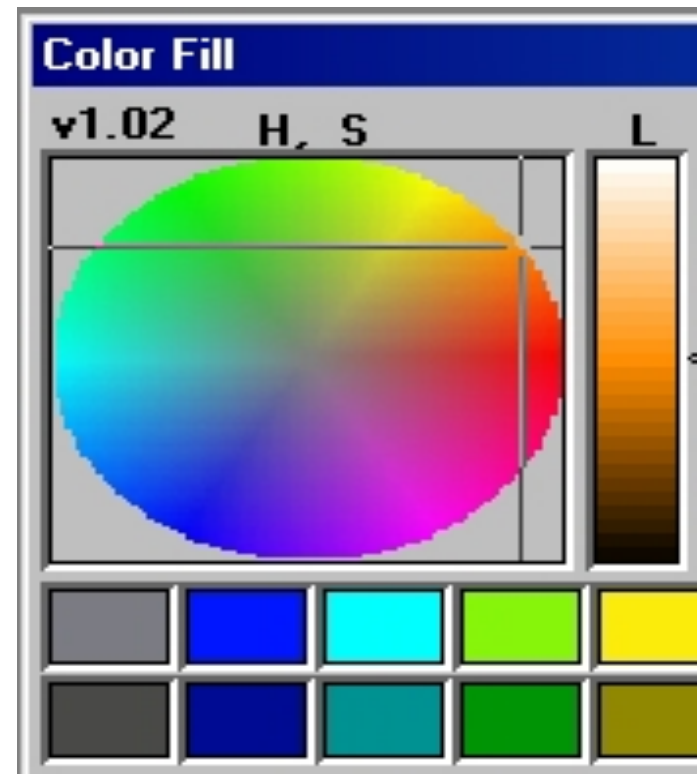
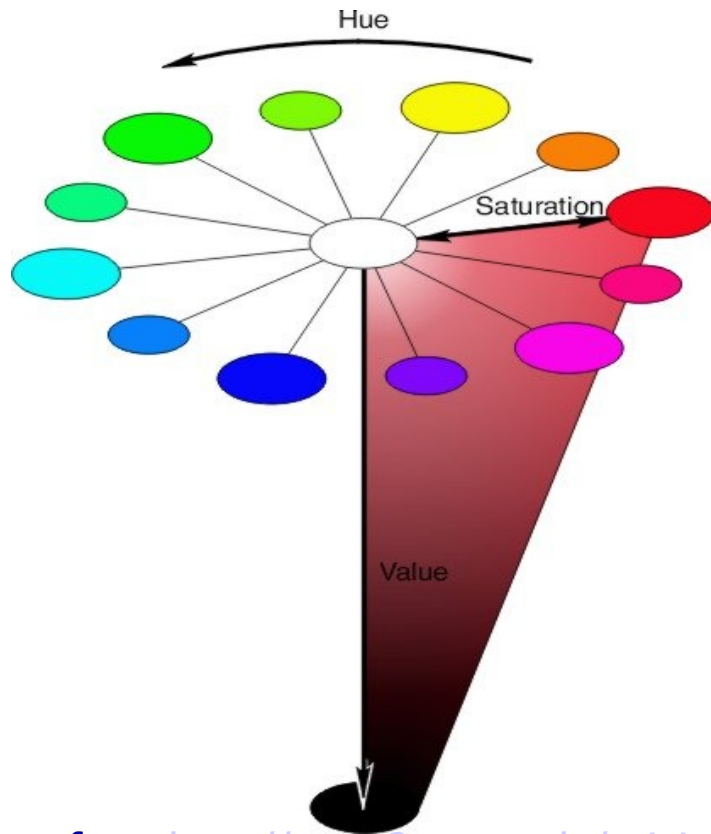
# HSI model

- Two values (H & S) encode *chromaticity*
- Convenient for *designing* colors
- Hue H is defined by an angle between 0 and  $2\pi$ :
  - “red” at angle of 0;
  - “green” at  $2\pi/3$ ;
  - “blue” at  $4\pi/3$
- Saturation S models the *purity* of the color
  - $S=1$  for a completely pure or saturated color
  - $S=0$  for a shade of “gray”



