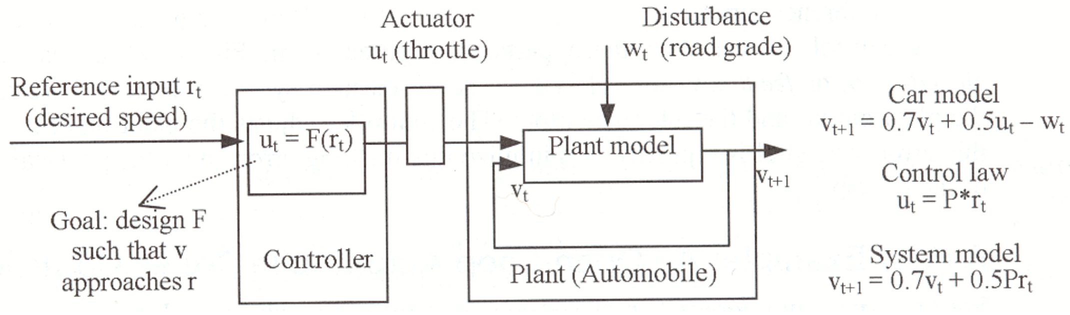
Embedded Systems Design: A Unified Hardware/Software Introduction

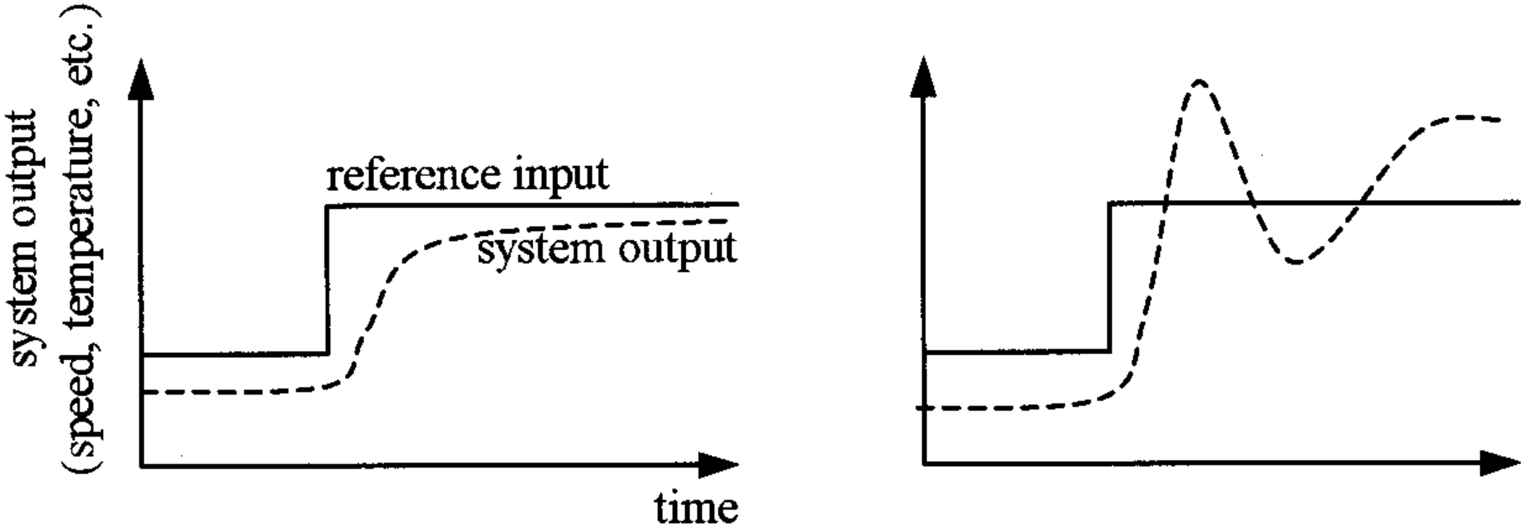
# Chapter 9: Control Systems

# Control System

- Control physical system’s output
  - By setting physical system’s input
- Tracking
- E.g.
  - Cruise control
  - Thermostat control
  - Disk drive control
  - Aircraft altitude control
- Difficulty due to
  - Disturbance: wind, road, tire, brake; opening/closing door…
  - Human interface: feel good, feel right…

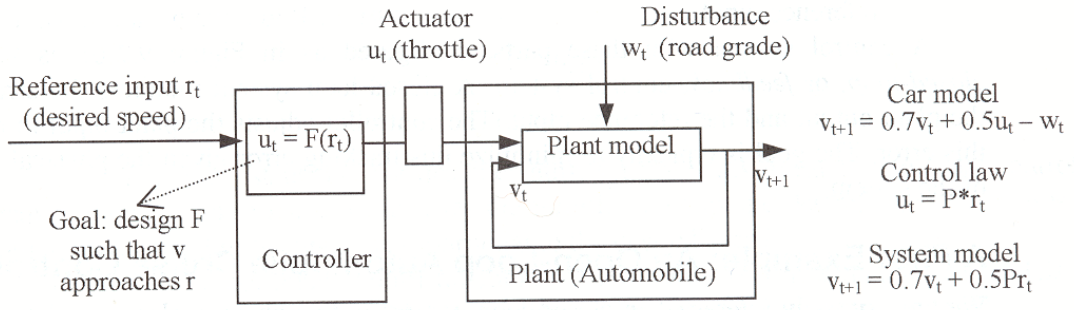


# Tracking


system output
(speed, temperature, etc.)
reference input
system output
time

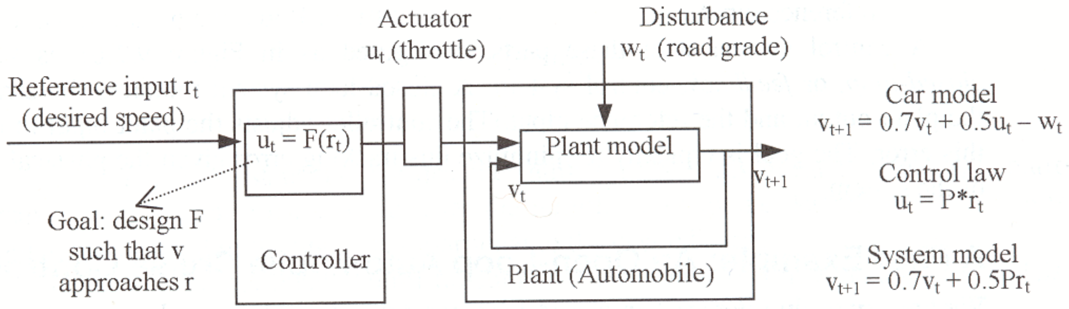
# Open-Loop Control Systems

- Plant
  - Physical system to be controlled
    - Car, plane, disk, heater,…
- Actuator
  - Device to control the plant
    - Throttle, wing flap, disk motor,…
- Controller
  - Designed product to control the plant



# Open-Loop Control Systems

- Output
  - The aspect of the physical system we are interested in
    - Speed, disk location, temperature
- Reference
  - The value we want to see at output
    - Desired speed, desired location, desired temperature
- Disturbance
  - Uncontrollable input to the plant imposed by environment
    - Wind, bumping the disk drive, door opening



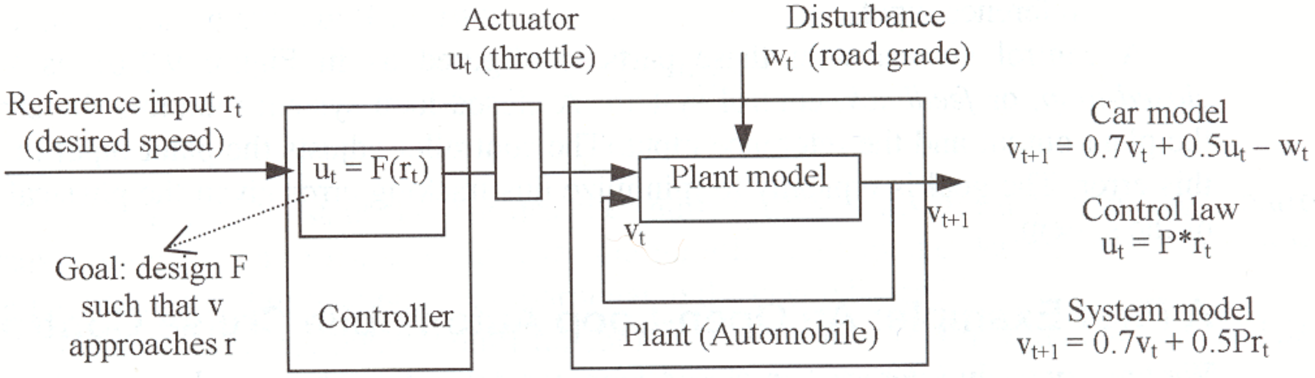
Car model
$v_{t+1} = 0.7v_t + 0.5u_t - w_t$

