$$y = f(x) = \frac{8}{3} x^3 \left(\log x - \frac{1}{3}\right) + x$$

ANALISI MATEMATICA I

1° modulo ~~Bioinformatica~~

a.a. 2007/08 Prof. M. Ipera

APPENDICE alla LEZIONE III

Altri studi di funzione

dominio $x > 0$

$$\lim_{x \to 0^+} f(x) = 0$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

non ci sono asintoti $\left(\because \lim_{x \to +\infty} \frac{f(x)}{x} = +\infty \right)$

$$f'(x) = (x\, g(x))' \qquad \left(g(x) = \frac{8}{3} x^2 \left(\log x - \frac{1}{3}\right) + 1 \right)$$

$$= \dots \; 8x^2 \log x + 1$$

$$f''(x) = 16x \log x + 8x^2 \frac{1}{x} = 16x\left(\log x + \frac{1}{2}\right)$$

segno di f''

$e^{-\frac{1}{2}}$ ← flesso ascendente

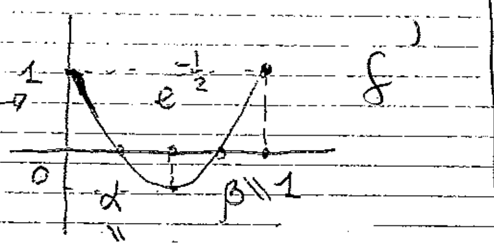
$f(e^{-\frac{1}{2}}) = \dots = \left(1 - \frac{20}{9e}\right) e^{-\frac{1}{2}} > 0$

$f'(e^{-\frac{1}{2}}) = \dots = 1 - \frac{4}{e} < 0$

$f'(0) = 1$; $f'(1) = 1$; $f(1) = 1 - \frac{8}{9} > 0$

→ def × continuità

$$\lim_{x \to 0^+} f''(x) = 0$$



⇒ Segno di

$$y = \frac{8}{3} x^3 \left(\log x - \frac{1}{3}\right) + x$$

Si ha, posto $\frac{1}{3} = \log e^{\frac{1}{3}}$

$$y = \dots = \frac{8}{3} x^3 \log\left(\frac{x}{e^{1/3}}\right) + x$$

poiché $x > 0$, studiamo

$$\frac{8}{3} x^2 \log\left(\frac{x}{e^{1/3}}\right) + 1 =$$

$$= \frac{8}{3} e^{2/3} \left(\frac{x}{e^{1/3}}\right)^2 \log\left(\frac{x}{e^{1/3}}\right) + 1$$

$$= \dots \frac{8}{3} e^{2/3} z^2 \log z + 1$$

Det. gli eventuali zeri $= 0$

$$z^2 \log z = \frac{-1}{\frac{8}{3} e^{2/3}} = \frac{-3}{8 e^{2/3}}$$

Studiamo $y = x^2 \log x \qquad x > 0$

⇒ $y(0) = \dots 0 \qquad \lim_{x \to +\infty} y = +\infty$

$$y'(x) = 2x \log x + x^2 \frac{1}{x} = 2x\left(\log x + \frac{1}{2}\right) \qquad x > 0$$

⇒ y': $\quad \overset{e^{-1/2}}{- - - - 0 \text{———}}$

minimo assoluto

$x = e^{-\frac{1}{2}}$ è

$$x^2 \log x = e^{-1} \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2e}$$