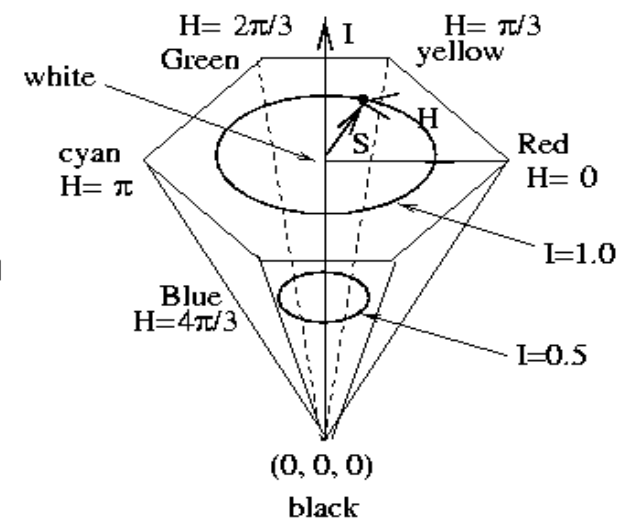
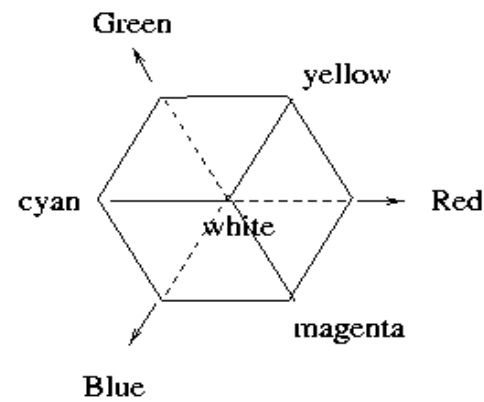
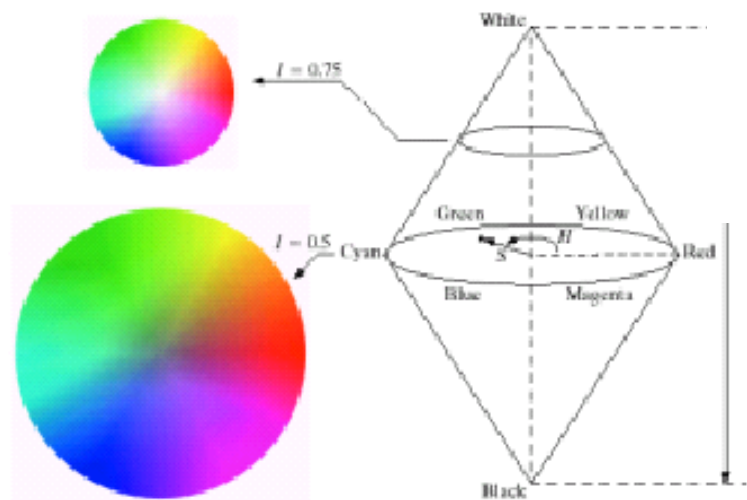
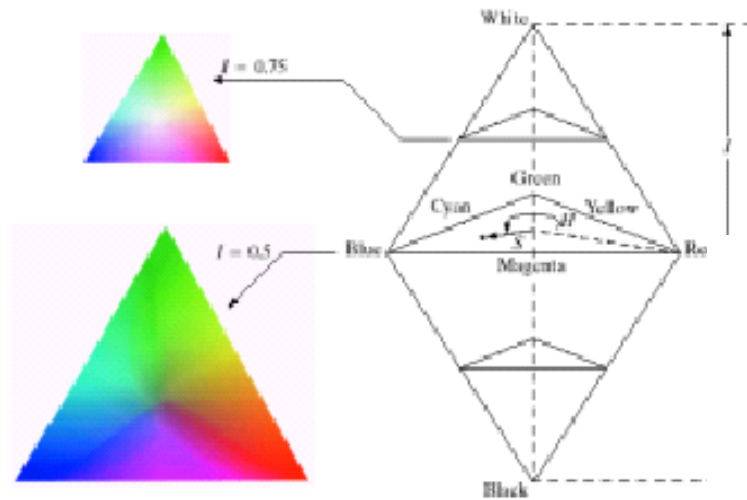
# HSI-like model