Control law
$u_t = P*r_t$

System model
$v_{t+1} = 0.7v_t + 0.5Pr_t$

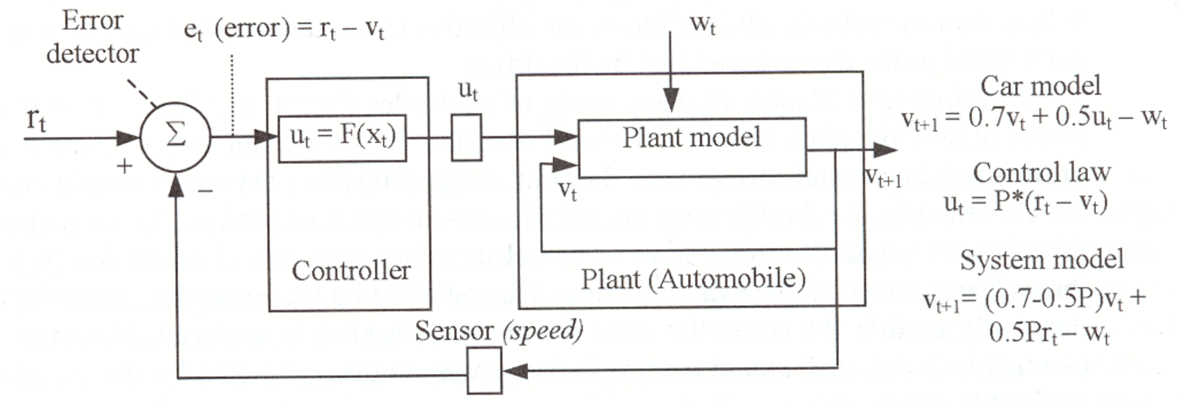
# Other Characteristics of open loop

- Feed-forward control
- Delay in actual change of the output
- Controller doesn’t know how well thing goes
- Simple
- Best use for predictable systems



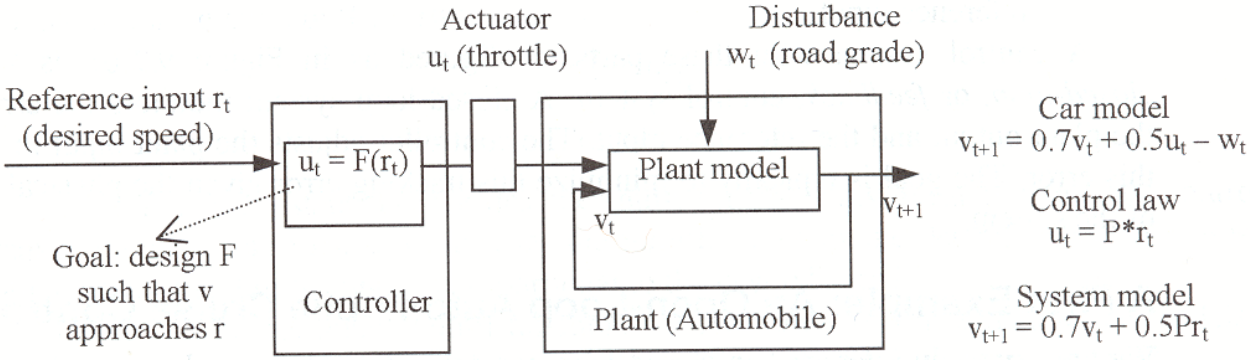
# Close Loop Control Systems

- Sensor
  - Measure the plant output
- Error detector
  - Detect Error
- Feedback control systems
- Minimize tracking error



# Designing Open Loop Control System

- Develop a model of the plant
- Develop a controller
- Analyze the controller
- Consider Disturbance
- Determine Performance
- Example:  Open Loop Cruise Control System

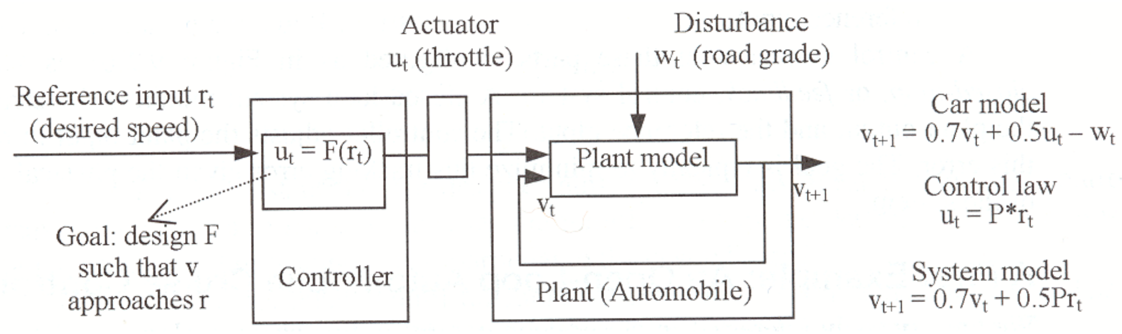


# Model of the Plant

- May not be necessary
  - Can be done through experimenting and tuning
- But,
  - Can make it easier to design
  - May be useful for deriving the controller
- Example: throttle that goes from 0 to 45 degree
  - On flat surface at 50 mph, open the throttle to 40 degree
  - Wait 1 “time unit”
  - Measure the speed, let’s say 55 mph
  - Then the following equation satisfy the above scenario
    - $v_{t+1}=0.7*v_t+0.5*u_t$
    - $55 = 0.7*50+0.5*40$
  - IF the equation holds for all other scenario
    - Then we have a model of the plant

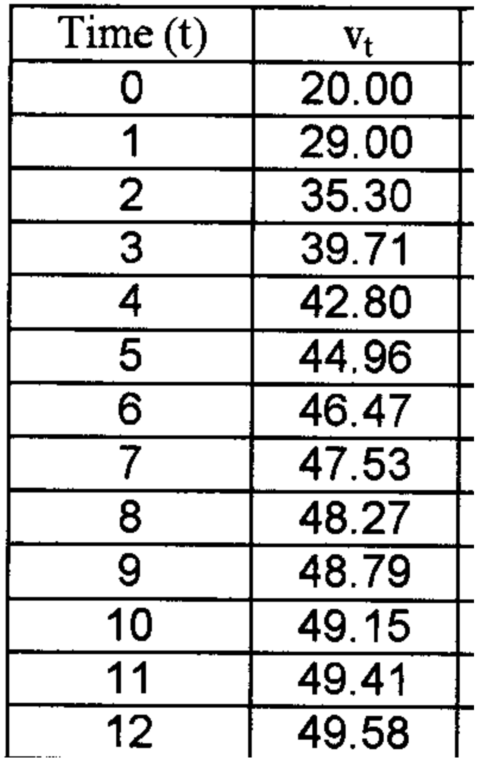
# Designing the Controller

- Assuming we want to use a simple linear function
  - $u_t = F(r_t) = P * r_t$
  - $r_t$ is the desired speed
- Linear proportional controller
- $v_{t+1} = 0.7 * v_t + 0.5 * u_t = 0.7 * v_t + 0.5P * r_t$
- Let $v_{t+1} = v_t$ at steady state = $v_{ss}$
- $v_{ss} = 0.7 * v_{ss} + 0.5P * r_t$
- At steady state, we want $v_{ss} = r_t$
- P=0.6
  - I.e. $u_t = 0.6 * r_t$



