Image interpolation

A reinterpretation of low-pass filtering

Image Interpolation

- Introduction
  - What is image interpolation? (D-A conversion)
  - Why do we need it?
- Interpolation Techniques
  - 1D zero-order, first-order, third-order
  - 2D = two sequential 1D (divide-and-conquer)
  - Directional(Adaptive) interpolation*
- Interpolation Applications
  - Digital zooming (resolution enhancement)
  - Image inpainting (error concealment)
  - Geometric transformations (where your imagination can fly)

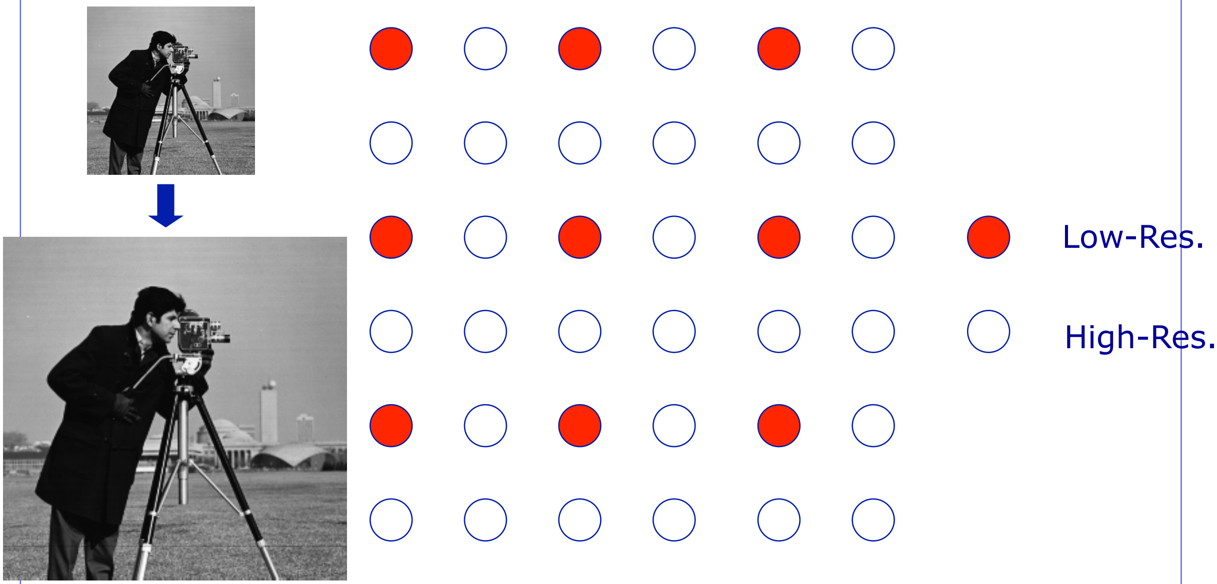
Introduction

- What is image interpolation?
  - An image f(x,y) tells us the intensity values at the integral lattice locations, i.e., when x and y are both integers
  - Image interpolation refers to the "guess" of intensity values at missing locations, i.e., x and y can be arbitrary
  - Note that it is just a guess (Note that all sensors have finite sampling distance)

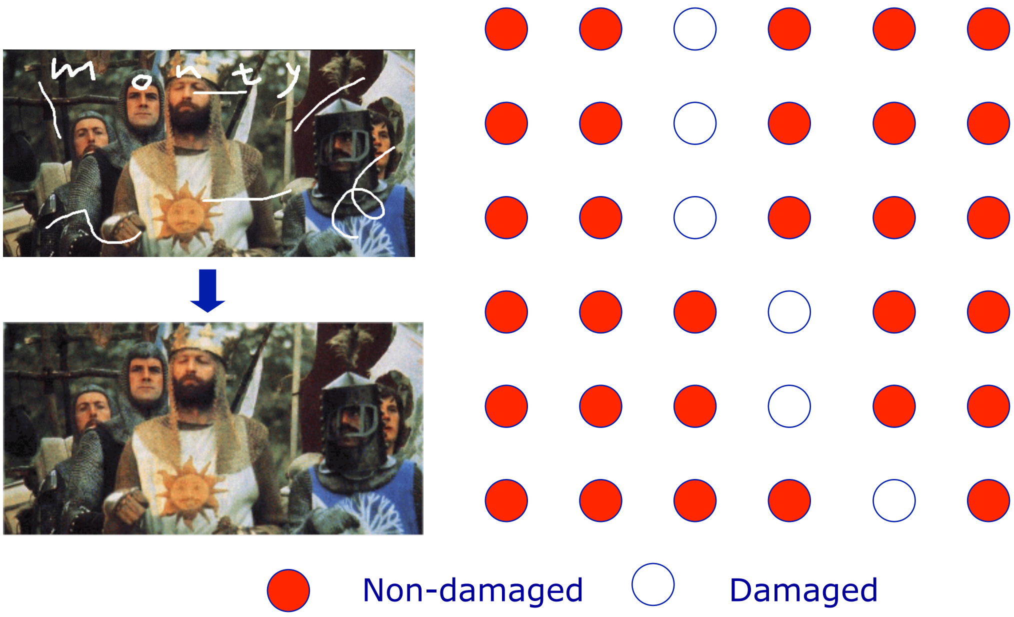
Engineering Motivations

- Why do we need image interpolation?
  - We want BIG images
    - When we see a video clip on a PC, we like to see it in the full screen mode
  - We want GOOD images
    - If some block of an image gets damaged during the transmission, we want to repair it
  - We want COOL images
    - Manipulate images digitally can render fancy artistic effects as we often see in movies

