$x + y$

$\underline{m} + n \qquad (\underline{m}+) : \mathbb{N} \to \mathbb{N}$

$$\boxed{\begin{aligned} &\underline{m} + 0 = \underline{m} \\ &\underline{m} + \sigma(n) = \sigma(\underline{m} + n) \end{aligned}}$$

$f_{\bar{m}} : \mathbb{N} \to \mathbb{N}$

$$\boxed{\begin{aligned} &f_{\bar{m}}(0) = \bar{m} \\ &f_{\bar{m}}(\sigma(n)) = \sigma(f_{\bar{m}}(n)) \end{aligned}}$$

$x + y \equiv$

$f_x(y)$

lezione 14 marzo 2014

$$\boxed{\begin{array}{l} m+0 = m \\ m+\sigma(n) = \sigma(m+n) \end{array}}$$

$$0+m = m \quad \forall m \in \mathbb{N}$$

BASE $\quad 0+0 = 0$

IND $\quad$ ASSUMIAMO CHE $0+m = m$

DIMOSTRIAMO CHE

$$0+\sigma(m) = \sigma(m)$$

$$0+\sigma(m) = \sigma(0+m) \underset{II}{=} \sigma(m)$$

$$\boxed{\begin{array}{l} m+0=m \\ m+\sigma(n)=\sigma(m+n) \end{array}}$$

$$\sigma(m)+n=\sigma(m+n)$$

$$\sigma(m)+0\overset{?}{=}\sigma(m+0)$$

$$\sigma(m)+0=\sigma(m)=\sigma(m+0)$$

P.I. ASSUMIAMO CHE $\sigma(m)+n=\sigma(m+n)$

DIMOSTR. CHE $\sigma(m)+\sigma(n)=\sigma(m+\sigma(n))$

$$\sigma(m+\sigma(n))=\sigma(\sigma(m+n))\overset{II}{=}\sigma(\sigma(m)+n)=$$

$$=\sigma(m)+\sigma(n)$$

$$\boxed{\begin{array}{l} m+0=m \\ m+\sigma(n)=\sigma(m+n) \end{array}}$$

$$m+n = n+m$$

$$\sigma(m)+n = \sigma(m+n)$$

BASE $m+0 = 0+m$

P. IND. $m+n = n+m$ II

VOGL. DIM. CHE $m+\sigma(n) = \sigma(n)+m$

$$m+\sigma(n) = \sigma(m+n) \overset{II}{=} \sigma(n+m) = \sigma(n)+m$$

$$m+n = m+\underbrace{1+\cdots+1}_{n} = \overbrace{\sigma(\sigma\ \cdots\ \sigma}^{m}(n)) = \sigma^m(n)$$

$$m \times n = \underbrace{m + \cdots + m}_{n \text{ volte}}$$

$$\begin{cases} m \times 0 = 0 \\ m \times \sigma(n) = m + (m \times n) \end{cases}$$

$$1 \stackrel{\text{def}}{=} \sigma(0)$$

$$m \times 1 = m \leftarrow \begin{aligned} m \times \sigma(0) &= m + (m \times 0) \\ &= m + 0 = m \end{aligned}$$

$$0 \times m = 0$$

$$m \times n = n \times m$$

$$m \oplus n \equiv \sigma^{n}(m)$$

$$m \oplus n = m + n$$

$$m \oplus 0 = m$$

$$m \oplus \sigma(n) = \sigma(m \oplus n)$$

$$m \oplus 0 = \sigma^{0}(m)$$

$$= \iota_{\mathbb{N}}(m) = m$$

$$f^{\sigma(n)} = f \circ f^{n} \qquad \sigma^{\sigma(n)} = \sigma \circ \sigma^{n}$$

$$m \oplus \sigma(n) = \sigma^{\sigma(n)}(m) = \sigma(\sigma^{n}(m)) = \sigma(m \oplus n)$$

$$f : \mathbb{N} \to \mathbb{N} \qquad \sum_{i=0}^{n} E_i$$

$$S(n) = \sum_{i=0}^{n} f(i) \equiv f(0) + \cdots + f(n)$$

?

$\Downarrow$

RIC. PR.

$$S(0) = f(0) \qquad \sum_{i=0}^{0} f(i) = f(0)$$

$$\sum_{i=0}^{\sigma(n)} f(i) = \sum_{i=0}^{n} f(i) + f(\sigma(n))$$