State Machines

EECS 20

Lecture 8 (February 2, 2001)

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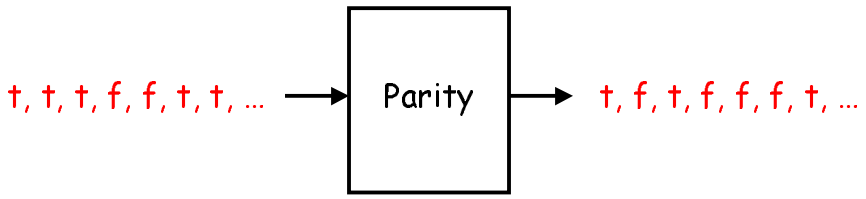
Finite-Memory Systems

Discrete-Time Delay:	remember last input value
Discrete-Time Moving Average:	remember last 2 input values
Parity:	remember if number of past inputs "true" is even

Infinite-Memory Systems

Count:	remember if number of past inputs "true" is greater than number of past inputs "false"



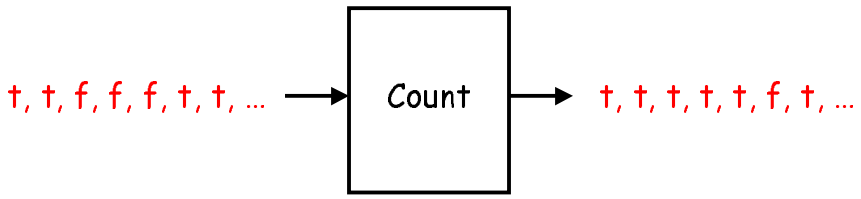
The Parity System

Parity : $[\ Nats_0 \rightarrow Bools\ ] \rightarrow [\ Nats_0 \rightarrow Bools\ ]$

such that $\forall\ x \in [\ Nats_0 \rightarrow Bools\ ]\ ,\ \forall\ y \in Nats_0\ ,$

$$(\ Parity\ (x)\ )\ (y) = \begin{cases} true & \text{if } |\ trueValues\ (x,y)\ |\ \text{is even} \\ false & \text{if } |\ trueValues\ (x,y)\ |\ \text{is odd} \end{cases}$$

where $trueValues\ (x,y) = \{\ z \in Nats_0 \mid z < y \wedge x\ (z) = true\ \}$



The Count System

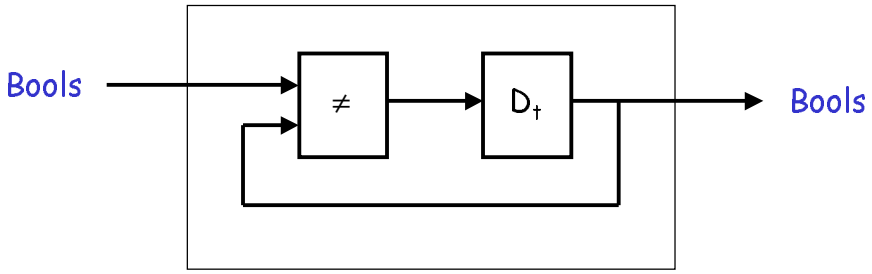
Count : $[\ Nats_0 \rightarrow Bools\ ] \rightarrow [\ Nats_0 \rightarrow Bools\ ]$

such that $\forall\ x \in [\ Nats_0 \rightarrow Bools\ ]\ ,\ \forall\ y \in Nats_0\ ,$