confrontiamo $-\dfrac{1}{2e}$ e $\dfrac{-3}{8e^{2/3}}$

$$-\frac{1}{2e} \overset{?}{<} -\frac{3}{8e^{2/3}}$$

$$\Big[\Updownarrow$$

$$\frac{3}{8e^{2/3}} < \frac{1}{2e} \iff \frac{8e^{2/3}}{3} > 2e$$

$$\iff e^{2/3} > \frac{3}{4}e \iff e^2 > \left(\frac{3}{4}\right)^3 e^3$$

$$1 > \left(\frac{3}{4}\right)^3 e \Big] \iff \left(\frac{4}{3}\right)^3 > e$$

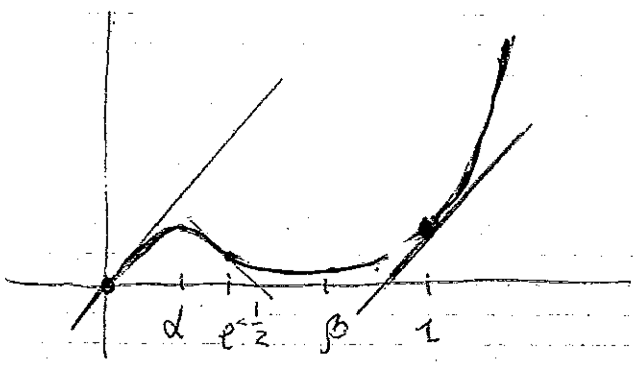
$$\parallel$$

$$\left(1+\frac{1}{3}\right)^3 > e \quad \underline{\underline{NO}}$$

[ricordare che $\left(1+\frac{1}{n}\right)^n \nearrow e$ (str. crescente)]

$\Rightarrow f(x) > 0 \quad x > 0$

☆ grafico di f



$$\boxed{f(x) = x\left|1+\frac{1}{\log x}\right|}$$

$x > 0 \quad x \neq 1$ (dominio)

1

• estremi del dominio

$$\lim_{x\to 0^+} f(x) = 0 \qquad \lim_{x\to 1} f(x) = +\infty$$

• asintoto verticale $x=1$

Segno: sempre positivo quando $x=0$, $x = e^{-1}$

• eventuali asintoti obliqui

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \left|1+\frac{1}{\log x}\right| = 1$$

$$\lim_{x\to+\infty} f(x)-x = \lim_{x\to+\infty} x\left(\left|1+\frac{1}{\log x}\right|-1\right)$$

ma $1+\frac{1}{\log x} - 1 = \frac{1}{\log x}$

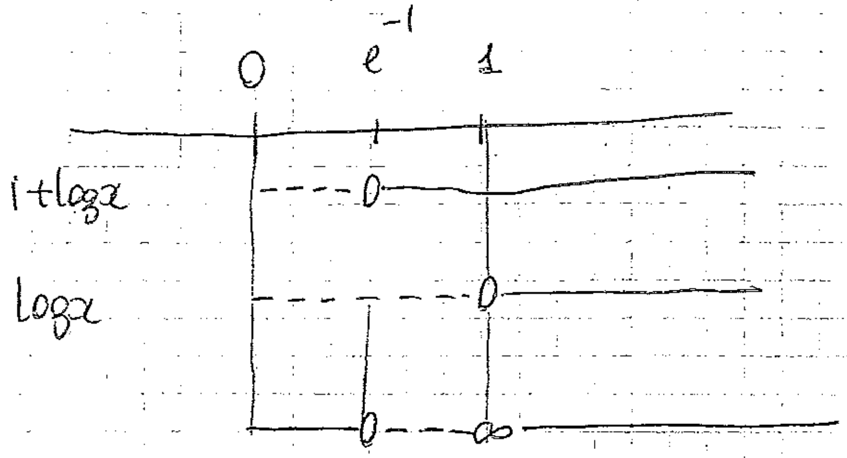
$$\text{e } \lim_{x\to+\infty} \frac{x}{\log x} = \lim_{x\to+\infty} \frac{1}{\frac{1}{x}} = +\infty$$

• non ci sono asintoti obliqui

• Derivabilità, max, min

Studiamo $\left|1+\frac{1}{\log x}\right|$

$$1+\frac{1}{\log x} \geq 0 \qquad \frac{\log x+1}{\log x} \geq 0$$



$$\Rightarrow f(x) = f_1(x) = x\left[1+(\log x)^{-1}\right] \qquad 0 \le x \le e^{-1},\quad x > 1$$

$$f(x) = f_2(x) = -x\left(1+\frac{1}{\log x}\right) \qquad e^{-1} \le x < 1$$

Studiamo f_1

$$f_1' = \left(1+\frac{1}{\log x}\right) + x\,D\left(\frac{1}{\log x}\right) = 1+\frac{1}{\log x} - x\,\frac{1}{x}\,\frac{1}{(\log x)^2}$$

$$= 1+\frac{1}{\log x} - \frac{1}{\log^2 x} = \frac{\log^2 x + \log x - 1}{\log^2 x} = \frac{t^2+t-1}{t^2}$$

