$$P(0) \;\&\; \forall x\,(P(x) \to P(x+1)) \to \forall z\, P(z)$$

$$n > 0$$

$$\boxed{P(n) \;\&\; \forall x \geq n\,(P(x) \to P(x+1)) \to \forall z \geq n\; P(z)}$$

$$Q(z) \equiv P(z+n)$$

$$P(0+n) \;\&\; \forall x\,(P(x+n) \to P(x+1+n)) \to$$

$$\forall y\,(P(y+n))$$

$$Q(0) \;\&\; \forall x\,(Q(x) \to Q(x+1))$$

$$\to \forall y\; Q(y)$$

$$P(0) \;\&\; \forall x\,(P(x) \to P(x+1)) \to \forall y\, P(y)$$

$$P(0) \;\&\; \forall n\,((\forall k \leq n\; P(k)) \to P(n+1)) \to \forall y\, P(y)$$

$$\cancel{P(0)} \;\&\; \boxed{\forall n\,((\forall k < n\; P(k)) \to P(n))} \to \forall y\, P(y)$$

$$\Updownarrow$$

$$\forall n\,(\forall k\,(k < n \Rightarrow P(k)) \to P(n))$$

$$\Downarrow$$

$$\underbrace{\underbrace{\forall k\,(\underbrace{k<0}_{\text{FALSA}} \Rightarrow P(0))}_{\text{VERA}} \to P(0)}_{\text{VERO}}$$

$$\Uparrow$$

$$P(0)$$

$B \subsetneq A$ A finito $|A| > 0$

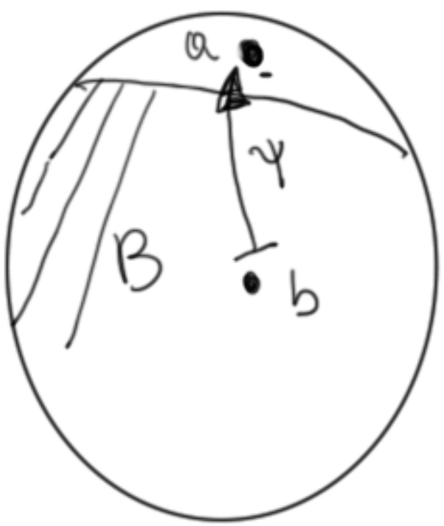
NON ESISTE $\psi : B \to A$ BIIETTIVA

PER INDUZIONE SU $|B|$

BASE $|B| = 0$ $B = \emptyset$ $\psi : \emptyset \to A$ BIIETTIVA

PASSO INDUTTIVO $|B| = n+1$

ASSUMIAMO CHE LA PROP. VALGA PER INSIEMI X CON $|X| = n$



$\exists\, \psi : B \to A$ BIIETT.

$B - \{b\}$ $A - \{a\}$ $\bar{\psi} = \psi - \{(b, a)\}$

$\bar{\psi} : B - \{b\} \to A - \{a\}$ BIIETTIVA

$|B - \{b\}| = n$

ASSURDO

$\mathbb{N}$ $\quad S \subseteq \mathbb{N}$ $\Rightarrow$ se S È INFINITO ALLORA
S È NUMERABILE

