Formal verification of hybrid systems using Ariadne

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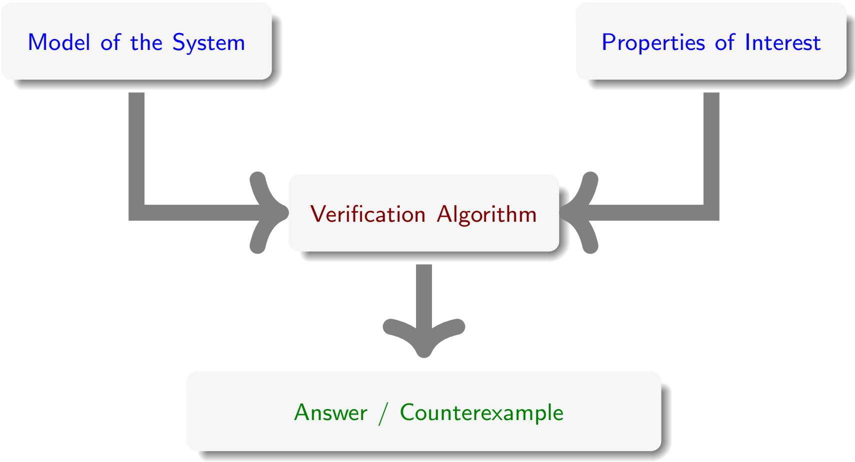
The formal verification flow



Hybrid systems

Many real systems have a double nature. They:

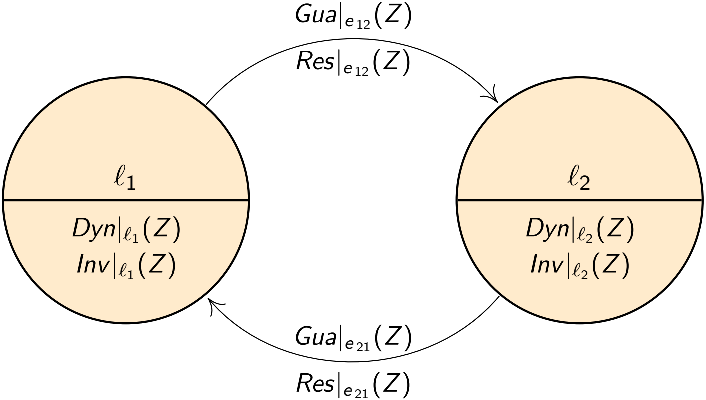
- evolve in a continuous fashion
- are controlled by a discrete system

Such systems are called hybrid systems and may be modeled by hybrid automata

Hybrid automata

The intuition

A hybrid automaton H is a finite-state automaton with continuous variables Z



A state is a couple (ℓ, r) where r is a valuation for Z

Hybrid automata

Functional representation

- dynamics $Dyn|_\ell$: evolution of the variables in location ℓ
- invariant $Inv|_\ell$: conditions under which continuous evolution is allowed in location ℓ
- guard $Gua|_e$: conditions under which discrete evolution is allowed according to event e
- reset $Res|_e$: transformation of the continuous state after event e

Reachability analysis

To verify whether a dynamical system satisfies some properties, we describe its behaviour by computing the set of reached states (reachable set) *Re*.

- It allows full observation of system evolution (compared to abstraction methods).

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Re is not computable in general, in particular for **nonlinear** systems.

Re can be approximated, but not in *both* an effective and efficient way.

- Some operations on accurate representations are still undecidable.
- Coarse approximations are problematic in terms of reliability of results.

How to approximate regions

Possible choices of approximating *Re*:

1. **Inner approximation I:** Re strictly contains I.
2. **Outer approximation O:** Re is strictly contained in O (an over-approximation of Re).
3. **ε-lower approximation L_ε:** every point of L_ε is at a distance less than ε from Re (an over-approximation of a subset of Re).

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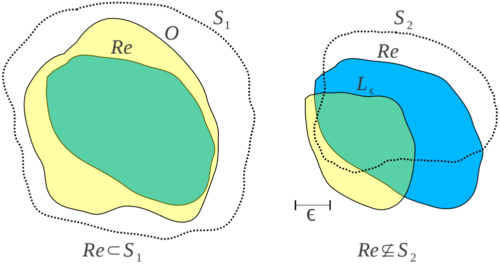
- Inner approximation is not computable in general.
- Outer and ε-lower approximations can be used to verify/falsify properties.

Property satisfaction in terms of sets



- *Re* is the reachable set, which is unobservable.
- S_1, S_2 are sets satisfying given properties.
- *O* is the outer approximation of *Re*.
- L_ε is the ε-lower approximation of *Re*.

A summary of Ariadne

- Developed by a joint team including the University of Verona, the University of Maastricht, the University of Padova, and the University of Barry (Florida)
- Uses the formalism of hybrid automata to describe nonlinear time-continuous systems.
- Based on rigorous semantics paired with interval arithmetics to guarantee correctness of verification over approximated sets.
- Written as a C++ library, released as an open source distribution: `http://www.ariadne-cps.org`

Semantics used for evolution

Upper semantics

When numerical inaccuracies make the transition undecidable, all possible choices are taken. Computed sets do not need to include a point of Re.

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Upper semantics

When numerical inaccuracies make the transition undecidable, all possible choices are taken. Computed sets do not need to include a point of Re.

Lower semantics

When numerical inaccuracies make the transition undecidable, evolution stops. Computed sets must include at least one point of Re.

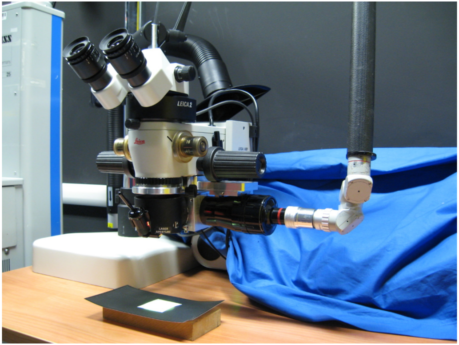
Reachable set approximations available

ARIADNE can compute the following approximations of the reachable set (available semantics under parentheses):

- An over-approximated subset, up to a given time t : for proving/disproving properties where a bound on the evolution time is identified [upper, lower];
- An outer approximation : for proving properties using infinite-time evolution [upper];
- For a given $\varepsilon > 0$, an ε-lower approximation : for disproving properties using infinite-time evolution [lower].

Current field of application: robotic surgery

- Very strict safety requirements.
- Increasing reliance on assisted control for improved accuracy.
- Traditionally focused on control theory specifications, recently adopting formal verification approaches.



Project proposal: Acquamondo

Combine hydraulic components to obtain a complex system.

- Focus on finite-time reachability
- Project instructions: `http://www.ariadne-cps.org/files/acquamondo.pdf`