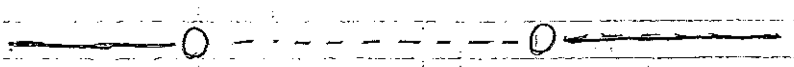
$t = \log x$

Segno di f_1' $\qquad t^2+t-1=0$

$$t = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1\pm\sqrt{5}}{2}$$

$$\lim_{x\to 0^+} f' = \lim_{t\to -\infty} \frac{t^2+t-1}{t^2} = 1$$

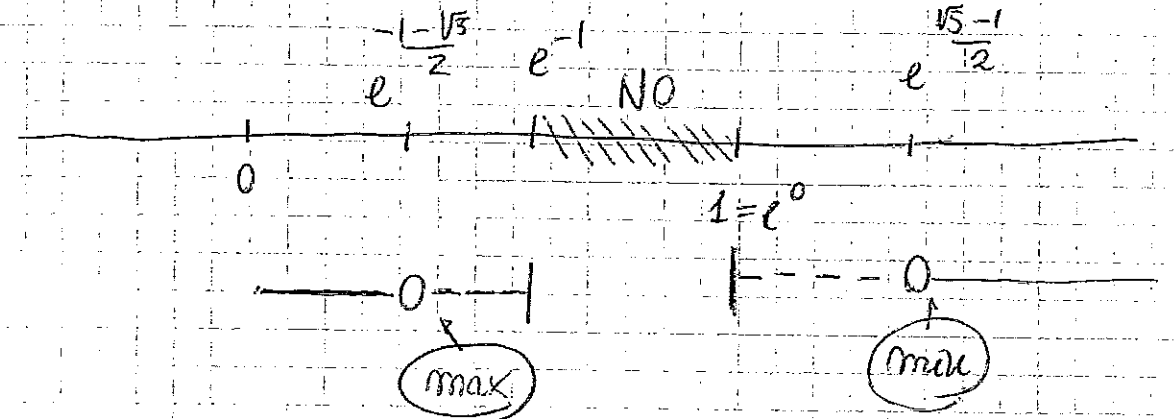
(t^2+t-1) $\qquad \frac{-1-\sqrt{5}}{2}$ $\qquad \frac{-1+\sqrt{5}}{2}$



$$\log x > \frac{\sqrt{5}-1}{2}$$

$x>1 \qquad x > e^{\frac{\sqrt{5}-1}{2}}$

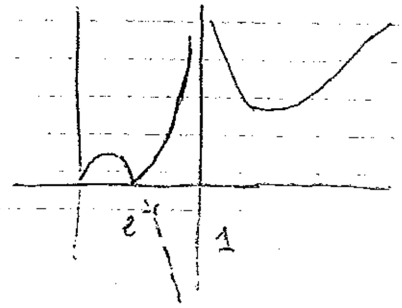
$0 \leq x \leq e^{-1} \qquad x < e^{\frac{-1-\sqrt{5}}{2}}$