Scenario I: Resolution Enhancement



Scenario II: Image Inpainting



Scenario III: Image Warping

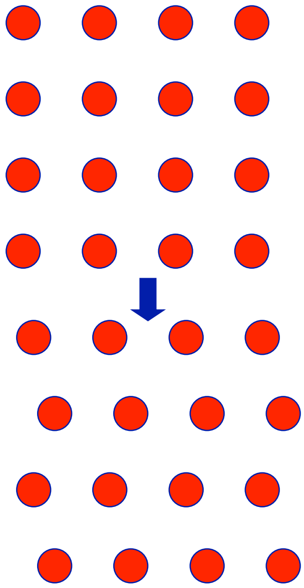


Image Interpolation

- Introduction
  - What is image interpolation?
  - Why do we need it?
- Interpolation Techniques
  - 1D linear interpolation (elementary algebra)
  - 2D = 2 sequential 1D (divide-and-conquer)
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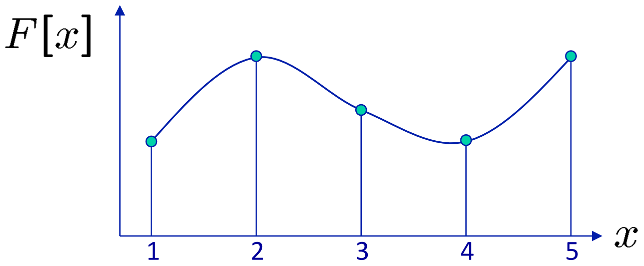
Upsampling

- This image is too small for this screen:

- How can we make it 10 times as big?
- Simplest approach:
- repeat each row
- and column 10 times
- ("Nearest neighbor
- interpolation")

Image interpolation



d = 1 in this example

Recall how a digital image is formed

$$F[x, y] = \text{quantize}\{f(xd, yd)\}$$

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

Adapted from: S. Seitz

Image interpolation

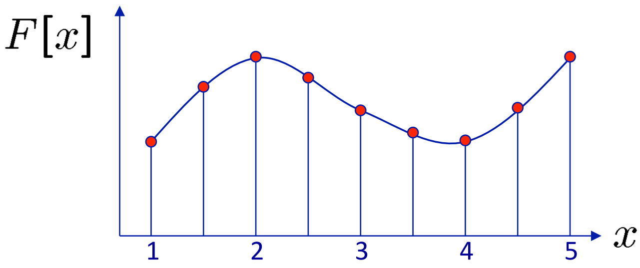


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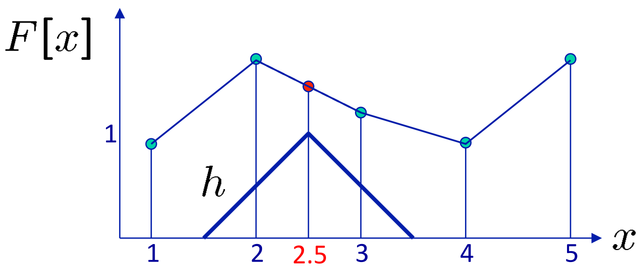
Image interpolation



d = 1 in this example

- What if we don't know f?
  - Guess an approximatio$\tilde{f}$:
  - Can be done in a principled way: filtering
  - Convert F to a continuous function:

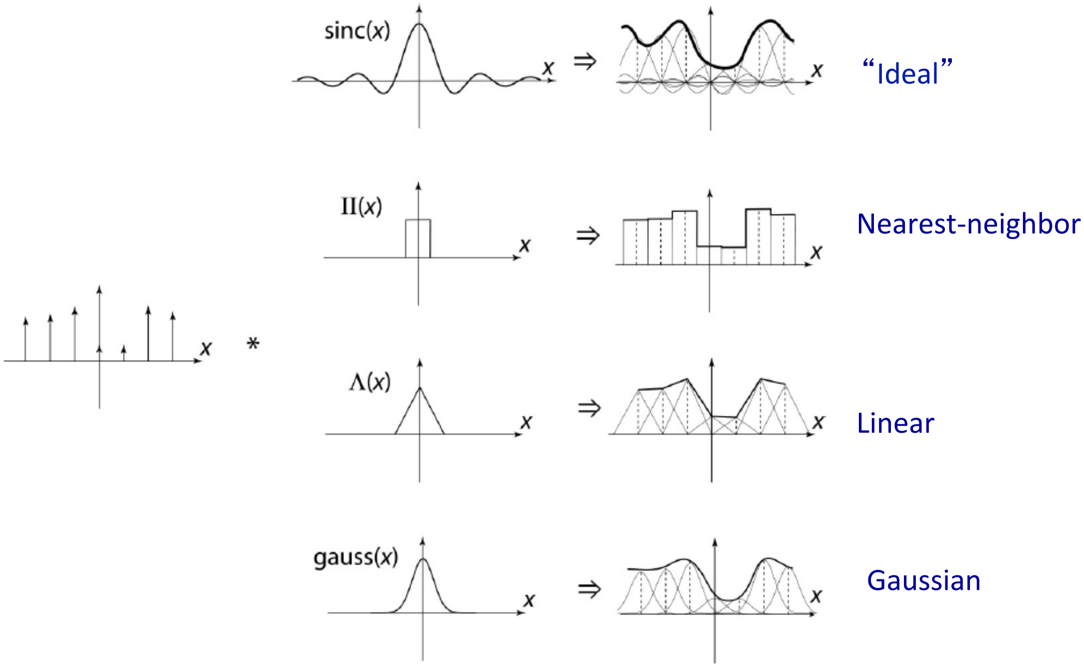
$$f_F(x) = F(\tfrac{x}{d}) \text{ when } \tfrac{x}{d} \text{ is an integer, 0 otherwise}$$