S infinito

$\varphi : \mathbb{N} \to S$ BIIETTIVA

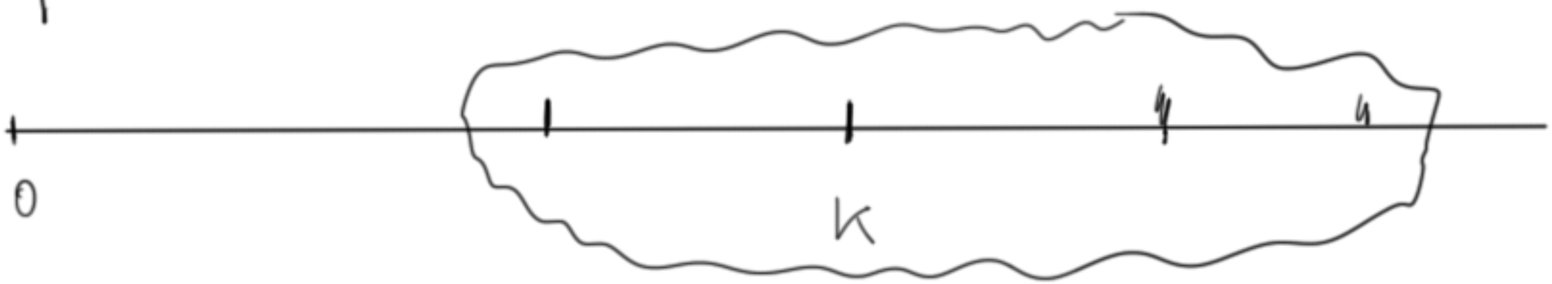
TEOREMA
OGNI SOTTOINSIEME $\neq \emptyset$ DEI NATURALI HA UN MINIMO