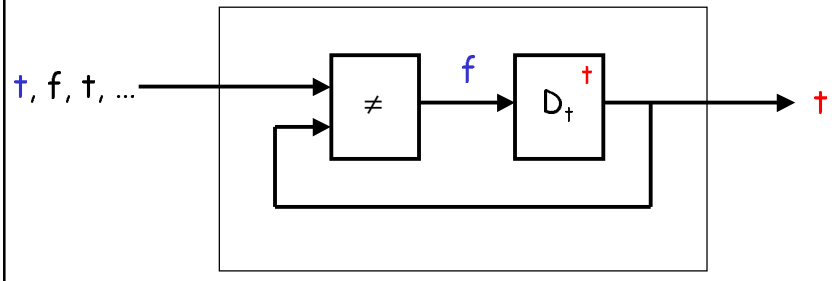
$$(\ Count\ (x)\ )\ (y) = \begin{cases} true & \text{if } |\ trueValues\ (x,y)\ | \geq |\ falseValues\ (x,y)\ | \\ false & \text{if } |\ trueValues\ (x,y)\ | < |\ falseValues\ (x,y)\ | \end{cases}$$

where $falseValues\ (x,y) = \{\ z \in Nats_0 \mid z < y \wedge x\ (z) = false\ \}$

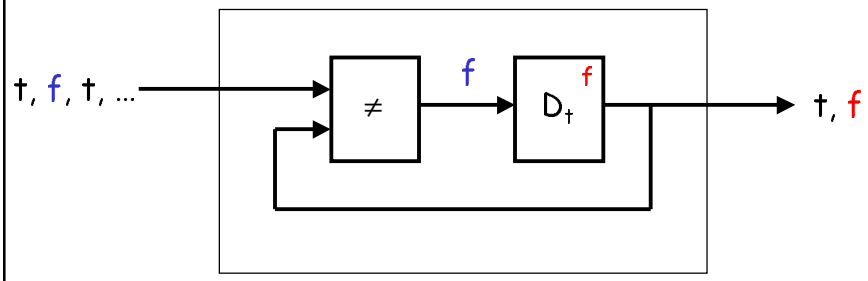
An Implementation of the Parity System



An Implementation of the Parity System



An Implementation of the Parity System



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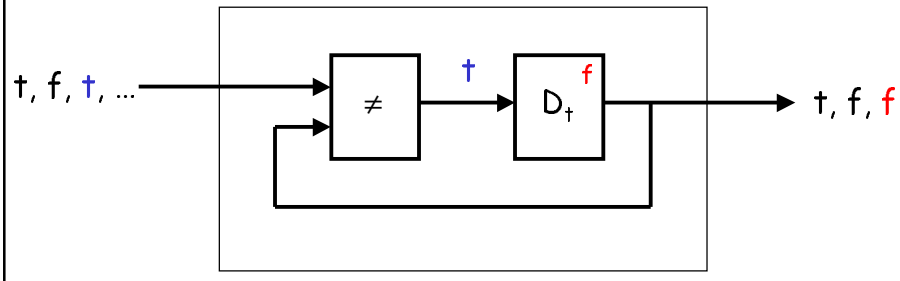

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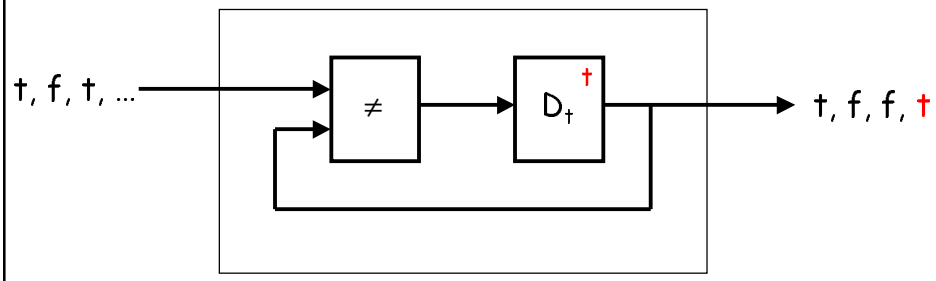
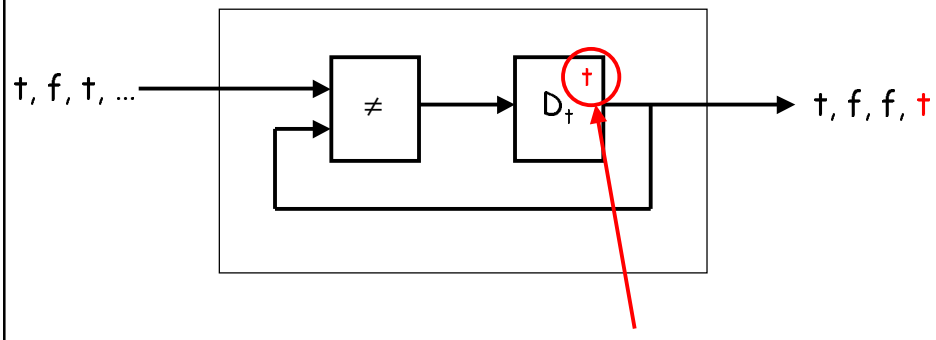