# Analyzing the Controller

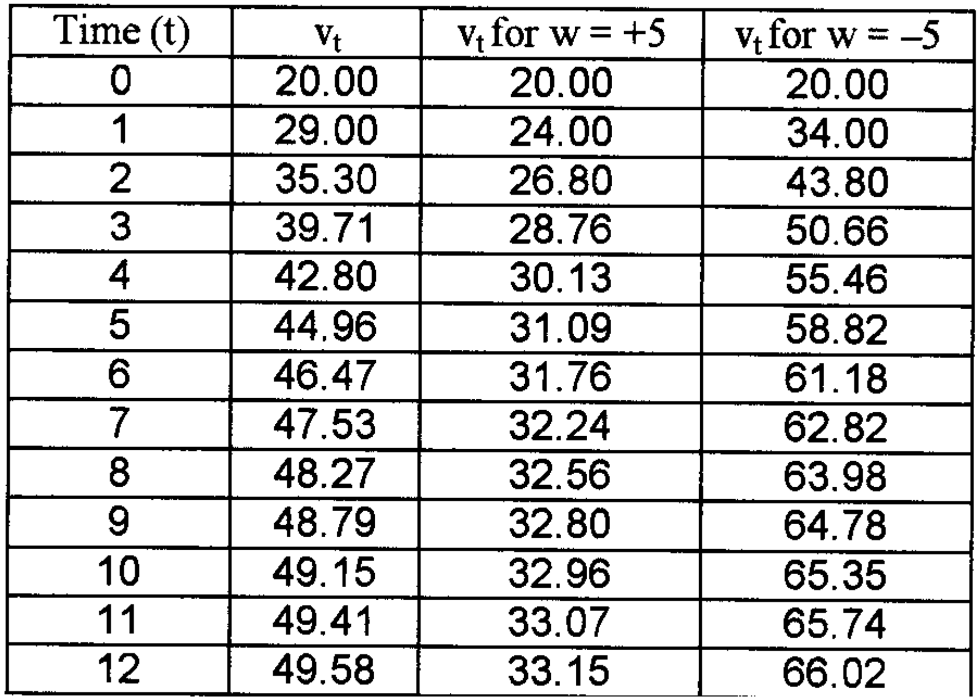
- Let $v_0$=20mph, $r_0$=50mph
- $v_{t+1}=0.7*v_t+0.5(0.6)*r_t=0.7*v_t+0.3*50=0.7*v_t+15$
- Throttle position is 0.6*50=30 degree



| Time (t) | $v_t$ |
|---|---|
| 0 | 20.00 |
| 1 | 29.00 |
| 2 | 35.30 |
| 3 | 39.71 |
| 4 | 42.80 |
| 5 | 44.96 |
| 6 | 46.47 |
| 7 | 47.53 |
| 8 | 48.27 |
| 9 | 48.79 |
| 10 | 49.15 |
| 11 | 49.41 |
| 12 | 49.58 |

# Considering the Disturbance

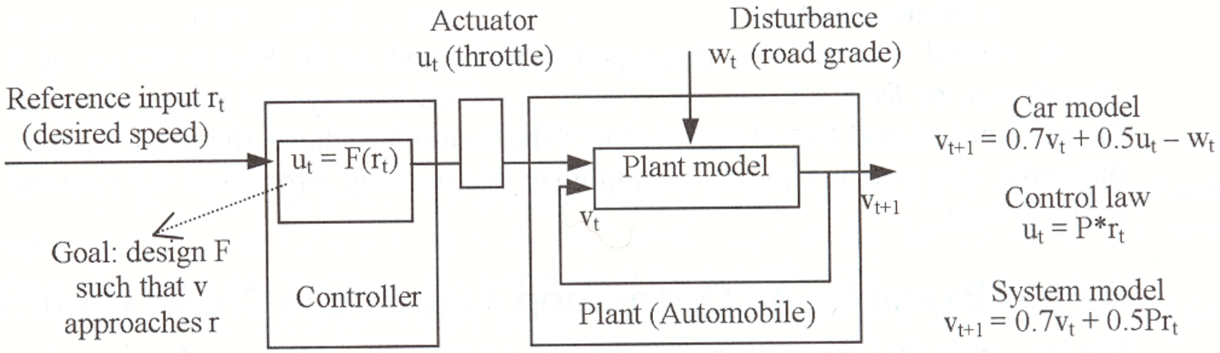
- Assume road grade can affect the speed
  - From –5mph to +5 mph
  - $v_{t+1}=0.7*v_t+10$
  - $v_{t+1}=0.7*v_t+20$



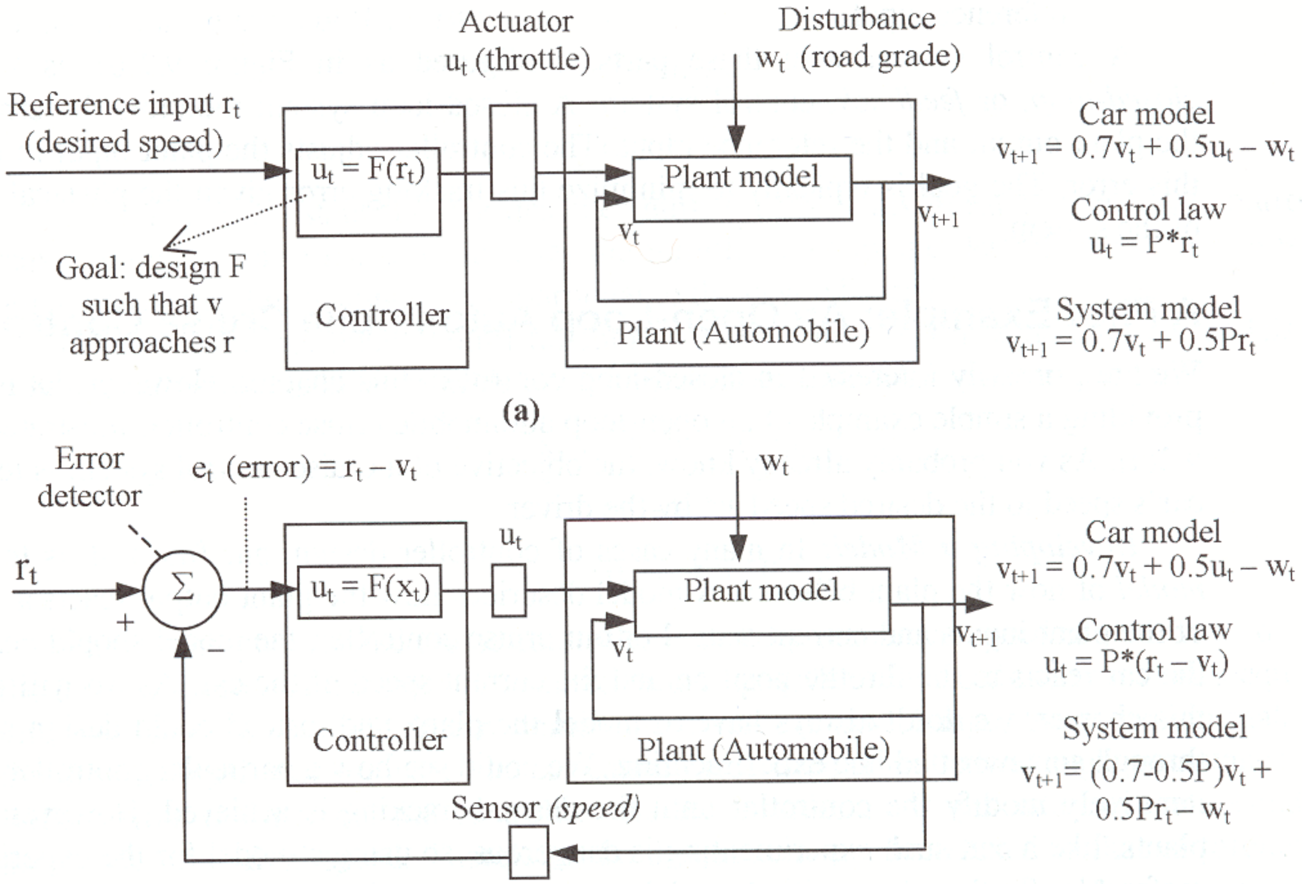
| Time (t) | $v_t$ | $v_t$ for w = +5 | $v_t$ for w = –5 |
|---|---|---|---|
| 0 | 20.00 | 20.00 | 20.00 |
| 1 | 29.00 | 24.00 | 34.00 |
| 2 | 35.30 | 26.80 | 43.80 |
| 3 | 39.71 | 28.76 | 50.66 |
| 4 | 42.80 | 30.13 | 55.46 |
| 5 | 44.96 | 31.09 | 58.82 |
| 6 | 46.47 | 31.76 | 61.18 |
| 7 | 47.53 | 32.24 | 62.82 |
| 8 | 48.27 | 32.56 | 63.98 |
| 9 | 48.79 | 32.80 | 64.78 |
| 10 | 49.15 | 32.96 | 65.35 |
| 11 | 49.41 | 33.07 | 65.74 |
| 12 | 49.58 | 33.15 | 66.02 |