Studiamo f_2 in $e^{-1} \leq x < 1$

$$f_2' = -f_1' \quad \text{in} \quad e^{-1} \leq x \leq 1$$

$\Rightarrow \boxed{\text{sempre} \geq 0}$



flessi

$$g(t) = \frac{t^2+t-1}{t^2}$$

$$f' = g(\log x) \qquad f'' = g' \cdot \frac{1}{x}$$

$$g' = \frac{(2t+1)t^2 - 2t(t^2+t-1)}{t^4} = \frac{2t^3+t^2-2t^3-2t^2+2t}{t^4}$$

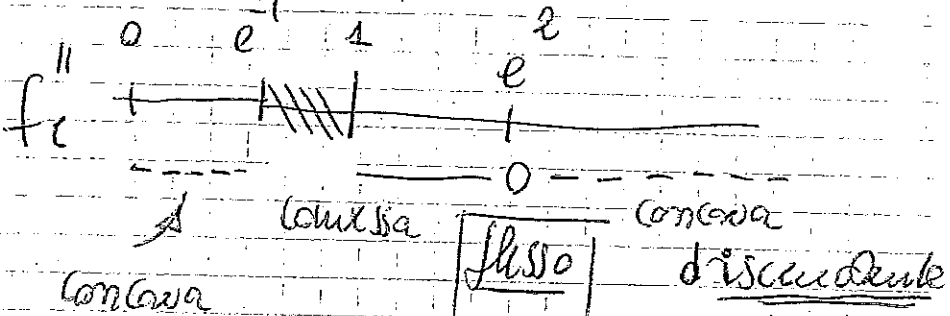
$$= \frac{-t^2+2t}{t^4} = \frac{-t+2}{t^3} =$$

$$= -\frac{t-2}{t^3}$$

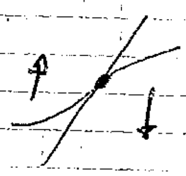
$$f_1''(x) = -\frac{\log x - 2}{\log^3 x} \cdot \frac{1}{x} \qquad x>1$$