- Hue, Saturation, Value (HSV) model



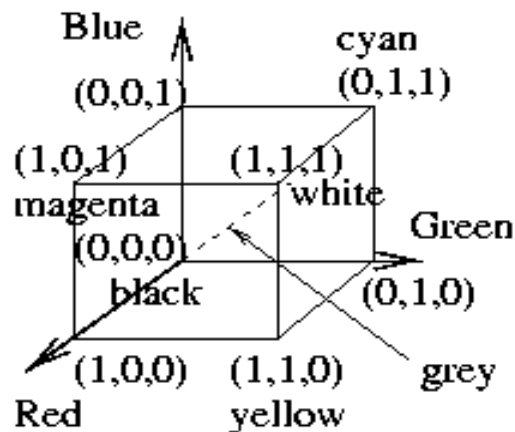
from [http://www2.ncsu.edu/scivis/lessons/colormodels/color\\_models2.html#saturation](http://www2.ncsu.edu/scivis/lessons/colormodels/color_models2.html#saturation).

# Variations on the theme

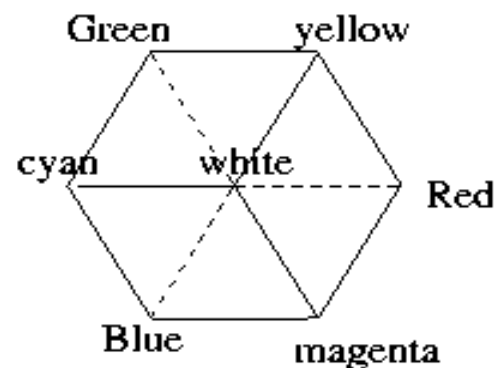


## Color hexagon for HSI (HSV)

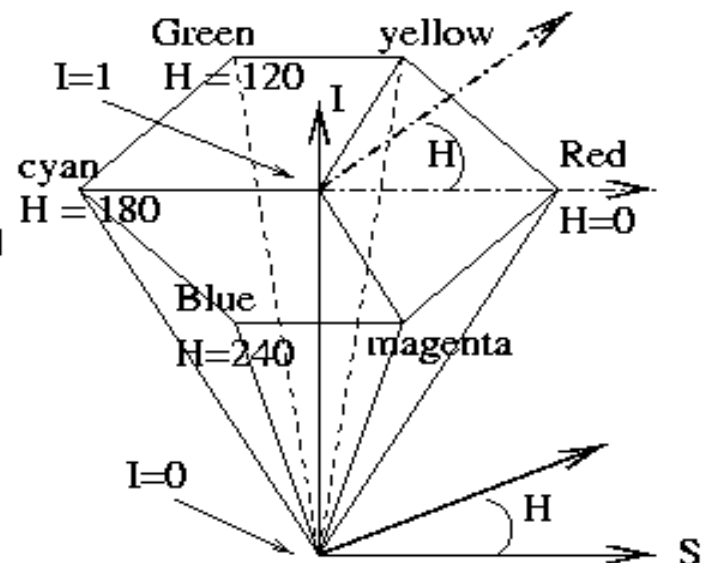
- Color is coded relative to the diagonal of the color cube. Hue is encoded as an angle, saturation is the relative distance from the diagonal, and intensity is height.