# Determining Performance

- $V_{t+1}=0.7*v_t+0.5P*r_0-w_0$
- $v_1=0.7*v_0+0.5P*r_0-w_0$
- $v_2=0.7*(0.7*v_0+0.5P*r_0-w_0)+0.5P*r_0-w_0=0.7*0.7*v_0+(0.7+1.0)*0.5P*r_0-(0.7+1.0)w_0$
- $v^t=0.7^t*v_0+(0.7^{t-1}+0.7^{t-2}+\ldots+0.7+1.0)(0.5P*r_0-w_0)$
- Coefficient of $v_t$ determines rate of decay of $v_0$
  - >1 or <-1, $v_t$ will grow without bound
  - <0, $v_t$ will oscillate



# Designing Close Loop Control System



# Stability

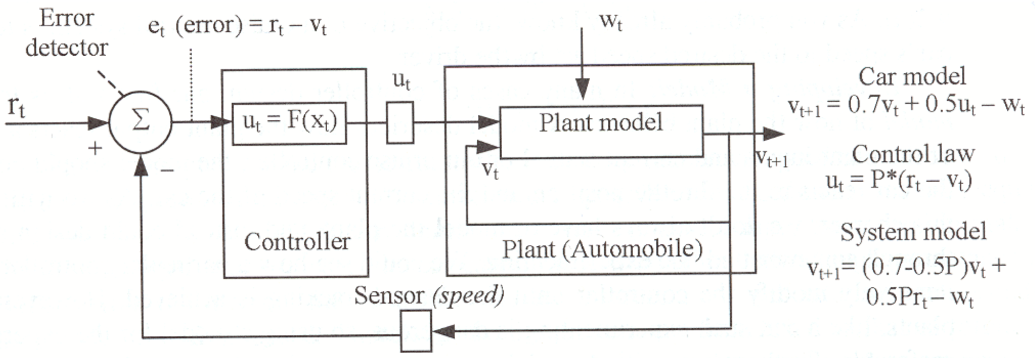
- $u_t = P * (r_t - v_t)$
- $v_{t+1} = 0.7v_t + 0.5u_t - w_t = 0.7v_t + 0.5P*(r_t - v_t) - w$
  $= (0.7-0.5P)*v_t + 0.5P*r_t - w_t$
- $v^t = (0.7-0.5P)^t * v_0 + ((0.7-0.5P)^{t-1} + (0.7-0.5P)^{t-2} + \ldots + 0.7-0.5P+1.0)(0.5P*r_0 - w_0)$

- Stability constraint (I.e. convergence) requires

$|0.7-0.5P|<1$

$-1<0.7-0.5P<1$

$-0.6<P<3.4$

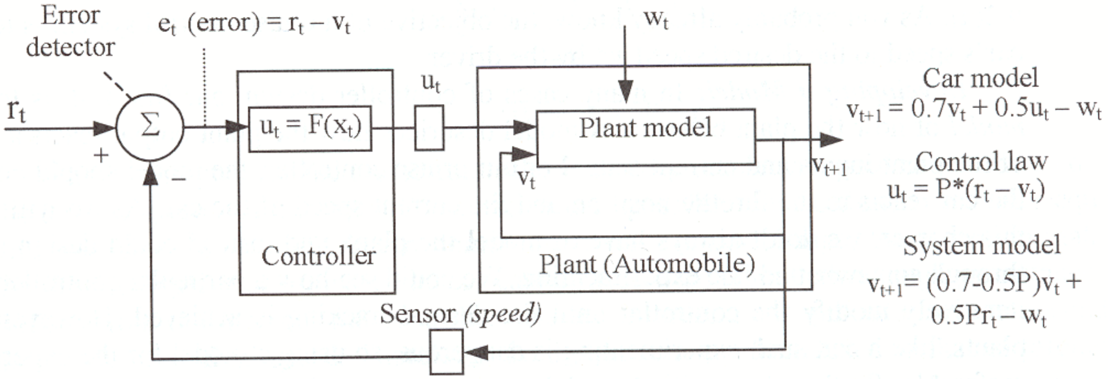


# Reducing effect of $v_0$

- $u_t = P * (r_t - v_t)$
- $v_{t+1} = 0.7v_t + 0.5u_t - w_t = 0.7v_t + 0.5P*(r_t - v_t) - w$

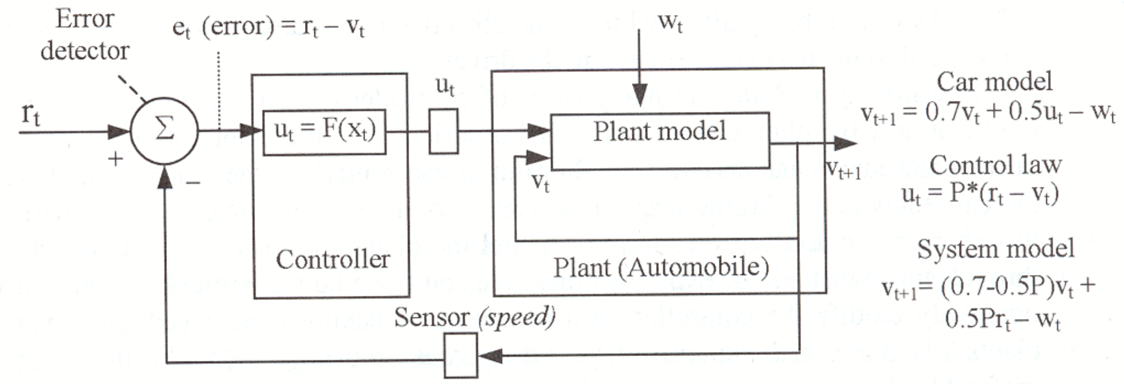
  $= (0.7 - 0.5P)*v_t + 0.5P*r_t - w_t$
- $v^t = (0.7-0.5P)^t * v_0 + ((0.7-0.5P)^{t-1} + (0.7-0.5P)^{t-2} + \ldots + 0.7 - 0.5P + 1.0)(0.5P*r_0 - w_0)$

- To reduce the effect of initial condition
  - 0.7-0.5P as small as possible
  - P=1.4



# Avoid Oscillation

- $u_t = P * (r_t - v_t)$
- $v_{t+1} = 0.7v_t + 0.5u_t - w_t = 0.7v_t + 0.5P*(r_t - v_t) - w$
  $= (0.7-0.5P)*v_t + 0.5P*r_t - w_t$
- $v^t = (0.7-0.5P)^t * v_0 + ((0.7-0.5P)^{t-1} + (0.7-0.5P)^{t-2} + \ldots + 0.7-0.5P+1.0)(0.5P*r_0 - w_0)$
- To avoid oscillation
  - $0.7-0.5P >= 0$
  - $P <= 1.4$



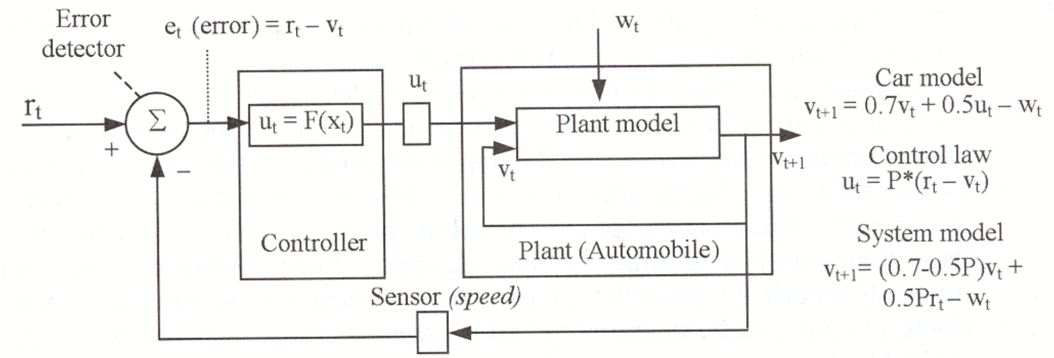
# Perfect Tracking

- $u_t = P * (r_t - v_t)$
- $v_{t+1} = 0.7v_t + 0.5u_t - w_t = 0.7v_t + 0.5P*(r_t - v_t) - w$

  $= (0.7 - 0.5P)*v_t + 0.5P*r_t - w_t$
- $v_{ss} = (0.7 - 0.5P)*v_{ss} + 0.5P*r_0 - w_0$