$$\log x = 2 \qquad x = e^2 > 1 \Rightarrow f_1'' = 0$$

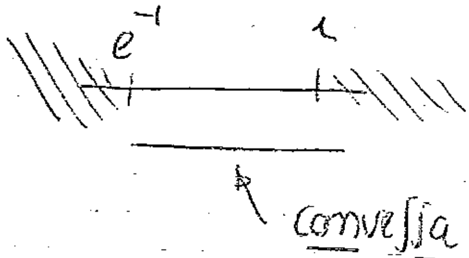
segno

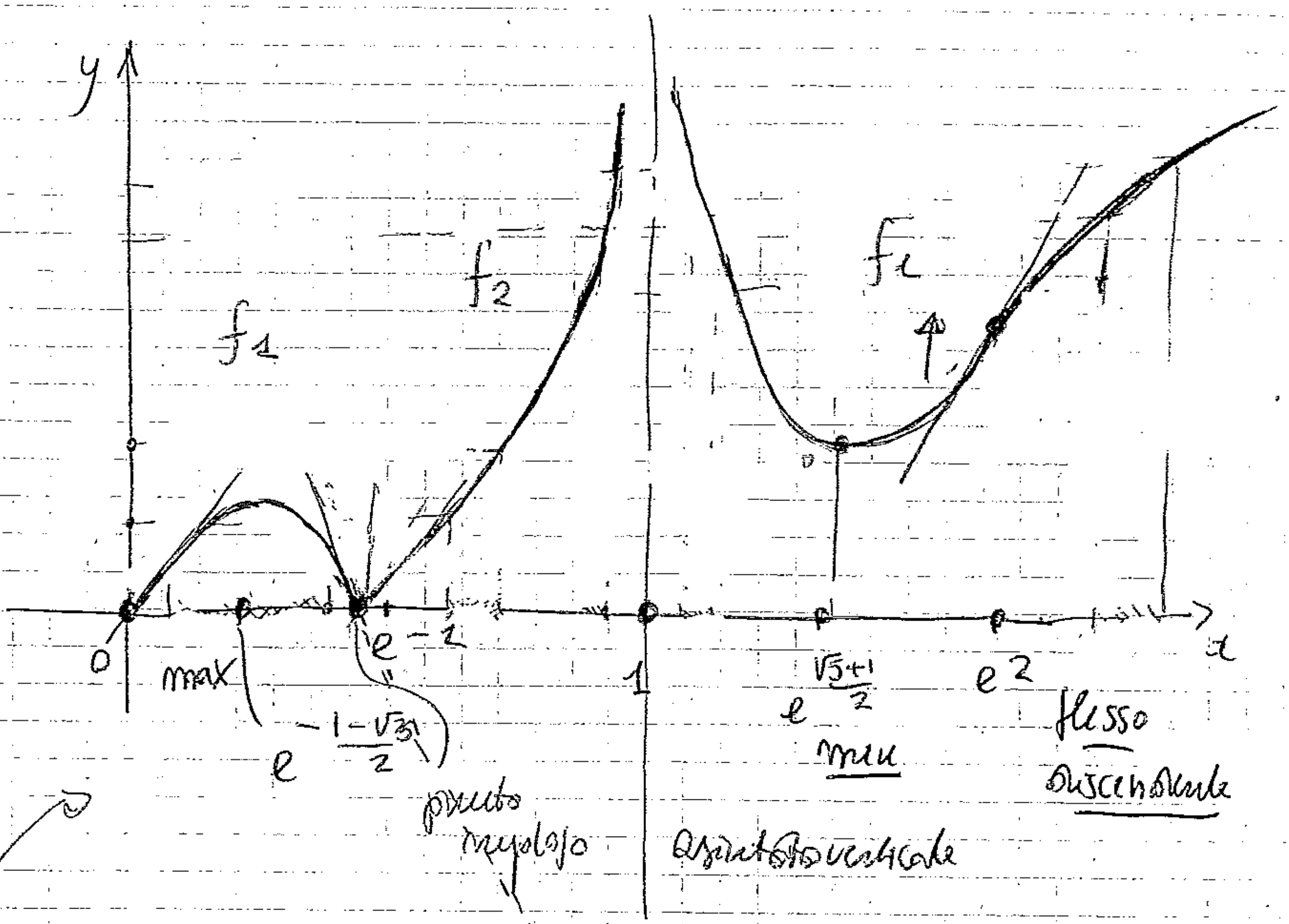


$$f(e^2) = e^2\left(1+\frac{1}{2}\right) = \frac{3}{2}e^2 \qquad f'(e^2) > 0$$



$$f_2''(x) = -\frac{2+\log x}{\log^3 x} \cdot \frac{1}{x} \qquad e^{-1} \le x < 1$$





Osservazione

Si può anche procedere riscrivendo

$$f(x) = x\left(1 + \frac{1}{\log x}\right)$$

e cambiare il segno a F in $(e^{-1}, 1)$ ...

ciò è più rapido

attacco

$$\lim_{x \to e^{-1-}} f_1' = \frac{(e^{-1})^2 + e^{-1} - 1}{e^{-2}} = e^2\left(\frac{1}{e^2} + \frac{1}{e} - 1\right)$$

$$= 1 + e - e^2$$

$$\lim_{x \to e^{-1+}} f_2' = -(1 + e - e^2)$$

punto angoloso

$$\boxed{y = \arctg\left(x\left|1+\log x^{-1}\right|\right)}$$

Importante

$f'(g(x))\cdot g'(x)$

$x \to 0^+ \quad \downarrow 1 \quad \downarrow 1$

$$y' = \arctg'(\quad)\, D(\quad)$$

>0

$$\left(\frac{1}{1+x^2}\right)$$

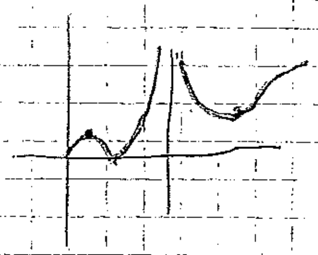
$$y = f(g(x))$$

se $f' > 0$

max e min stessa struttura

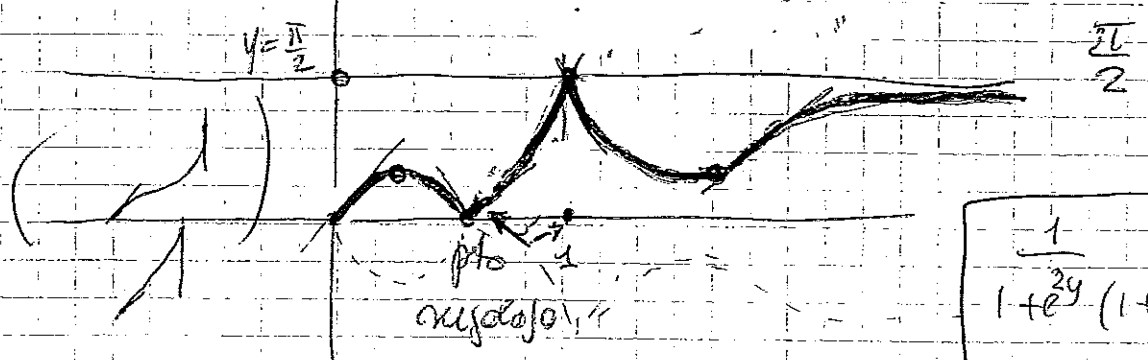
$$y' = f'(g(x))\cdot g'(x)$$

$$y'' = f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$$



$$\left(\arctg(+\infty) = \frac{\pi}{2}\right)$$

☆ è sparito l'asintoto verticale



$$y' = \frac{1}{1+(\quad)^2} D(\quad)$$

$\log x \to 0$

$$\frac{1}{1+\left(x\left|1+\frac{1}{\log x}\right|\right)^2}\ \frac{\log x^2 + \log x - 1}{\log x^2}$$