(a) RGB color cube



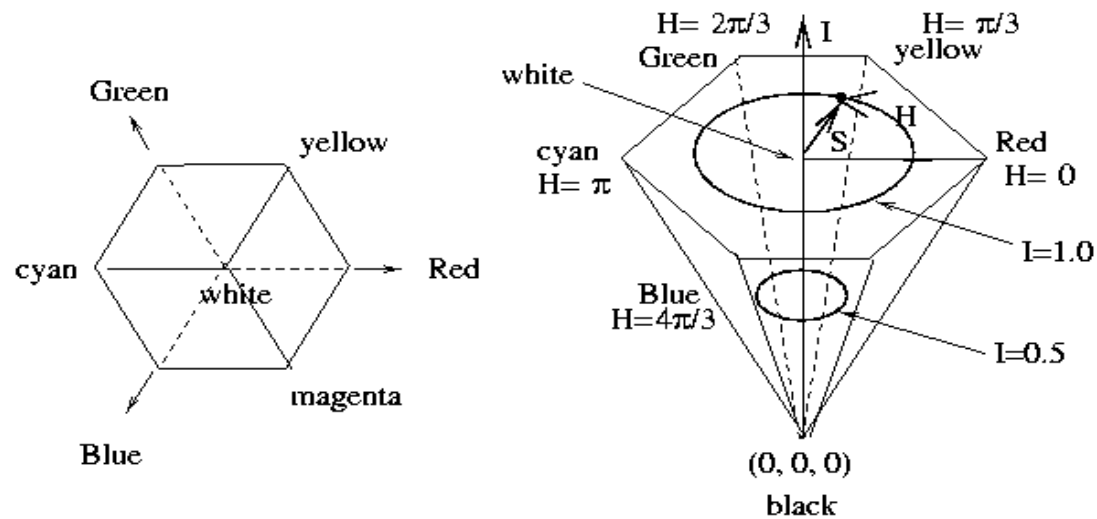
(b) view on diagonal from white to black



(c) single hexacone HSI model

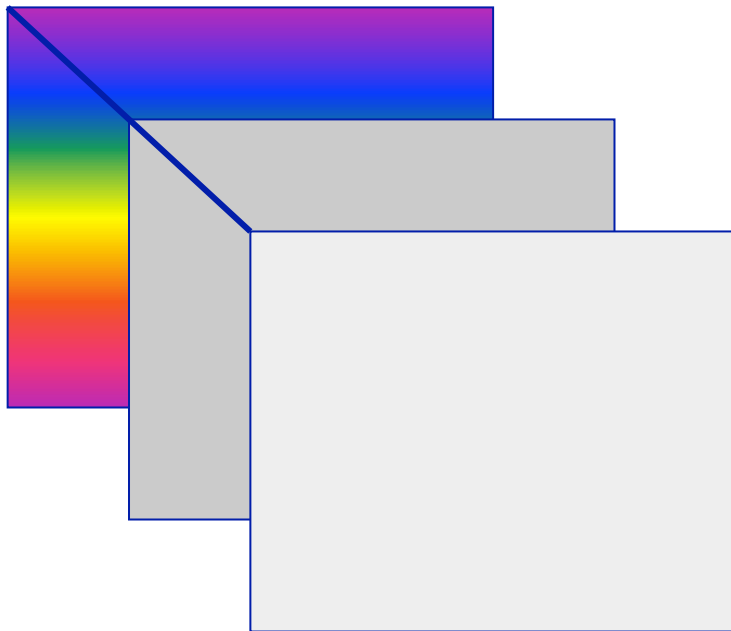
# Color hexacone for HSI (HSV)

- (Left) Projection of RGB cube perpendicular to the diagonal  $(0,0,0) - (1,1,1)$ .
- Color names now at vertices of a hexagon.
- Colors in HIS :
  - intensity  $I$  is vertical axis
  - hue  $H$  is angle with  $R$  at  $0$
  - saturation is  $1$  at periphery and  $0$  on  $I$  axis



# HSI Representation

(0,0)



A single pixel consists of three components.

Each pixel is a **Vector / Array**.

|     |     |    |
|-----|-----|----|
| 128 | 251 | 60 |
|-----|-----|----|

=



Pixel-Vector in the  
computer memory

Final pixel in  
the image

Caution! Sometimes pixels are not stored as vectors. Instead, first is stored the complete hue component, then the complete sat., then the intensity.