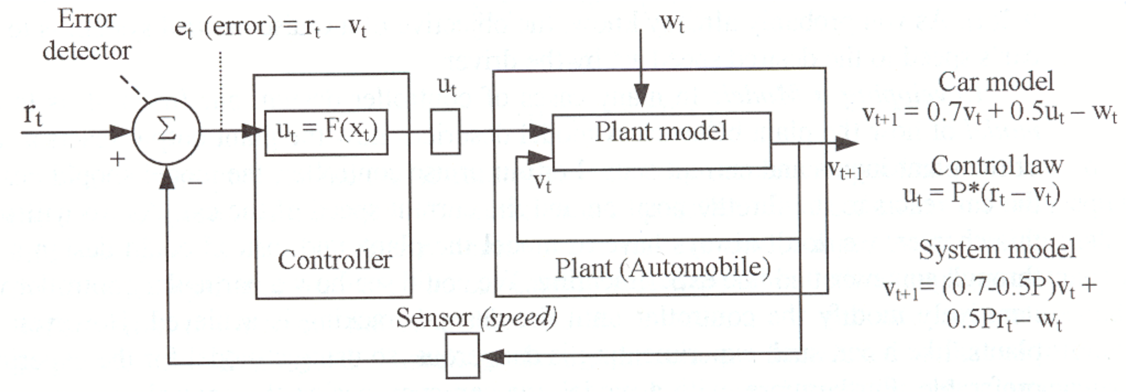
  $(1 - 0.7 + 0.5P)v_{ss} = 0.5P*r_0 - w_0$

  $v_{ss} = (0.5P/(0.3 + 0.5P)) * r_0 - (1.0/(0.3 + 0.5P)) * w_o$
- To make $v_{ss}$ as close to $r_0$ as possible
  - P should be as large as possible



# Close-Loop Design

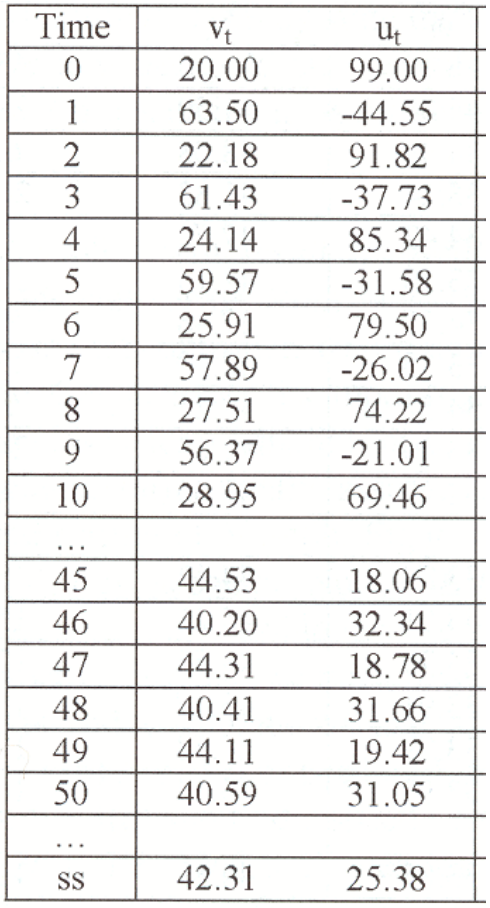
- $u_t = P * (r_t - v_t)$
- Finally, setting P=3.3
  - Stable, track well, some oscillation
  - $u_t = 3.3 * (r_t - v_t)$



# Analyze the controller

- $v_0$=20 mph, $r_0$=50 mph, w=0
- $v_{t+1} = 0.7v_t+0.5P*(r_t-v_t)-w$
  $= 0.7v_t+0.5*3.3*(50-v_t)$
- $u_t = P * (r_t-v_t)$
  $= 3.3 * (50-v_t)$

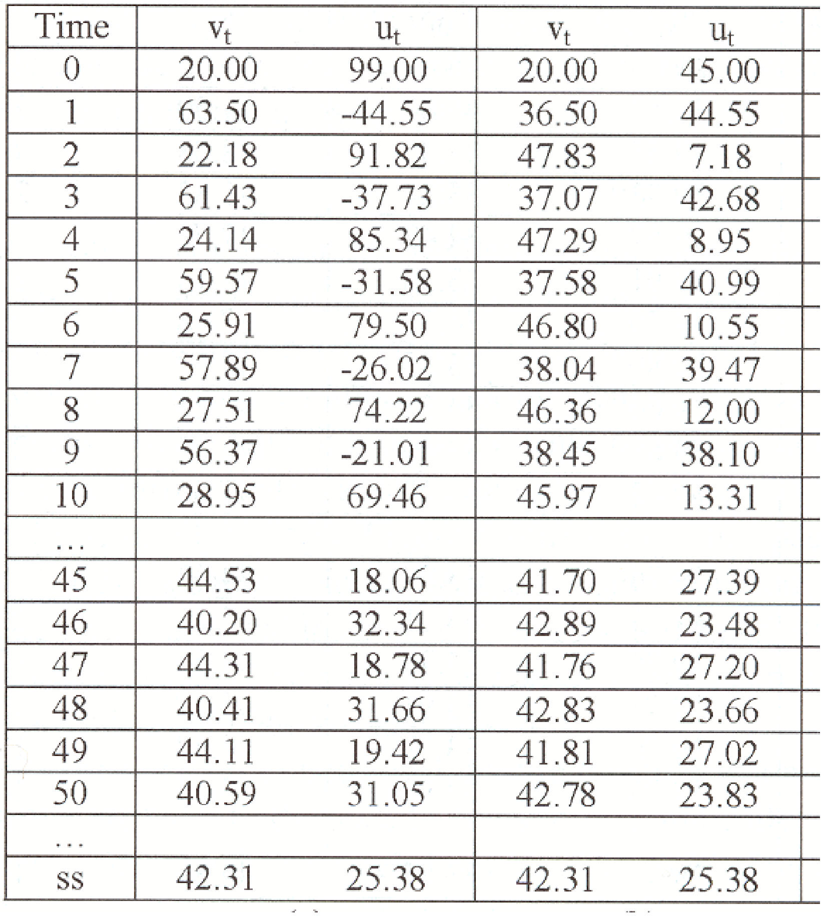
- But $u_t$ range from 0-45
- Controller saturates



| Time | $v_t$ | $u_t$ |
|---|---|---|
| 0 | 20.00 | 99.00 |
| 1 | 63.50 | -44.55 |
| 2 | 22.18 | 91.82 |
| 3 | 61.43 | -37.73 |
| 4 | 24.14 | 85.34 |
| 5 | 59.57 | -31.58 |
| 6 | 25.91 | 79.50 |
| 7 | 57.89 | -26.02 |
| 8 | 27.51 | 74.22 |
| 9 | 56.37 | -21.01 |
| 10 | 28.95 | 69.46 |
| … | | |
| 45 | 44.53 | 18.06 |
| 46 | 40.20 | 32.34 |
| 47 | 44.31 | 18.78 |
| 48 | 40.41 | 31.66 |
| 49 | 44.11 | 19.42 |
| 50 | 40.59 | 31.05 |
| … | | |
| ss | 42.31 | 25.38 |