$$\lim_{y\to 0} \frac{1}{1+\left(e^y\left|1+y^{-1}\right|\right)^2}\ \frac{y^2+y-1}{y^2} =$$

$$= \frac{1}{1+e^{2y}\left(1+\frac{1}{y}\right)^2} = \frac{1}{1+e^{2y}\frac{(y+1)^2}{y^2}}\ \frac{y^2+y-1}{y^2}$$

$$= \frac{y^2+y-1}{y^2+e^{2y}(y+1)^2} \to -1$$

$$☆ \quad y = f(x) = \frac{1}{4e^x(x-1)+x}$$

Sia $g = \frac{1}{f}$ $(f \neq 0)$ $f = \frac{1}{g}$

$$f' = -\frac{g'}{g^2}$$

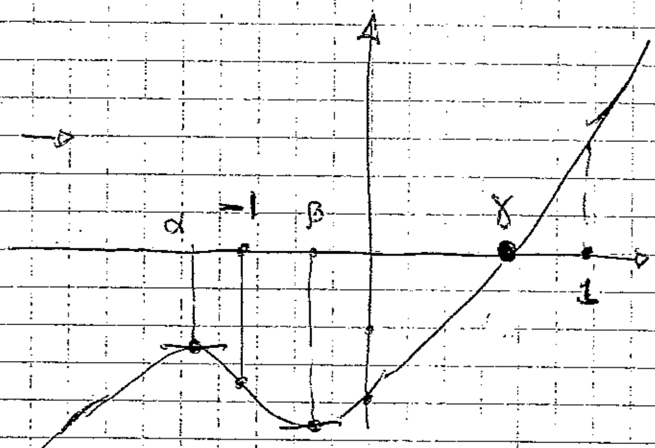
$$f'' = -\frac{g''g^2 - g'2gg'}{g^4} = -\frac{g''g - 2g'^2}{g^3}$$