  - Reconstruct by convolution with a *reconstruction filter, h*

$$\tilde{f} = h * f_F$$

Adapted from: S. Seitz

Image interpolation



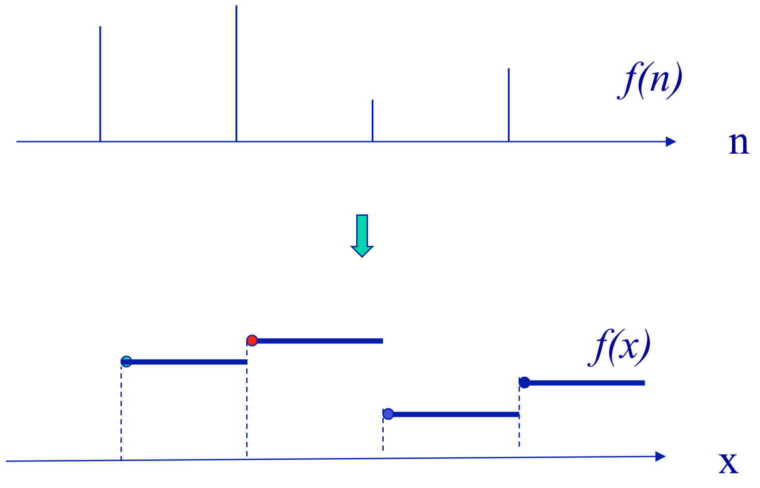
Source: B. Curless

Ideal reconstruction

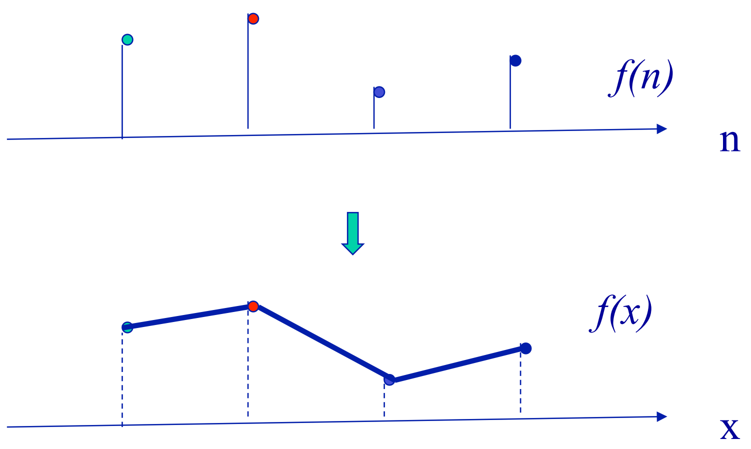
- The ideal reconstruction filter is a square window in the F-domain and a sinc function in the signal domain
- Its implementation is unpractical since it is prone to truncation artifacts and has relatively high computational complexity.
- Other low-pass filters can be used instead

Interpolation: LP filtering

1D Zero-order (Replication)

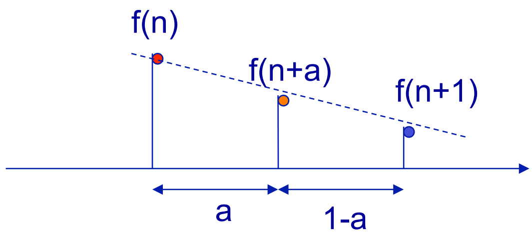


1D First-order Interpolation (Linear)



Linear Interpolation Formula

Heuristic: the closer to a pixel, the higher weight is assigned
Principle: line fitting to polynomial fitting (analytical formula)



$$f(n+a)=(1-a)\times f(n)+a\times f(n+1),\ 0<a<1$$

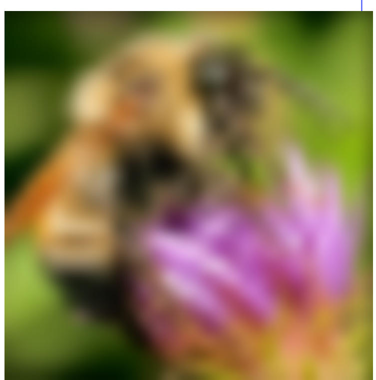
Note: when a=0.5, we simply have the average of two

Image interpolation

Original image: 
x 10


Nearest-neighbor interpolation


Bilinear interpolation


Bicubic interpolation

Numerical Examples

f(n)=[0,120,180,120,0]

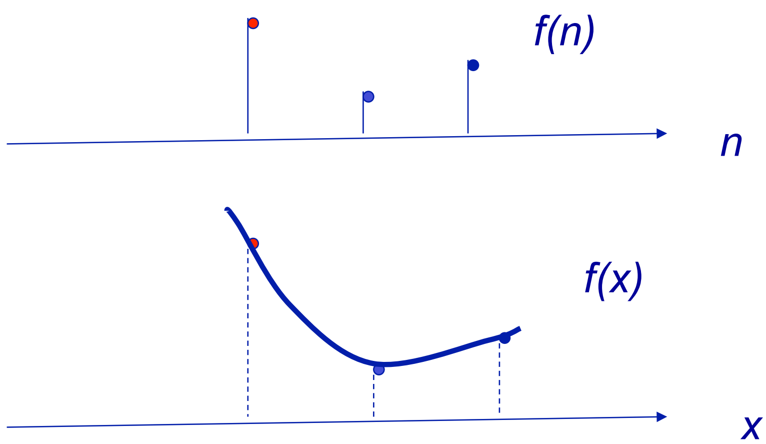
↓ Interpolate at 1/2-pixel

f(x)=[0,60,120,150,180,150,120,60,0], x=n/2

↓ Interpolate at 1/3-pixel

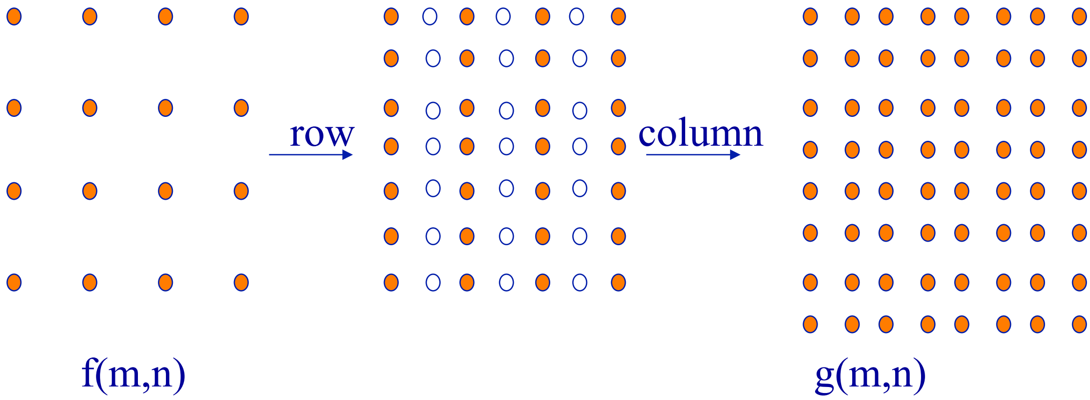
f(x)=[0,20,40,60,80,100,120,130,140,150,160,170,180,…], x=n/6