# Analyze the controller

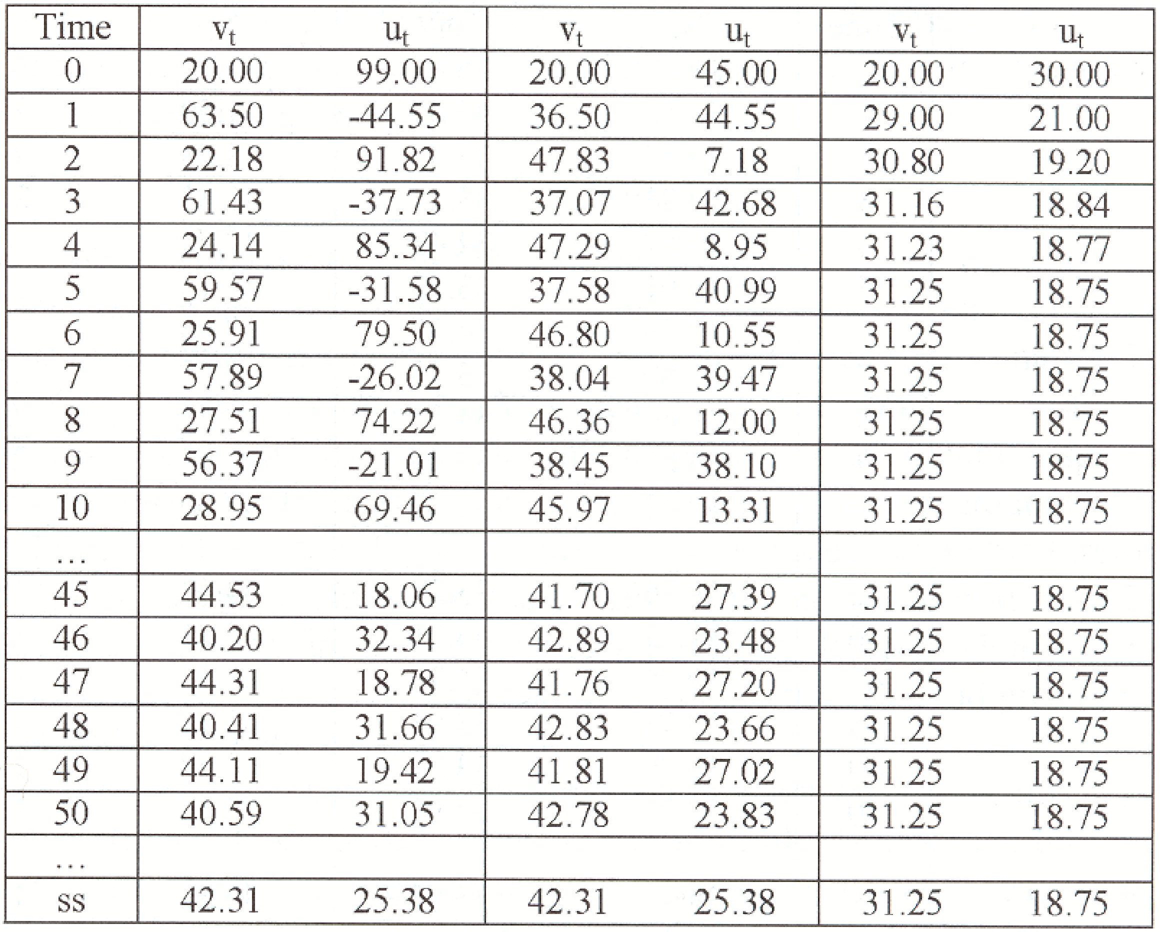
- $v_0$=20 mph, $r_0$=50 mph, w=0
- $v_{t+1} = 0.7v_t+0.5*u_t$
- $u_t = 3.3 * (50-v_t)$
  - Saturate at 0, 45
- Oscillation!
  - "feel bad"



| Time | $v_t$ | $u_t$ | $v_t$ | $u_t$ |
|---|---|---|---|---|
| 0 | 20.00 | 99.00 | 20.00 | 45.00 |
| 1 | 63.50 | -44.55 | 36.50 | 44.55 |
| 2 | 22.18 | 91.82 | 47.83 | 7.18 |
| 3 | 61.43 | -37.73 | 37.07 | 42.68 |
| 4 | 24.14 | 85.34 | 47.29 | 8.95 |
| 5 | 59.57 | -31.58 | 37.58 | 40.99 |
| 6 | 25.91 | 79.50 | 46.80 | 10.55 |
| 7 | 57.89 | -26.02 | 38.04 | 39.47 |
| 8 | 27.51 | 74.22 | 46.36 | 12.00 |
| 9 | 56.37 | -21.01 | 38.45 | 38.10 |
| 10 | 28.95 | 69.46 | 45.97 | 13.31 |
| ... | | | | |
| 45 | 44.53 | 18.06 | 41.70 | 27.39 |
| 46 | 40.20 | 32.34 | 42.89 | 23.48 |
| 47 | 44.31 | 18.78 | 41.76 | 27.20 |
| 48 | 40.41 | 31.66 | 42.83 | 23.66 |
| 49 | 44.11 | 19.42 | 41.81 | 27.02 |
| 50 | 40.59 | 31.05 | 42.78 | 23.83 |
| ... | | | | |
| ss | 42.31 | 25.38 | 42.31 | 25.38 |