# HSI Examples

Original Image



Hue



Saturation



Intensity



## Editing saturation of colors



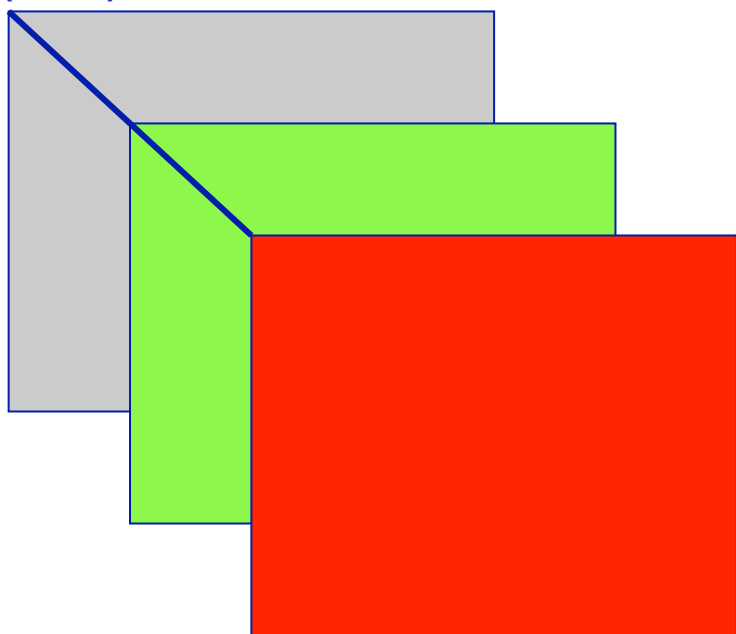
(Left) Image of food originating from a digital camera;  
(center) saturation value of each pixel decreased 20%;  
(right) saturation value of each pixel increased 40%.

# YUV Color model

- PAL TV standard
  - YCbCr similar, used in JPEG and MPEG
  - YIQ (similar) used in NTSC
- Color channels
  - Y: luminance
  - UV: chrominance. These are often downsampled exploiting the lower cutting frequency and sensitivity of the human visual system with respect to the luminance component

# YUV representation

(0,0)



A single pixel consists of three components.

Each pixel is a Vector / Array.

|     |     |    |
|-----|-----|----|
| 128 | 251 | 60 |
|-----|-----|----|



Pixel-Vector in the  
computer memory

Final pixel in  
the image

Same Caution as before applies here!

# YUV example

Original Image



U-Component



Y-Component



V-Component



# YIQ model

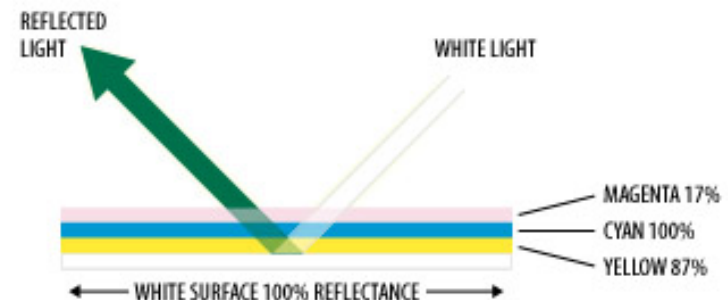
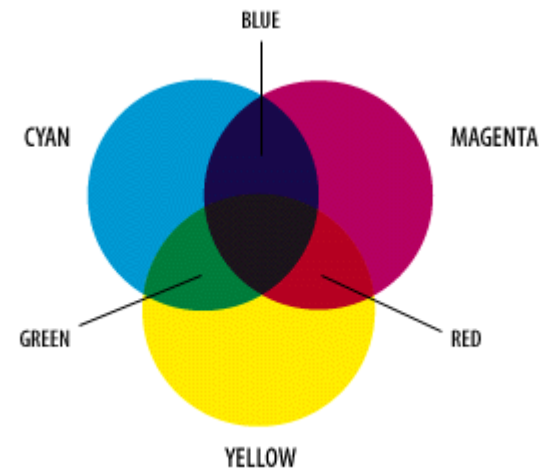
- NTSC
- Y is the luminance
- Chromaticity is represented by I and Q
  - in phase and in quadrature components
- RGB2YIQ

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.528 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

# Device-oriented color models: CMY(K)

- Color subtraction
  - Cyan, Magenta, Yellow filters
    - The Y filter removes B and transmits the R and G
    - The M filter removes G and transmits R and B
    - The C filter removes R and transmits G and B
  - Adjusting the transparency of these filters the amounts of R, G and B can be controlled