1D Third-order Interpolation (Cubic)*

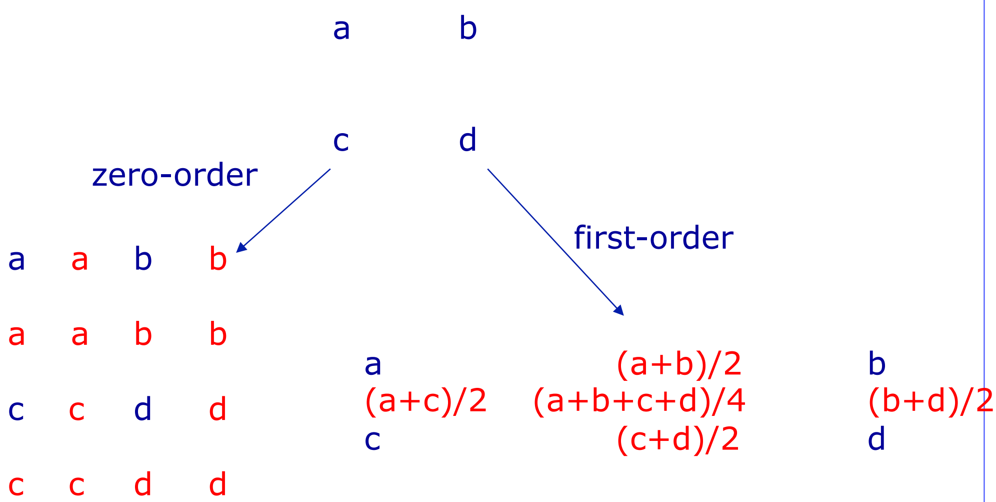


Cubic spline fitting

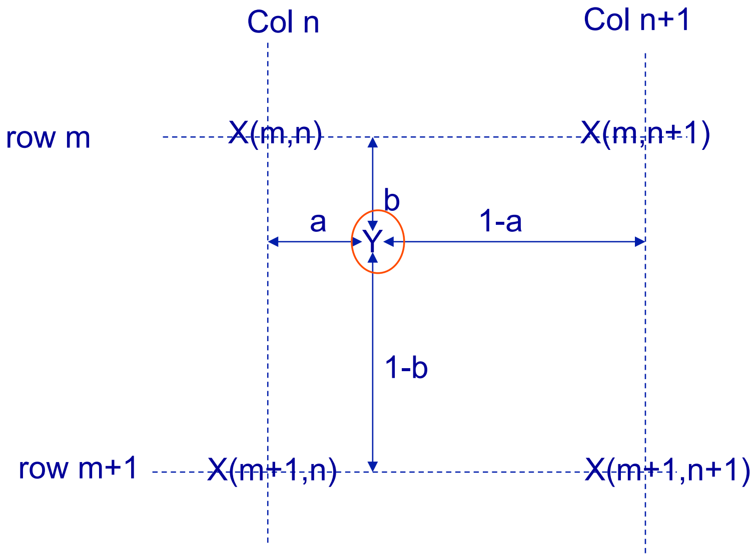
Graphical Interpretation of Interpolation at Half-pel



Numerical Examples



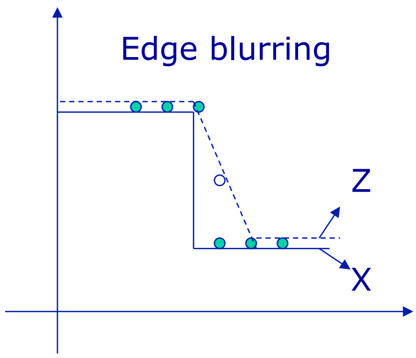
Numerical Examples (Con't)

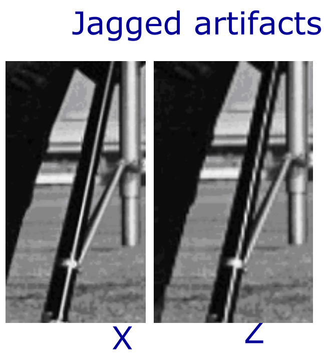


Q: what is the interpolated value at Y?
Ans.: abX(m,n)+(1-a)bX(m+1,n) +(1-a)bX(m,n+1)+a(1-b)X(m+1,n+1)

Limitation with bilinear/bicubic

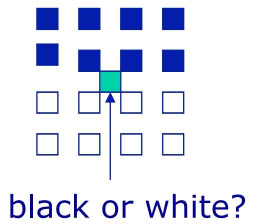
- Edge blurring
- Jagged artifacts

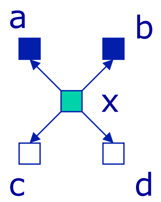




Edge-Sensitive Interpolation

Step 1: interpolate the missing pixels along the diagonal

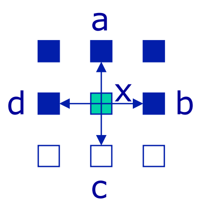




Since |a-c|=|b-d|

x has equal probability of being black or white

Step 2: interpolate the other half missing pixels



Since |a-c|>|b-d|

x=(b+d)/2=black

Applications

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- Interpolation Techniques
  - 1D zero-order, first-order, third-order
  - 2D zero-order, first-order, third-order
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Pixel Replication

low-resolution
image (100×100)

high-resolution
image (400×400)

Bilinear Interpolation

low-resolution
image (100×100)

high-resolution
image (400×400)

Bicubic Interpolation

low-resolution
image (100×100)