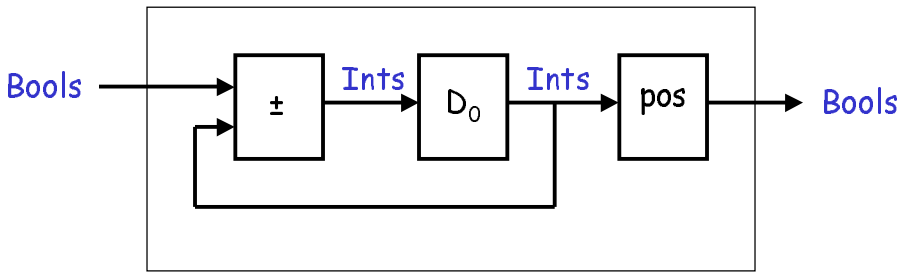
An Implementation of the Parity System



Memory = "state" of the system

Systems with finite memory
are naturally implemented as
finite state machines
(or finite transition systems) .

An Implementation of the Count System



$\pm$: Bools $\times$ Ints $\rightarrow$ Ints

such that $\forall x \in$ Bools, $\forall y \in$ Ints,

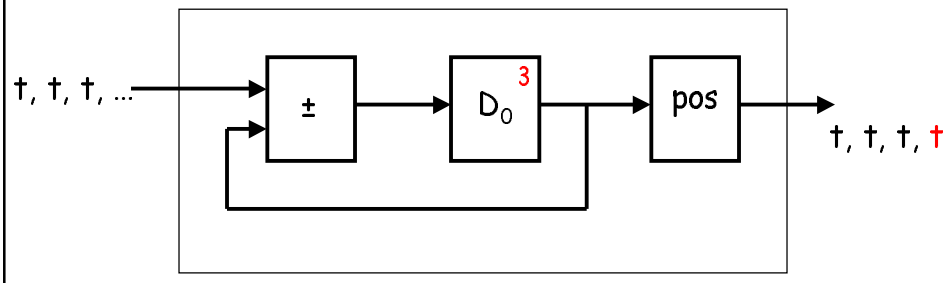
$$\pm(x,y) = \begin{cases} y+1 & \text{if } x = \text{true} \\ y-1 & \text{if } x = \text{false} \end{cases}$$

pos : Ints $\rightarrow$ Bools

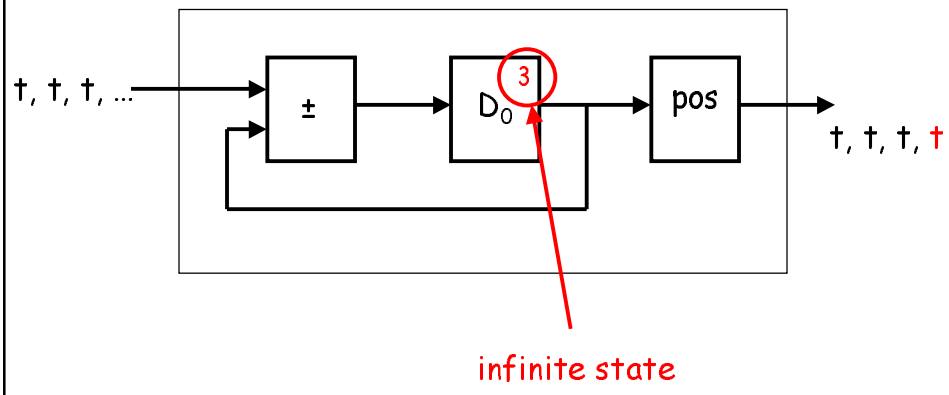
such that $\forall x \in$ Ints,

$$\text{pos}(x) = \begin{cases} \text{true} & \text{if } x \geq 0 \\ \text{false} & \text{if } x < 0 \end{cases}$$