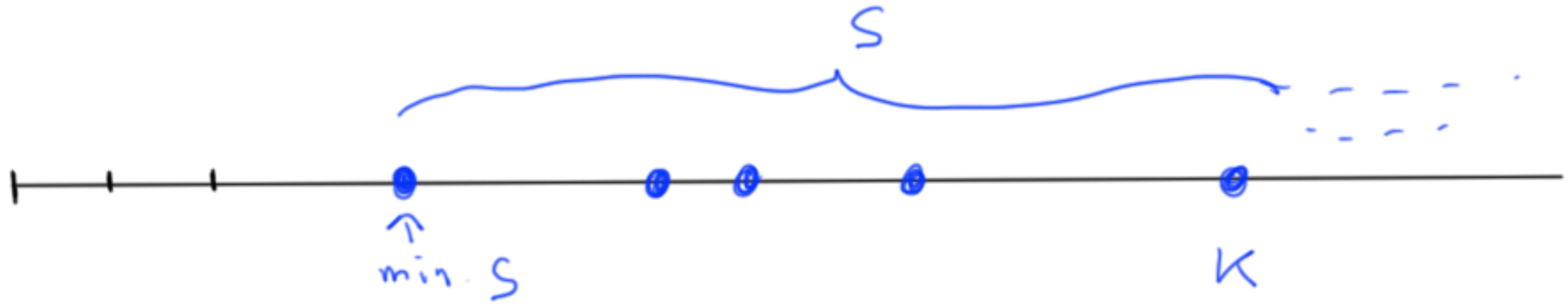
$\varphi(0) = \min S$

$\varphi(1) = \min(S - \{\varphi(0)\})$

$\vdots$

$\varphi(n) = \min(S - \{\varphi(0), \varphi(1), \dots, \varphi(n-1)\})$





$K \in S$

$\bar{m} = \min S$

$K - \bar{m}$

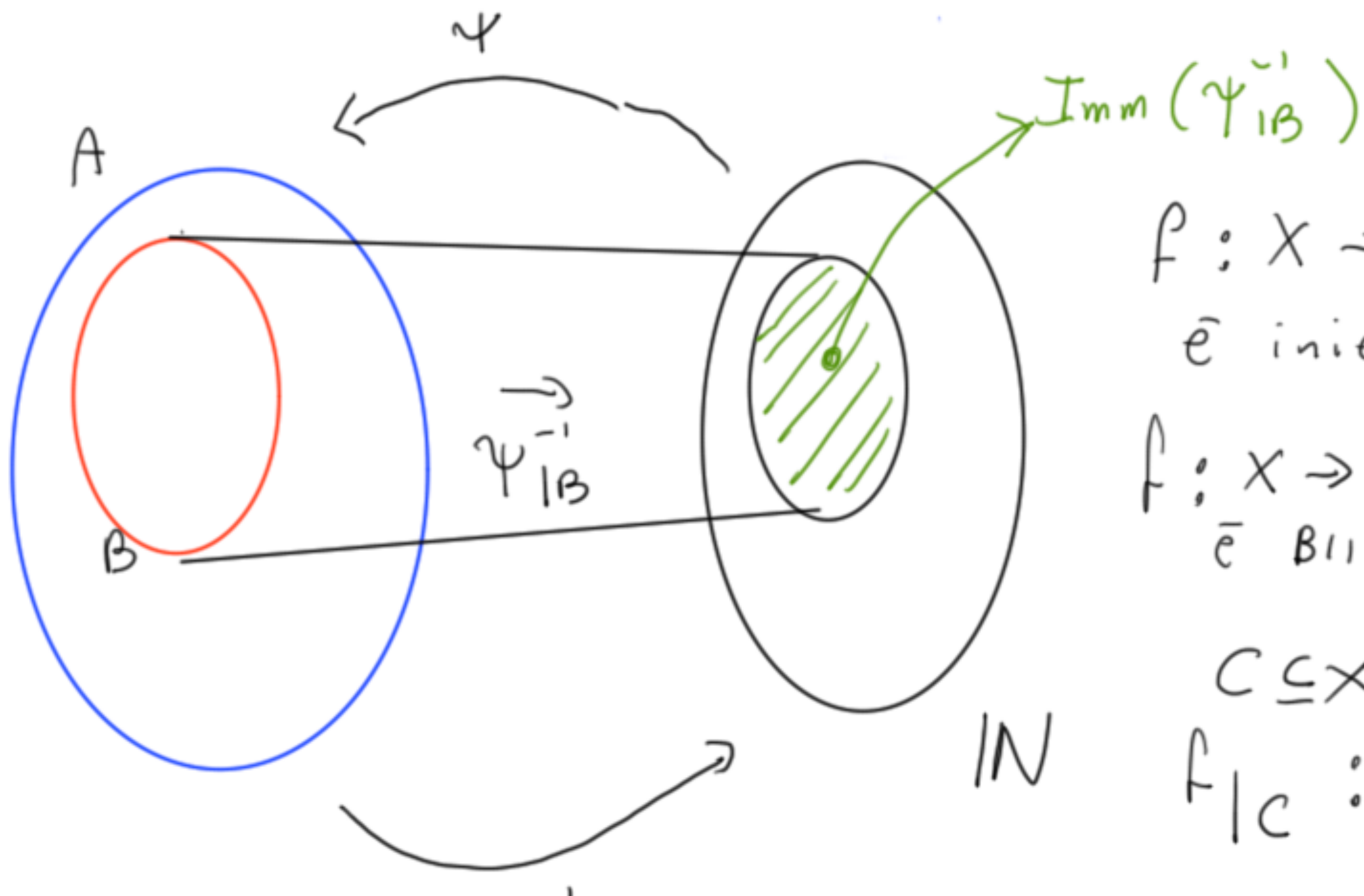
$\varphi(0) < \varphi(1) < \varphi(2)$

$\bar{m} = m_0$

$\bar{m} = m_0 < m_1 \dots < m_i = K$

$\varphi(0) = m_0 \quad \varphi(1) = m_1 \quad \dots \quad \varphi(i) = m_i = K$

A è numerabile $B \subseteq A$ & B infinito $\Rightarrow |B| = \aleph_0$



$f: X \to Y$
è iniettiva

$f: X \to Im(f)$
è BIIETT.