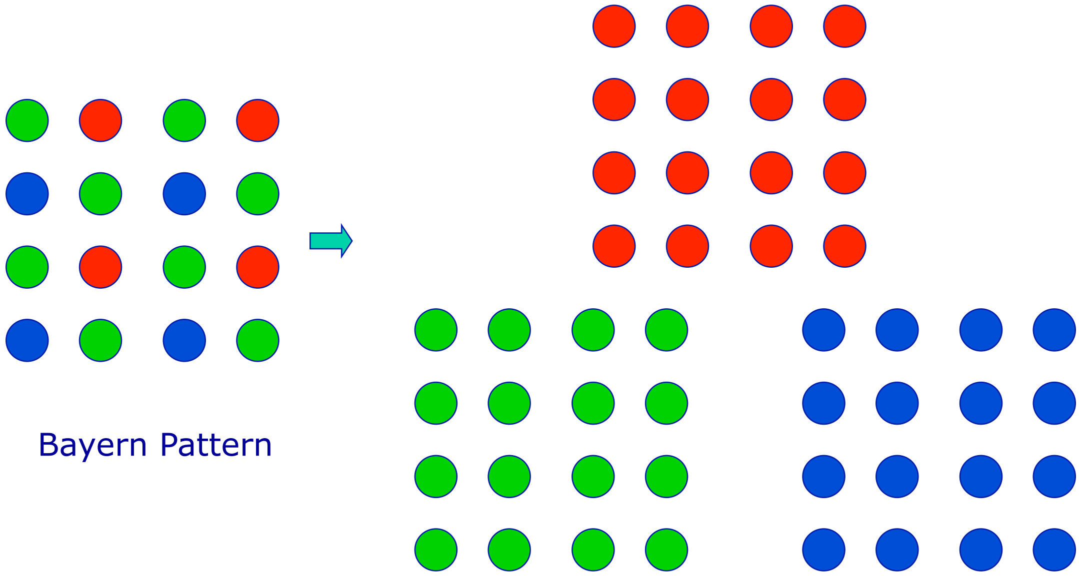
high-resolution
image (400×400)

Edge-Directed Interpolation (Li&Orchard'2000)

low-resolution
image (100×100)

high-resolution
image (400×400)

Image Demosaicing (Color-Filter-Array Interpolation)



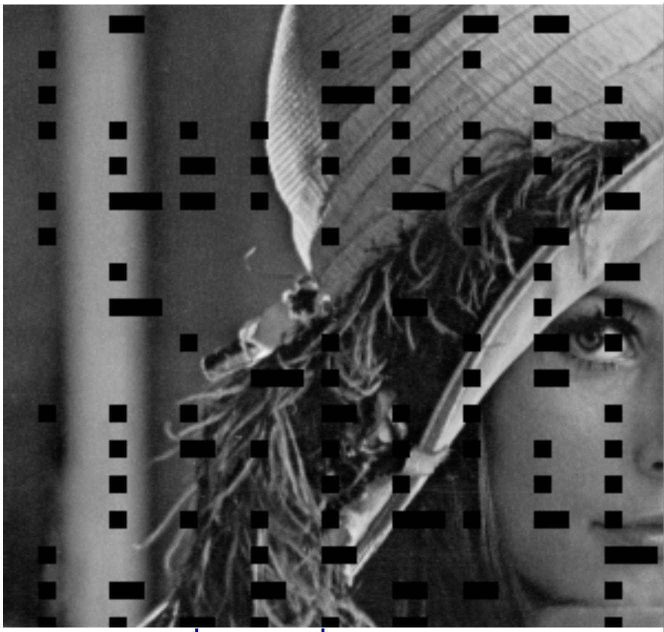
Bayern Pattern

Image Example

Ad-hoc CFA Interpolation

Advanced CFA Interpolation

Error Concealment*



damaged

interpolated

Image Inpainting

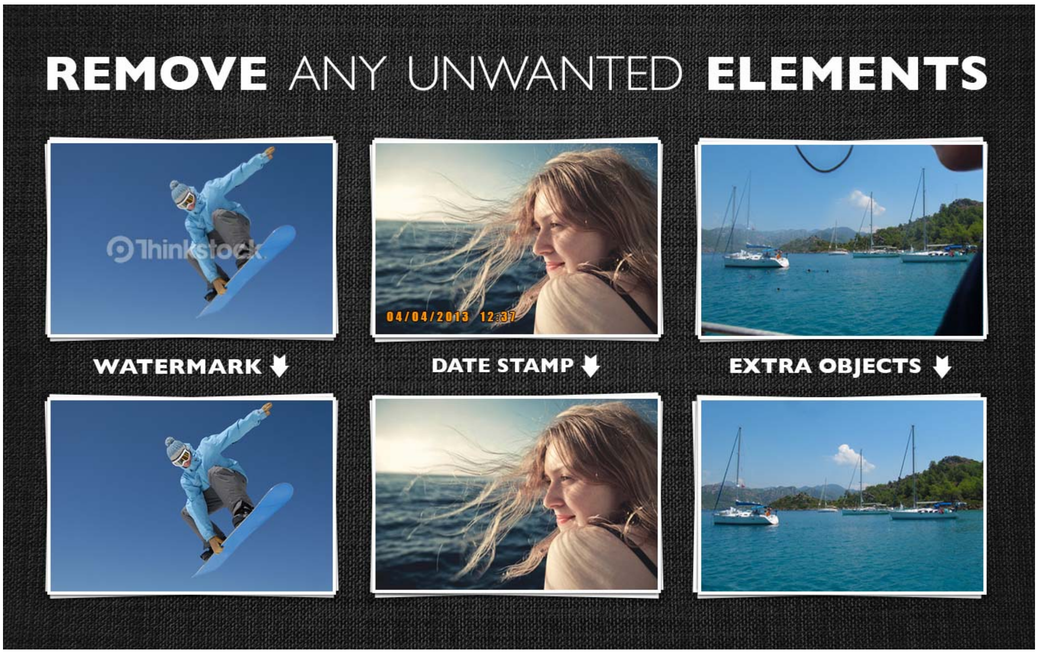
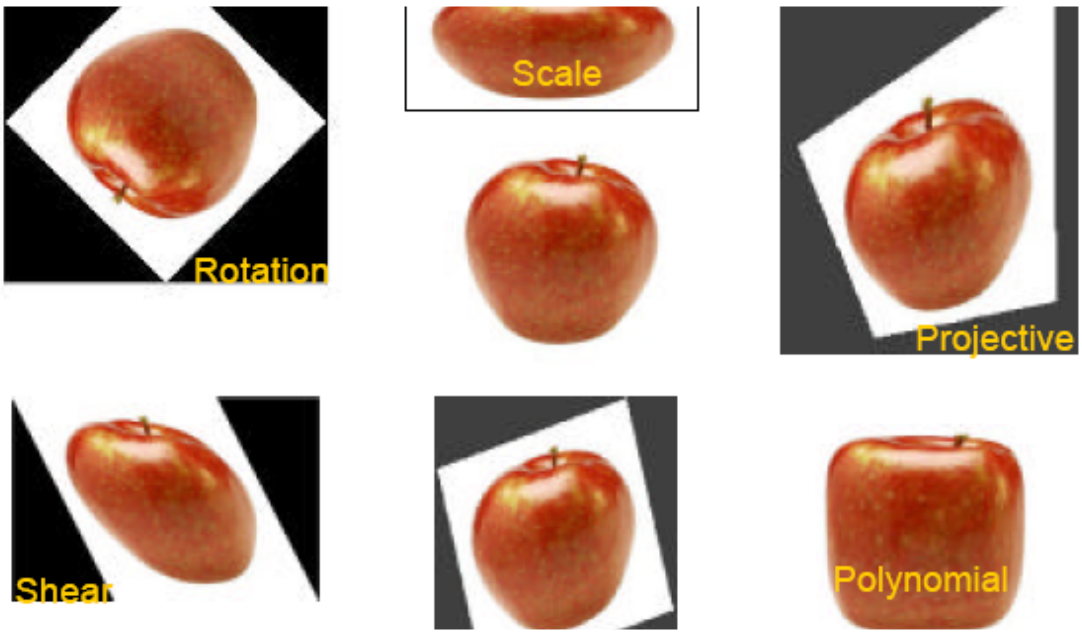