An Implementation of the Count System



An Implementation of the Count System



Systems with countable (integer) memory

are naturally implemented as

infinite state machines

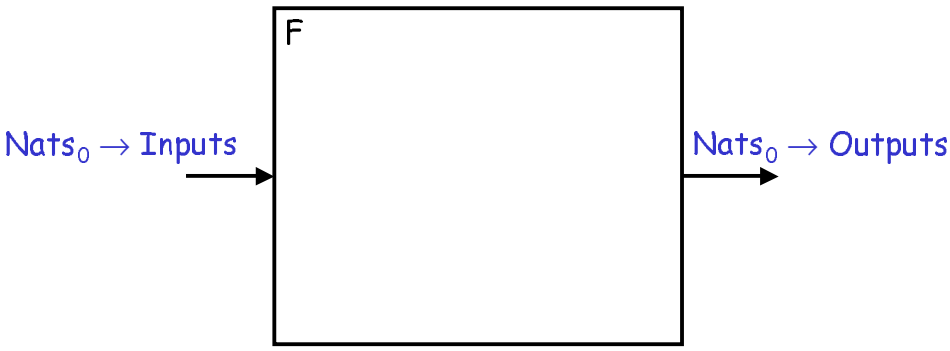
(or infinite transition systems) .

(Systems with uncountable (real) memory,

such as continuous-time delay,

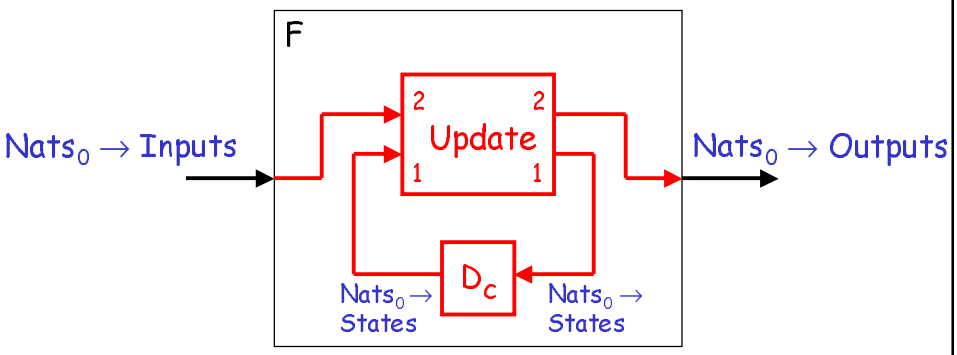
are not naturally viewed naturally as transition systems.)

A Discrete-Time Reactive System



$F : [\ Nats_0 \to Inputs\ ] \to [\ Nats_0 \to Outputs\ ]$

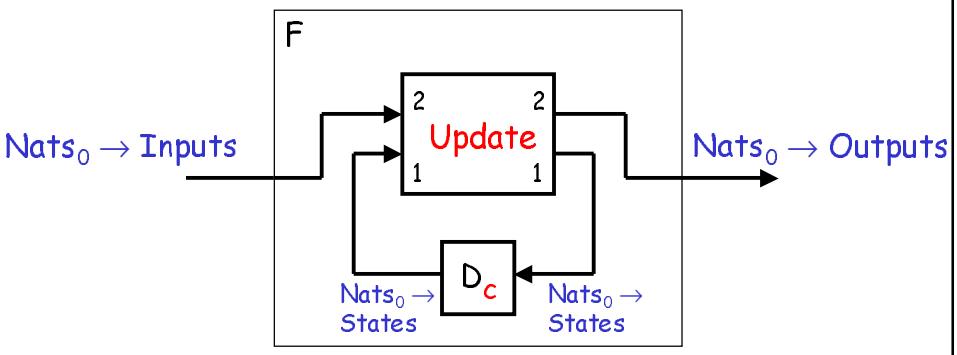
State-Based Implementation



Update: memory-free

Memory = States

State Implementation



update : $States \times Inputs \to States \times Outputs$

$c \in States$ ("initial state")