$C \subseteq X$

$f_{|C}: C \to Y$

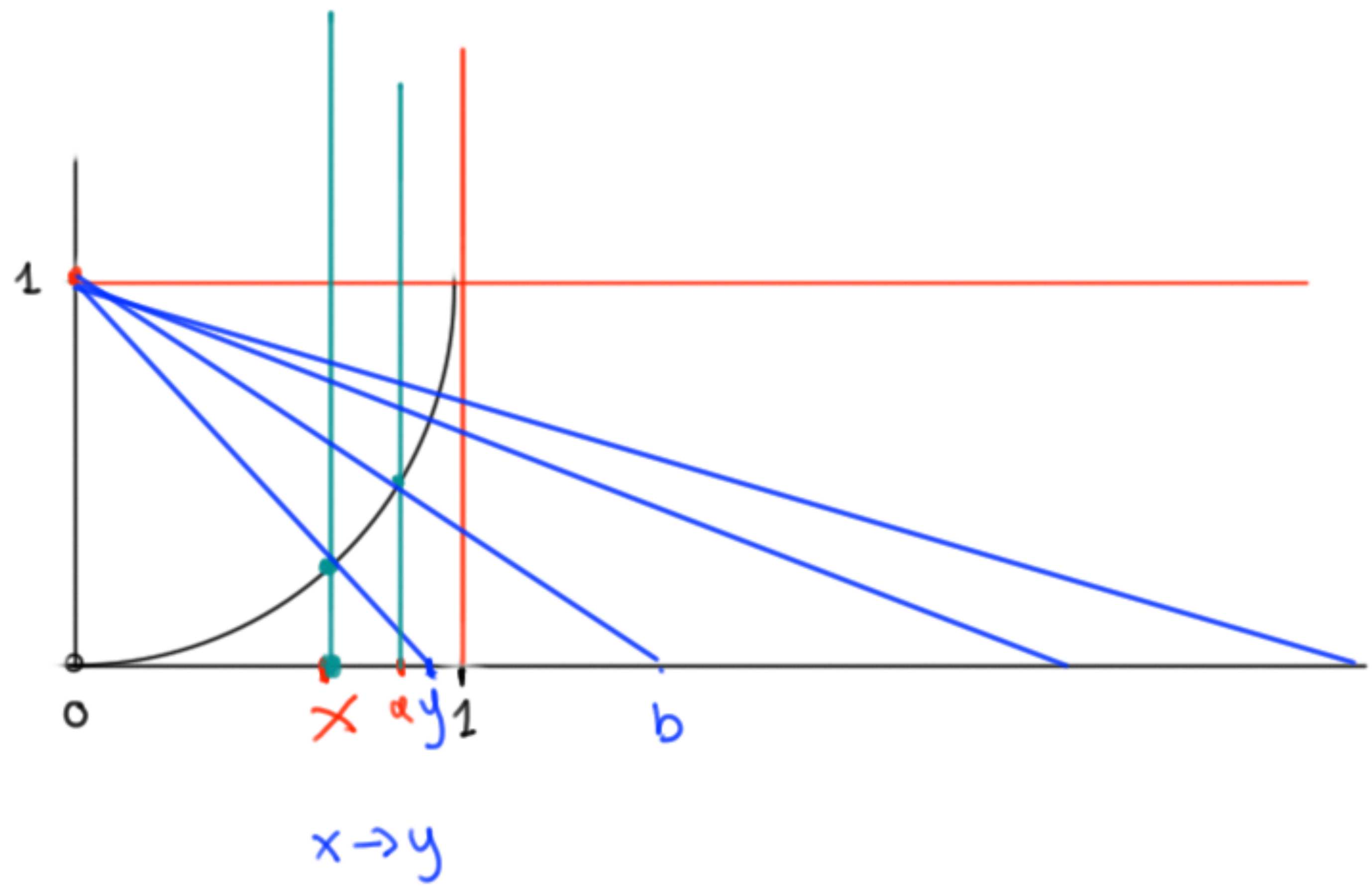
$\psi^{-1}_{|B}: B \to \mathbb{N}$

$\psi^{-1}_{|B}: B \to Imm(\psi^{-1}_{|B}) \rightsquigarrow$ è biiettiva.

$\mathbb{R}$ NON SONO NUMERABILI !!!!

$\mathbb{Q}$ SONO NUMERABILI

$|\mathbb{R}| \neq \aleph_0$ $\quad |\mathbb{Q}| = \aleph_0$



$$[0,1)_{\mathbb{R}} \xrightarrow{\Psi} \mathbb{R}^{\geq 0}$$

$[0,1)$

$0, 1, 0, 1 \ldots$

$[0,1) = \{ b \mid b : \mathbb{N} \to \{0, 1\} \}$ CANTOR

$b[i]$ i-sima cifra binaria