Image mosaicing

Geometric Transformation



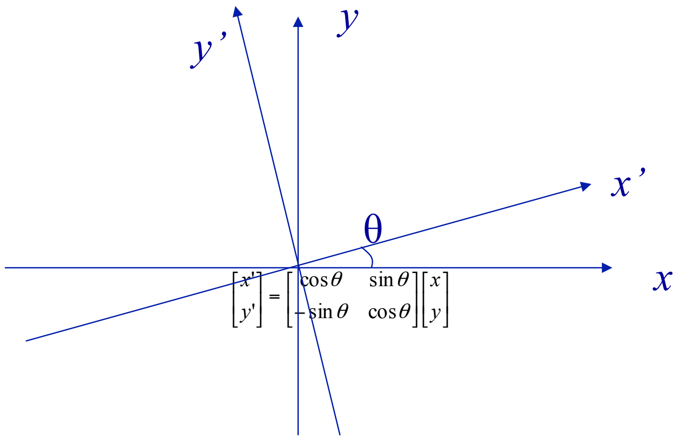
MATLAB functions: griddata, interp2, maketform, imtransform

Basic Principle

- (x,y) → (x',y') is a geometric transformation
- We are given pixel values at (x,y) and want to interpolate the unknown values at (x',y')
- Usually (x',y') are not integers and therefore we can use linear interpolation to guess their values

MATLAB implementation: interp2

Rotation



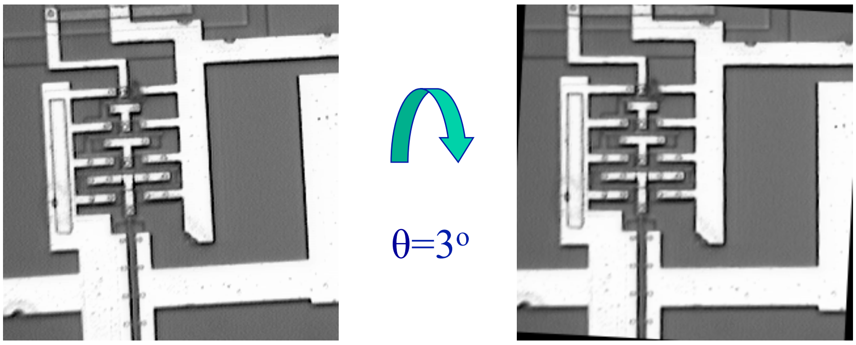
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

MATLAB Example

```
z=imread('cameraman.tif');
% original coordinates
[x,y]=meshgrid(1:256,1:256);

% new coordinates
a=2;
for i=1:256;for j=1:256;
x1(i,j)=a*x(i,j);
y1(i,j=y(i,j)/a;
end;end
% Do the interpolation
z1=interp2(x,y,z,x1,y1,'cubic');
```


Rotation Example

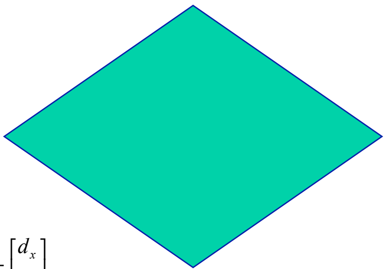


Scale

a=1/2

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Affine Transform



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

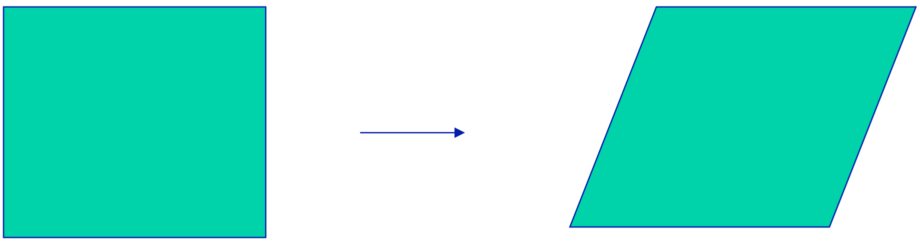
square

parallelogram

Affine Transform Example

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} .5 & 1 \\ .5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Shear