$$g = 4e^x(x-1) + x$$

$$g' = 4e^x(x-1) + 4e^x + 1 = 4e^x x + 1$$

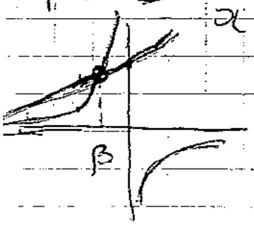
$$g'' = 4e^x x + 4e^x = 4e^x(x+1)$$

$g \rightarrow$



$g' = 0$

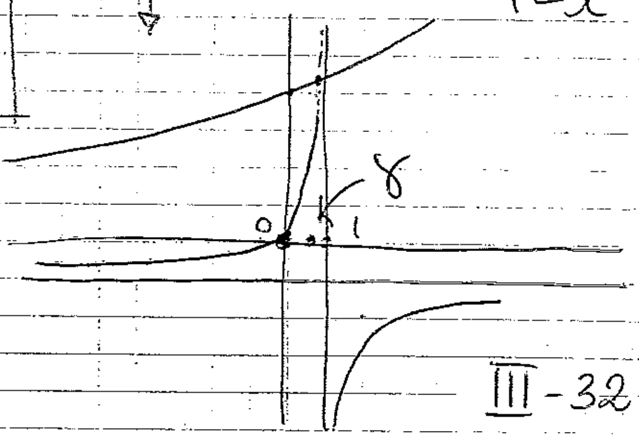
$4e^x = -\frac{1}{x}$



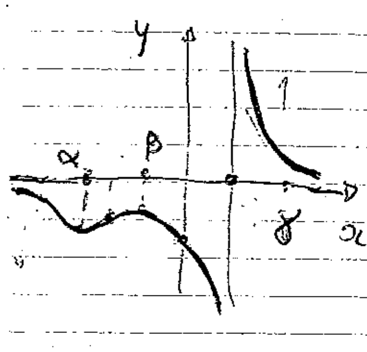
$g = 0$

$$4e^x(x-1) + x = 0$$

$$4e^x = \frac{x}{1-x}$$



☆☆☆ grafico di f

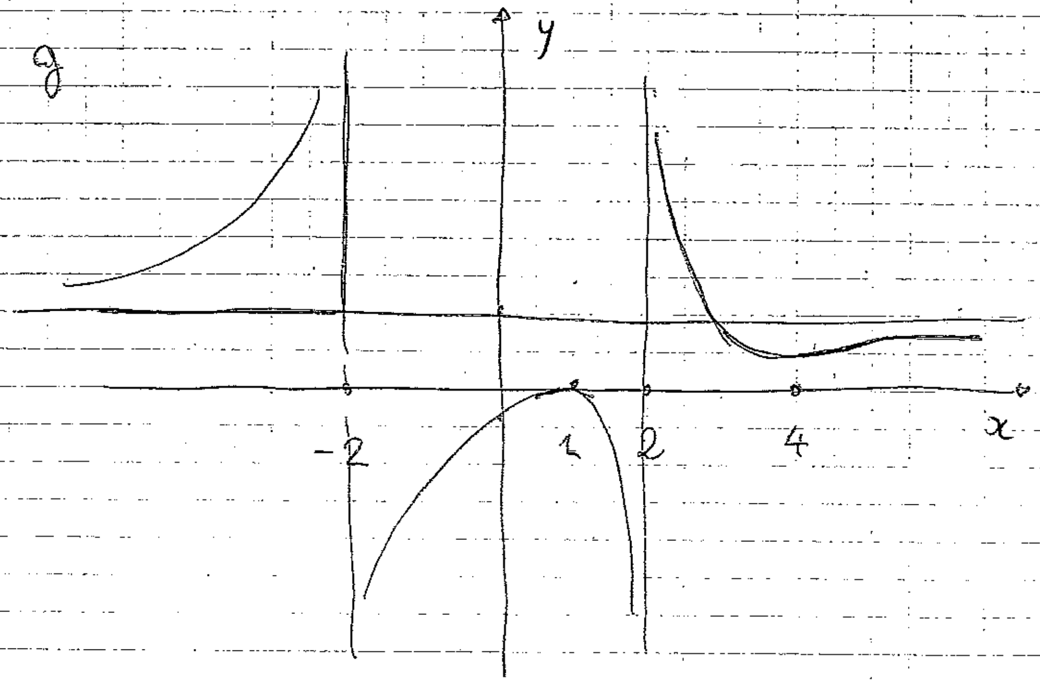


$$y = f(x) = \operatorname{arctg}\left(\frac{x^2-2x+1}{x^2-4}\right)$$

$$g(x) = \frac{x^2-2x+1}{x^2-4} = \frac{(x-1)^2}{x^2-4}$$

$$\left[\begin{array}{r|l} x^2-2x+1 & x^2-4 \\ \underline{-x^2 \qquad +4} & 1 \\ -2x+5 & \end{array}\right]$$

$$g(x) = 1 + \frac{-2x+5}{x^2-4}$$



$$g'(x) = \frac{2(x^2-4) - (-2x+5)\,2x}{(\quad)^2}$$

$$= \frac{-2x^2+8+4x^2-10x}{(\quad)} =$$

$$= \frac{2x^2-10x+8}{(\quad)^2} = \frac{2}{(\;)^2}(x^2-5x+4)$$

$$= \frac{2}{(\;)^2}(x-1)(x-4)$$

-2 1 2 4

$-\infty$ 0 $-\infty$ 0

$$g(4) = 1 + \frac{-8+5}{12} = 1 - \frac{3}{12} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$g(1) = 1 + \frac{-2+5}{-3} = 1 - 1 = 0$$

$$g(0) = 1 - \frac{5}{4} = -\frac{1}{4}$$

$$f' = \frac{1}{1+g(x)^2}\,g'(x) = \frac{1}{1+\left(1+\frac{-2x+5}{x^2-4}\right)^2}\,(x-1)(x-4)\,\frac{2}{(x^2-4)^2}$$