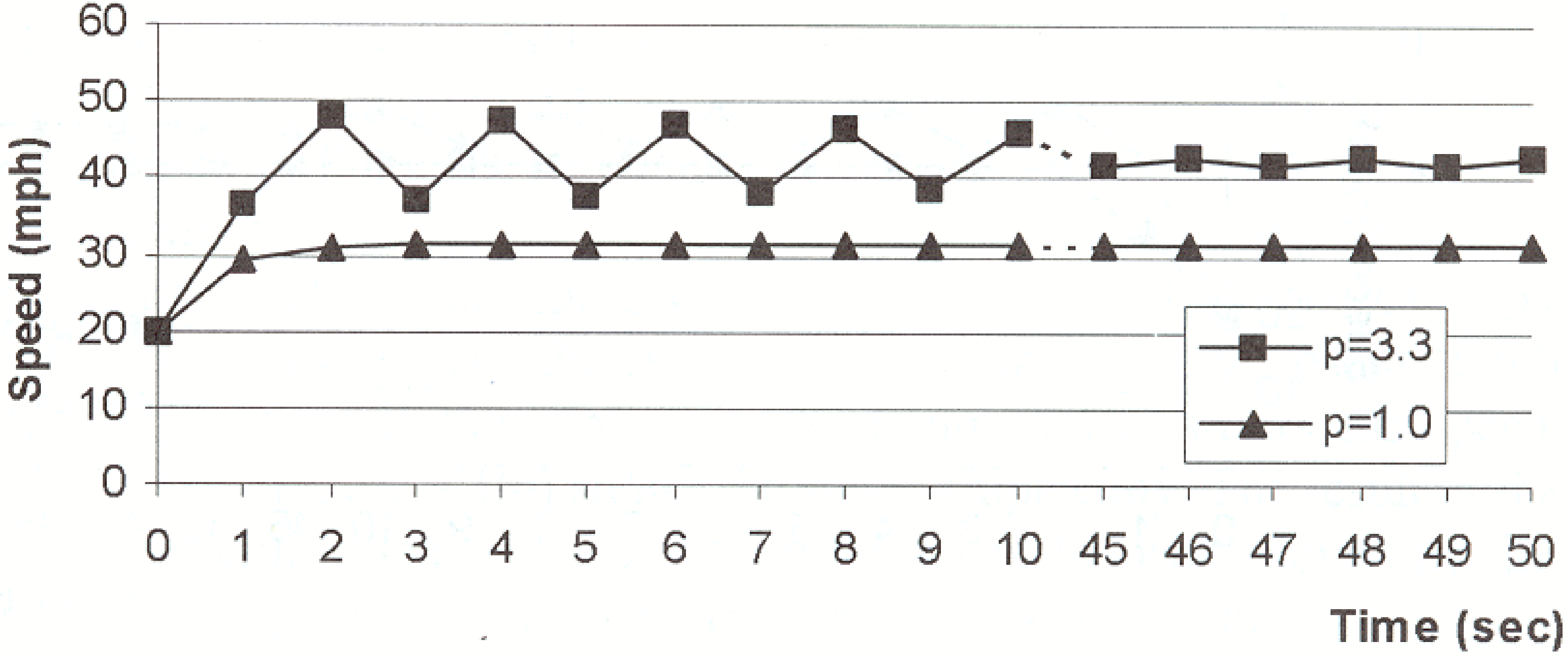
# Analyze the controller

- Set P=1.0 to void oscillation
  - Terrible SS performance



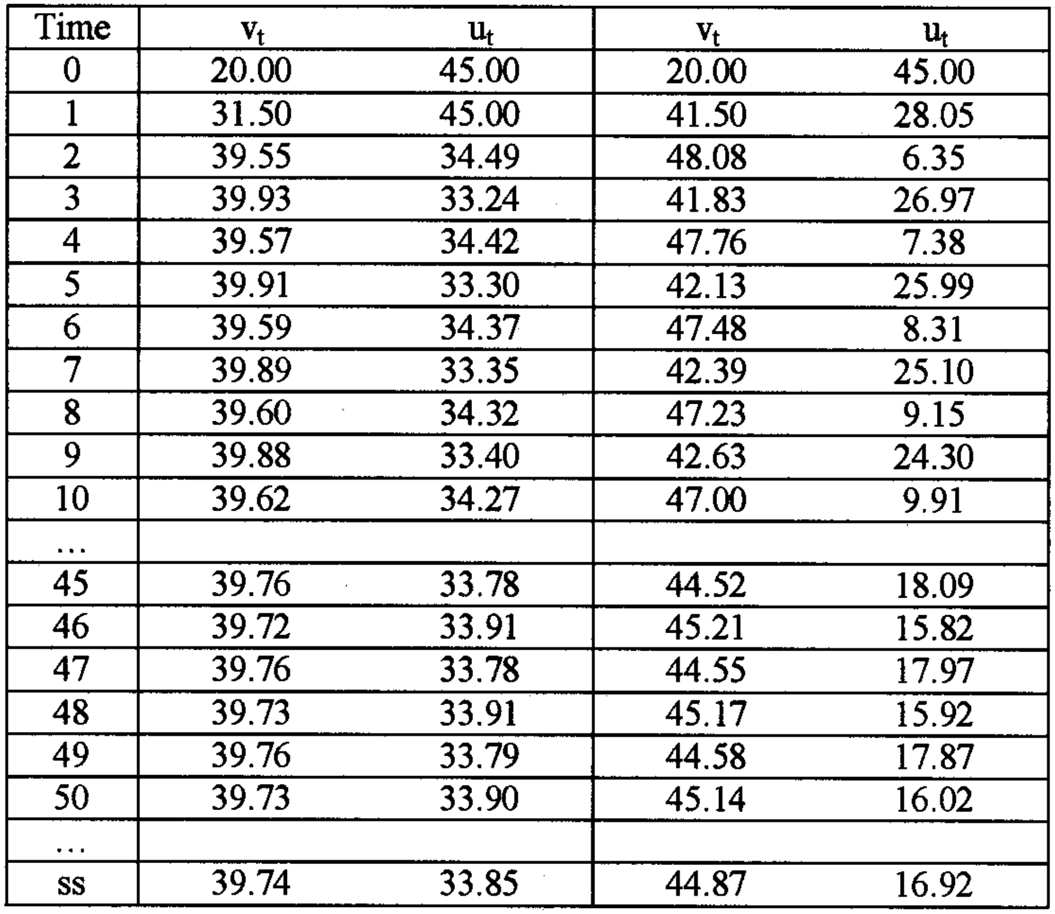
| Time | $v_t$ | $u_t$ | $v_t$ | $u_t$ | $v_t$ | $u_t$ |
|---|---|---|---|---|---|---|
| 0 | 20.00 | 99.00 | 20.00 | 45.00 | 20.00 | 30.00 |
| 1 | 63.50 | -44.55 | 36.50 | 44.55 | 29.00 | 21.00 |
| 2 | 22.18 | 91.82 | 47.83 | 7.18 | 30.80 | 19.20 |
| 3 | 61.43 | -37.73 | 37.07 | 42.68 | 31.16 | 18.84 |
| 4 | 24.14 | 85.34 | 47.29 | 8.95 | 31.23 | 18.77 |
| 5 | 59.57 | -31.58 | 37.58 | 40.99 | 31.25 | 18.75 |
| 6 | 25.91 | 79.50 | 46.80 | 10.55 | 31.25 | 18.75 |
| 7 | 57.89 | -26.02 | 38.04 | 39.47 | 31.25 | 18.75 |
| 8 | 27.51 | 74.22 | 46.36 | 12.00 | 31.25 | 18.75 |
| 9 | 56.37 | -21.01 | 38.45 | 38.10 | 31.25 | 18.75 |
| 10 | 28.95 | 69.46 | 45.97 | 13.31 | 31.25 | 18.75 |
| ... | | | | | | |
| 45 | 44.53 | 18.06 | 41.70 | 27.39 | 31.25 | 18.75 |
| 46 | 40.20 | 32.34 | 42.89 | 23.48 | 31.25 | 18.75 |
| 47 | 44.31 | 18.78 | 41.76 | 27.20 | 31.25 | 18.75 |
| 48 | 40.41 | 31.66 | 42.83 | 23.66 | 31.25 | 18.75 |
| 49 | 44.11 | 19.42 | 41.81 | 27.02 | 31.25 | 18.75 |
| 50 | 40.59 | 31.05 | 42.78 | 23.83 | 31.25 | 18.75 |
| ... | | | | | | |
| ss | 42.31 | 25.38 | 42.31 | 25.38 | 31.25 | 18.75 |

# Analyzing the Controller



# Minimize the effect of disturbance

- $v_{t+1} = 0.7v_t + 0.5*3.3*(r_t - v_t) - w$
  - w=-5 or +5
- 39.74
  - Close to 42.31
  - Better than
    - 33
    - 66
- Cost
  - SS error
  - oscillation



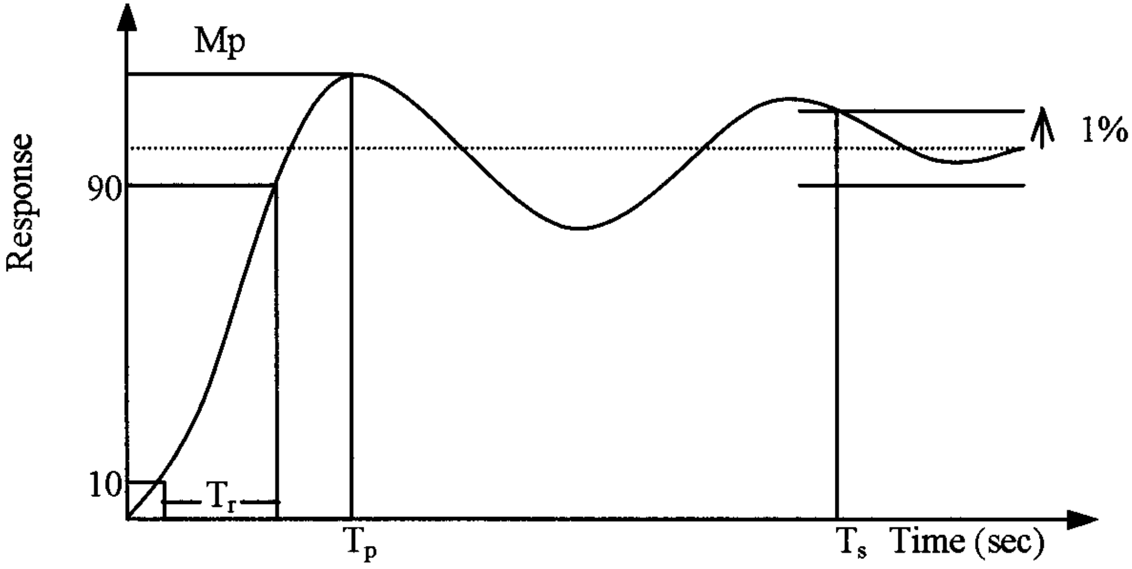
| Time | $v_t$ | $u_t$ | $v_t$ | $u_t$ |
|---|---|---|---|---|
| 0 | 20.00 | 45.00 | 20.00 | 45.00 |
| 1 | 31.50 | 45.00 | 41.50 | 28.05 |
| 2 | 39.55 | 34.49 | 48.08 | 6.35 |
| 3 | 39.93 | 33.24 | 41.83 | 26.97 |
| 4 | 39.57 | 34.42 | 47.76 | 7.38 |
| 5 | 39.91 | 33.30 | 42.13 | 25.99 |
| 6 | 39.59 | 34.37 | 47.48 | 8.31 |
| 7 | 39.89 | 33.35 | 42.39 | 25.10 |
| 8 | 39.60 | 34.32 | 47.23 | 9.15 |
| 9 | 39.88 | 33.40 | 42.63 | 24.30 |
| 10 | 39.62 | 34.27 | 47.00 | 9.91 |
| … | | | | |
| 45 | 39.76 | 33.78 | 44.52 | 18.09 |
| 46 | 39.72 | 33.91 | 45.21 | 15.82 |
| 47 | 39.76 | 33.78 | 44.55 | 17.97 |
| 48 | 39.73 | 33.91 | 45.17 | 15.92 |
| 49 | 39.76 | 33.79 | 44.58 | 17.87 |
| 50 | 39.73 | 33.90 | 45.14 | 16.02 |
| … | | | | |
| ss | 39.74 | 33.85 | 44.87 | 16.92 |