square

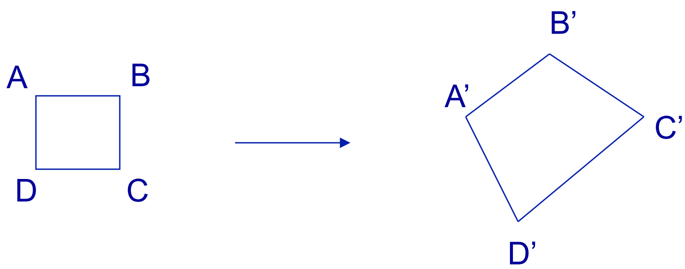
parallelogram

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

Shear Example

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ .5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Projective Transform

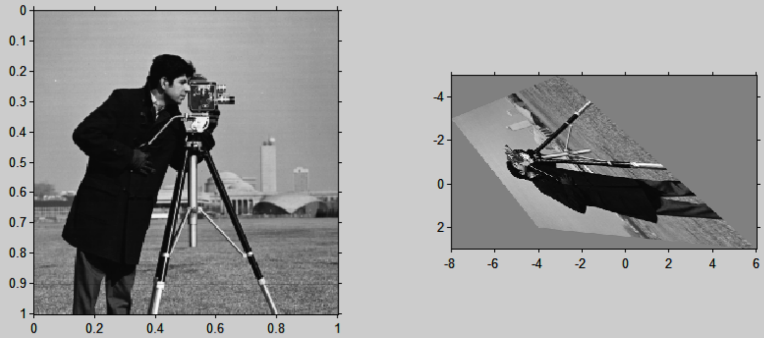


square quadrilateral

$$x' = \frac{a_1 x + a_2 y + a_3}{a_7 x + a_8 y + 1}$$

$$y' = \frac{a_4 x + a_5 y + a_6}{a_7 x + a_8 y + 1}$$

Projective Transform Example



[0 0; 1 0; 1 1; 0 1] ⟶ [-4 2; -8 -3; -3 -5; 6 3]

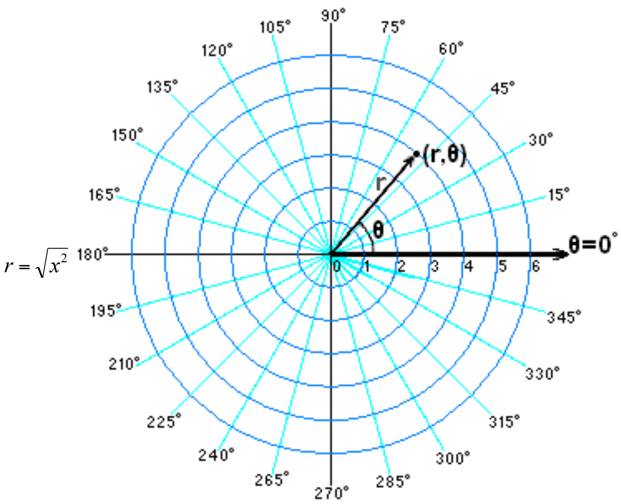
Polar Transform

$$r = \sqrt{x^2 + y^2}$$

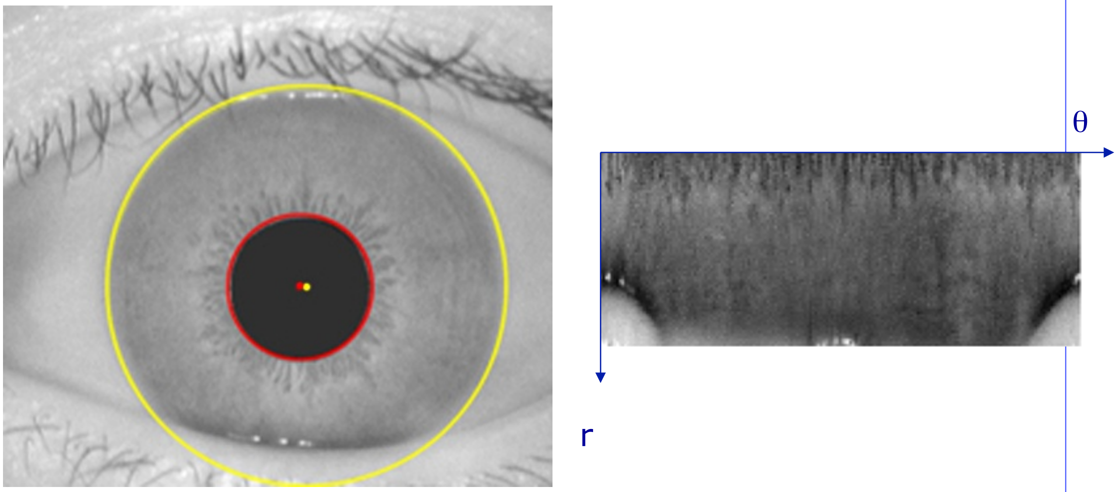
$$\theta = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\vartheta$$