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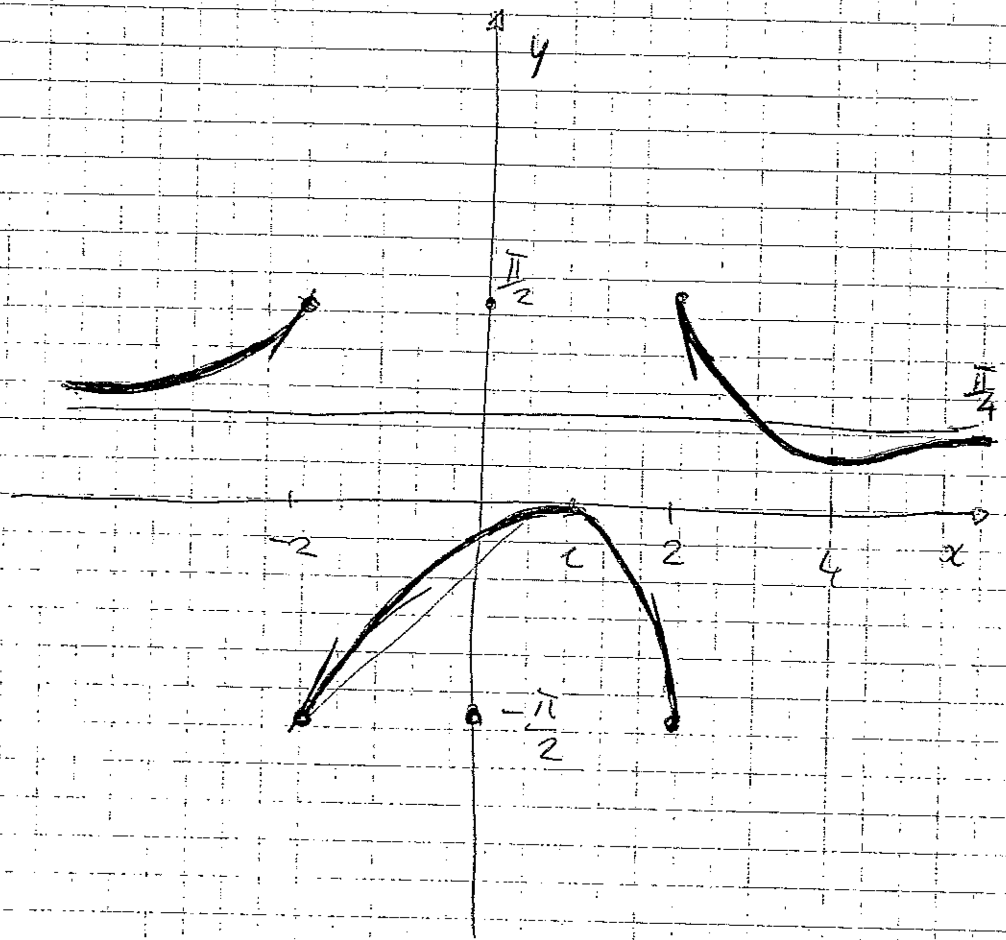
$$= \frac{1}{1+(x^2-2x+1)^2}\,\frac{2(x-1)(x-4)}{(x^2-4)^2} = \frac{2(x-1)(x-4)}{(x^2-4)^2+(x^2-2x+1)^2}$$

$f(x) = \arctan g(x)$



$$g'(x) = \frac{2}{(x^2-4)^2}(x-1)(x-4)$$

$$\lim_{x\to -2} g'(x) = +\infty$$

$$x \to +2 = -\infty$$

$$g' = \frac{2(x-1)(x-4)}{(x^2-4)^2+(x^2-2x+1)^2}$$

$x \to 2$

$$\frac{2\cdot 1\cdot(-2)}{0+1} = -2$$

$x \to -2$

$$\frac{2(-3)(-6)}{0+(4+4+1)^2} = +\frac{36}{9^2} = +\left(\frac{6}{9}\right)^2 = +\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

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