State Implementation of the Parity System

Inputs = Bools

Outputs = Bools

States = Bools

initialState = true

update : States $\times$ Inputs $\rightarrow$ States $\times$ Outputs

such that $\forall\, q \in$ States, $\forall\, x \in$ Inputs,

update $(q,x)_1$ = $(q \neq x)$

update $(q,x)_2$ = q

State Implementation of the Count System

Inputs = Bools

Outputs = Bools

States = Ints

initialState = 0

update : States $\times$ Inputs $\rightarrow$ States $\times$ Outputs

such that $\forall\, q \in$ States, $\forall\, x \in$ Inputs,

update $(q,x)_1$ = $\pm\,(x,q)$

update $(q,x)_2$ = pos (q)

A State Machine

Inputs (set of possible input values)

Outputs (set of possible output values)

States (set of states)

$initialState \in States$

$update : States \times Inputs \rightarrow States \times Outputs$

A state machine is

finite

iff

States is a finite set.

Parity : can be implemented by a two-state machine.

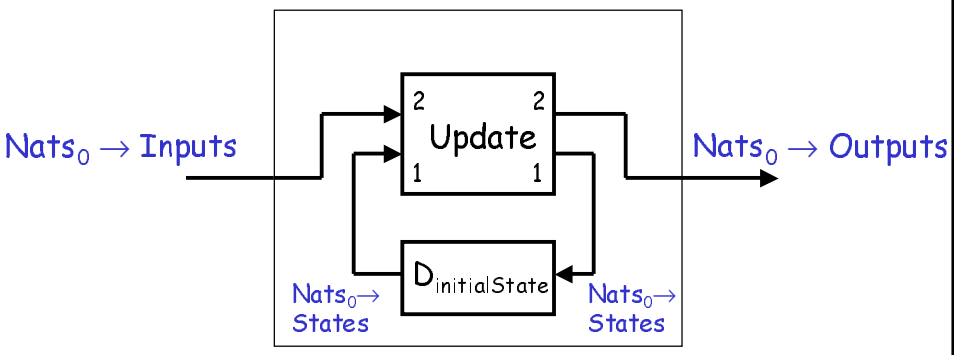
Count : cannot be implemented by finite-state machine.

Discrete-Time Reactive Systems

Every memory-free system can be implemented by a one-state machine.

Every causal system can be implemented by a state machine (take as state the entire history of inputs), and every system that can be implemented by a state machine is causal.

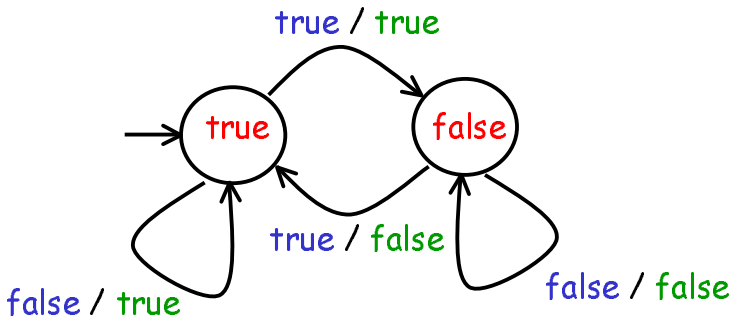
The Block Diagram of a State Machine



The Transition Table of a Finite-State Machine (Parity)

Current state	Input	Next state	Output
true	true	false	true
true	false	true	true
false	true	true	false
false	false	false	false

The Transition Diagram of a State Machine (Parity)



States = Bools
Inputs = Bools
Outputs = Bools