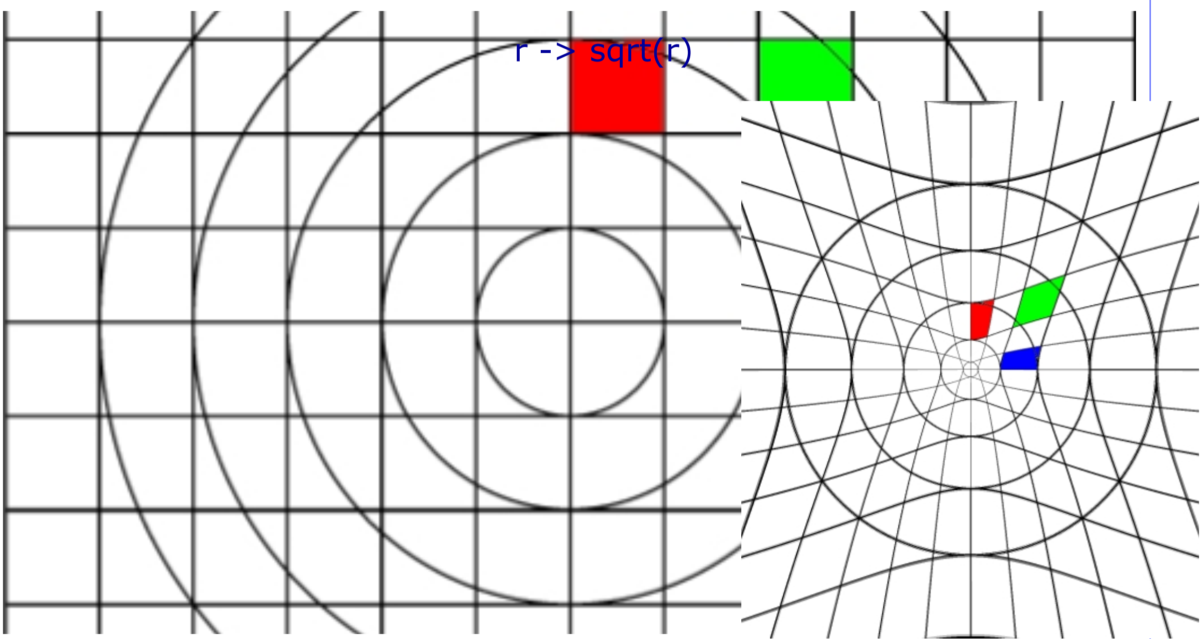
$$y = r\sin\vartheta$$



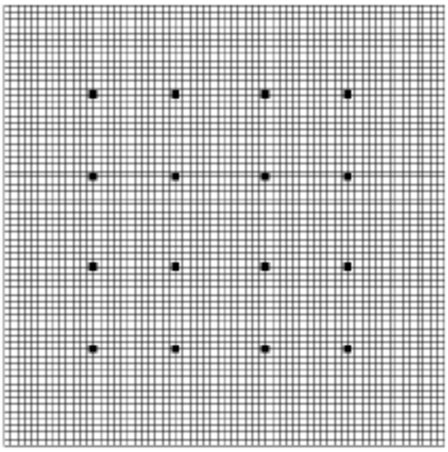
Iris Image Unwrapping

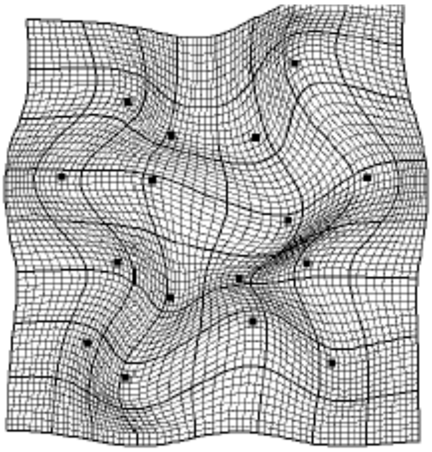


Use Your Imagination



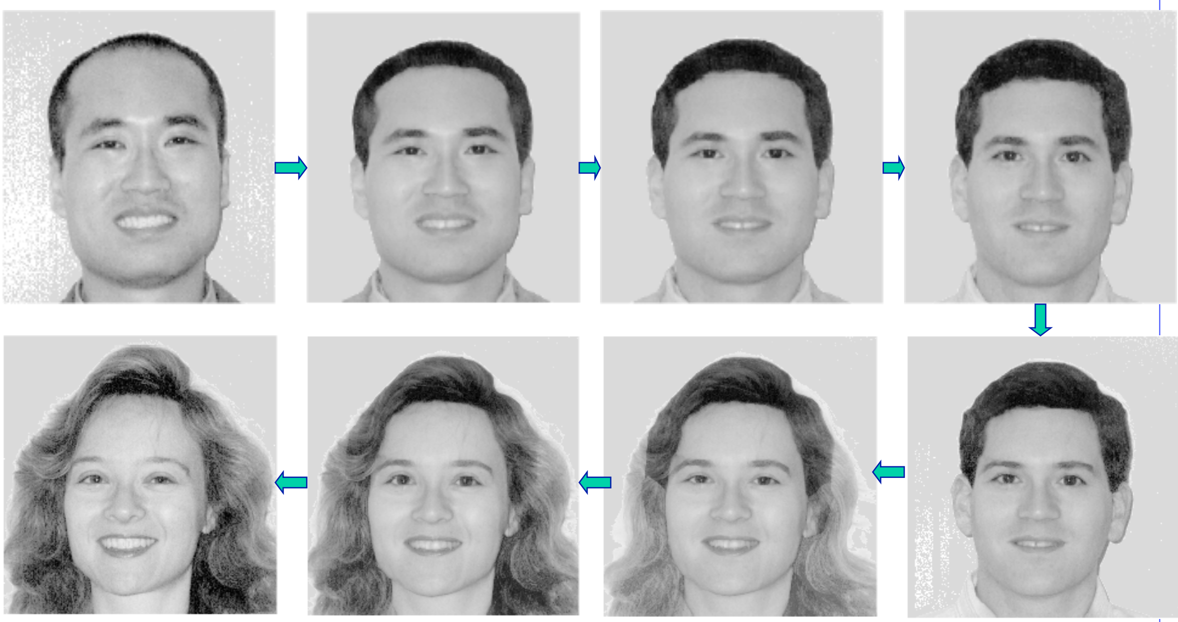
Free Form Deformation





Seung-Yong Lee et al., "Image Metamorphosis Using Snakes and Free-Form Deformations,"*SIGGRAPH'1985,* Pages 439-448

Application into Image Metamorphosis



Summary of Image Interpolation

- A fundamental tool in digital processing of images: bridging the continuous world and the discrete world
- Wide applications from consumer electronics to biomedical imaging
- Remains a hot topic after the IT bubbles break