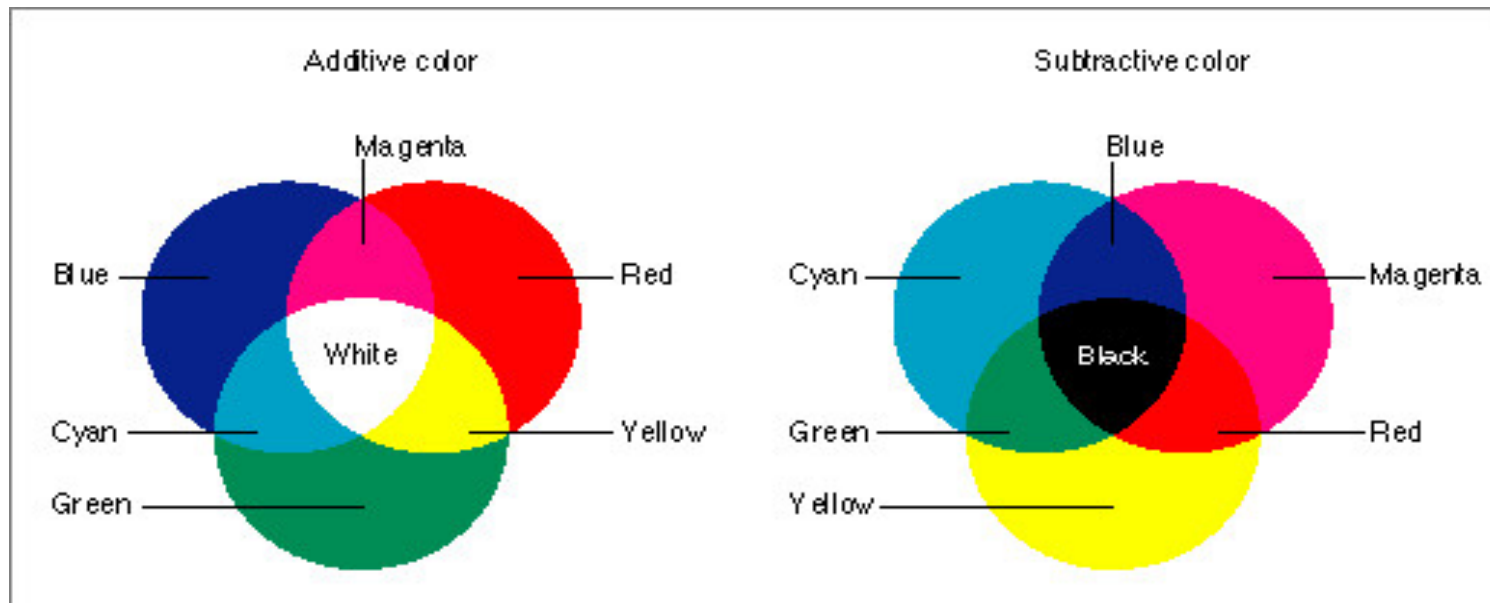
cyan=white-red  
magenta=white-green  
yellow=white-blue



# CMY model

- CMY (Cyan, Magenta, Yellow)
- Used in printing devices
- Subtractive color synthesis
- CMYK: adding the black ink
  - Equal amounts of C,M and Y should produce black, but in practice a dark brown results. A real black ink is then added to the printer

# CYM(K)



- cyan (C) absorbs red
- magenta (M) absorbs green
- yellow (Y) absorbs blue

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

# CMY(K) model

- Red, Green, Blue are the primary colors of light
- Cyan, Magenta, Yellow are the
  - Secondary colors of light
  - Primary colors of pigments
- When a cyan-colored object is illuminated with white light, no red light will be reflected from its surface! *Cyan subtracts red!*
- *The pigment when illuminated with white light absorbs its complementary color and reflects the others*

# Summary

- References
  - B. Wandell, “Foundations of vision”
  - Wyszecki&Stiles, “Color science, concepts, methods, quantitative data and formulae”, Wiley Classic Library
  - D. Malacara, “Color vision and colorimetry, theory and applications”, SPIE Press