# General Control System

- Objective
  - Causing output to track a reference even in the presence of
    - Measurement noise
    - Model error
    - Disturbances
- Metrics
  - Stability
    - Output remains bounded
  - Performance
    - How well an output tracks the reference
  - Disturbance rejection
  - Robustness
    - Ability to tolerate modeling error of the plant

# Performance (generally speaking)

- Rise time
  - Time it takes form 10% to 90%
- Peak time
- Overshoot
  - Percentage by which Peak exceed final value
- Settling time
  - Time it takes to reach 1% of final value



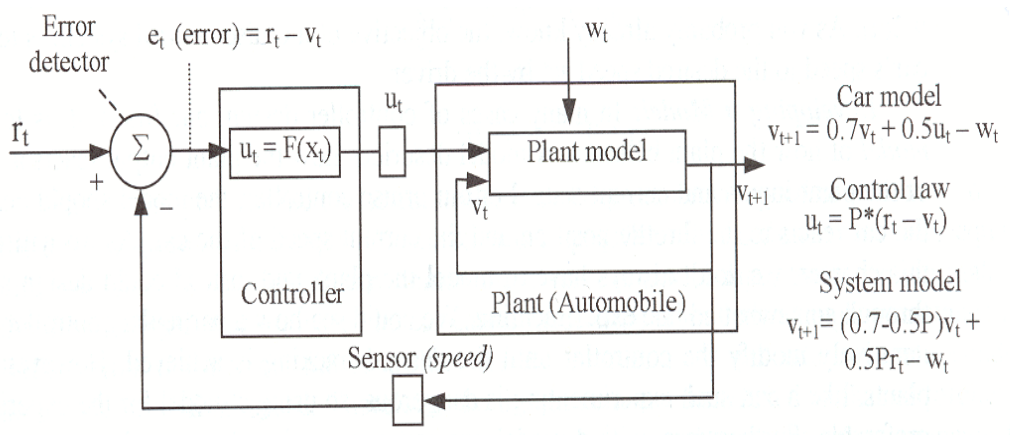
# Plant modeling is difficult

- May need to be done first
- Plant is usually on continuous time
  - Not discrete time
    - E.g. car speed continuously react to throttle position, not at discrete interval
  - Sampling period must be chosen carefully
    - To make sure "nothing interesting" happen in between
    - I.e. small enough
- Plant is usually non-linear
  - E.g. shock absorber response may need to be 8th order differential

- Iterative development of the plant model and controller
  - Have a plant model that is "good enough"

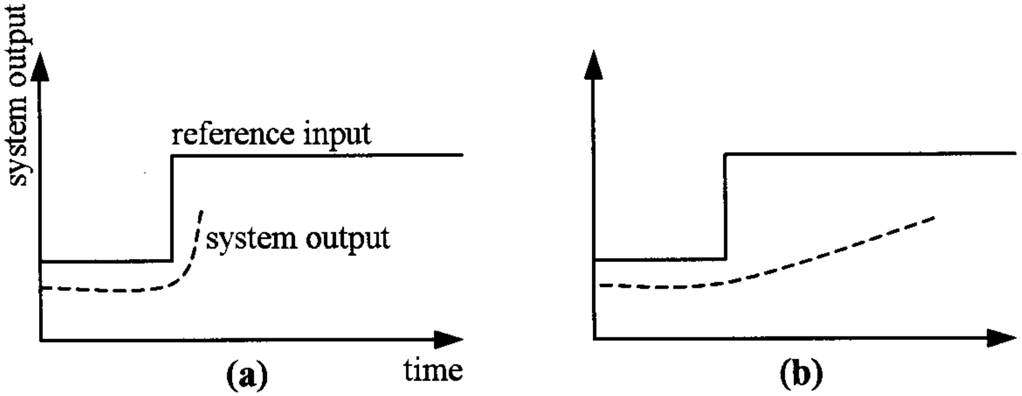
# Controller Design: P

- Proportional controller
  - A controller that multiplies the tracking error by a constant
    - $u_t = P * (r_t - v_t)$
  - Close loop model with a linear plant
    - E.g. $v_{t+1} = (0.7-0.5P)*v_t+0.5P*r_t-w_t$
- P affects
  - Transient response
    - Stability, oscillation
  - Steady state tacking
    - As large as possible
  - Disturbance rejection
    - As large as possible



# Controller Design: PD

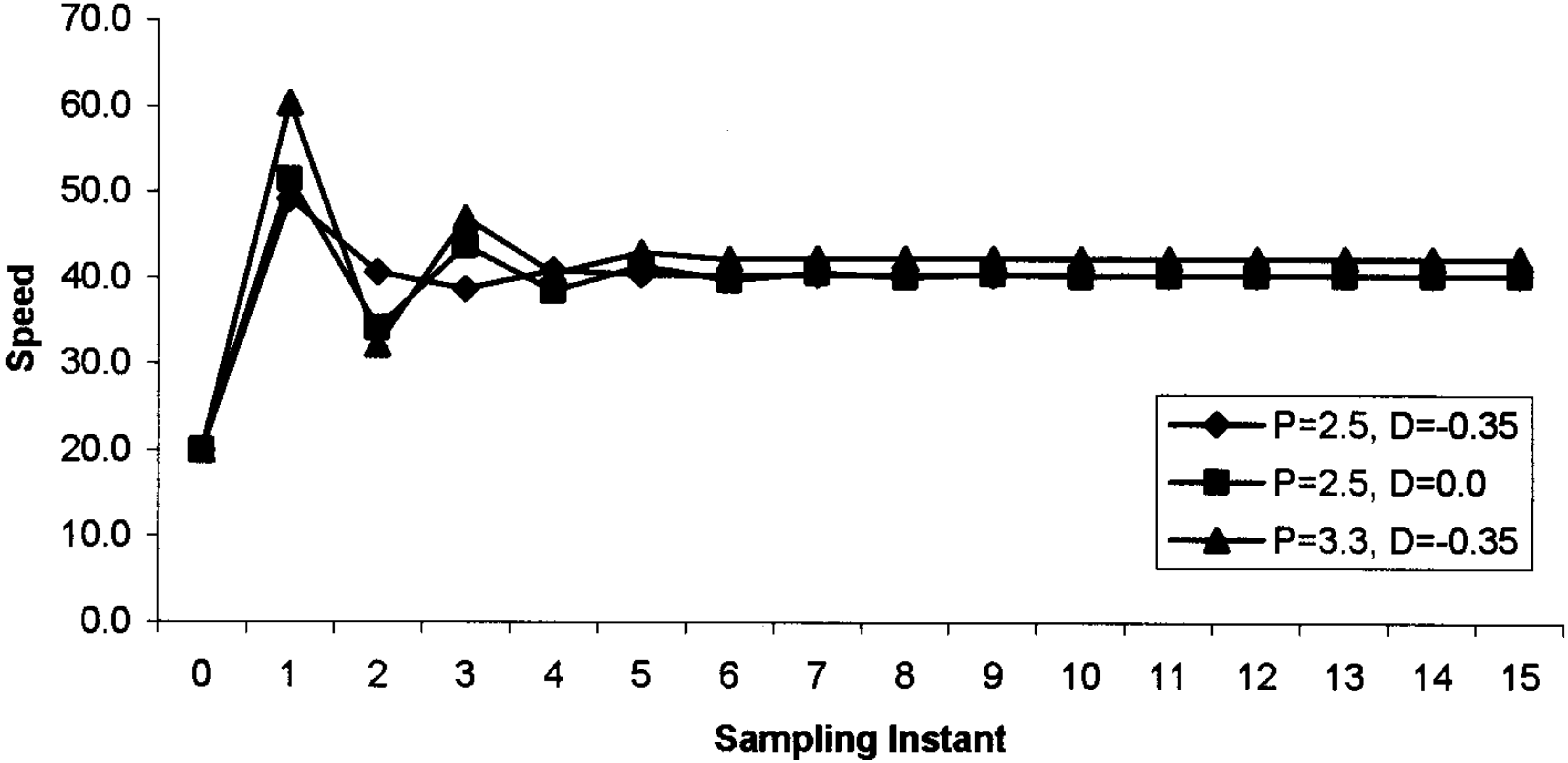
- Proportional and Derivative control
    - $u_t = P * (r_t - v_t) + D * ((r_t - v_t) - (r_{t-1} - v_{t-1})) = P * e_t + D * (e_t - e_{t-1})$
- Consider the size of error over time
- Intuitively
    - – Want to "push" more if the error is not reducing fast enough
    - – Want to "push" less if the error is reducing really fast



# PD Controller

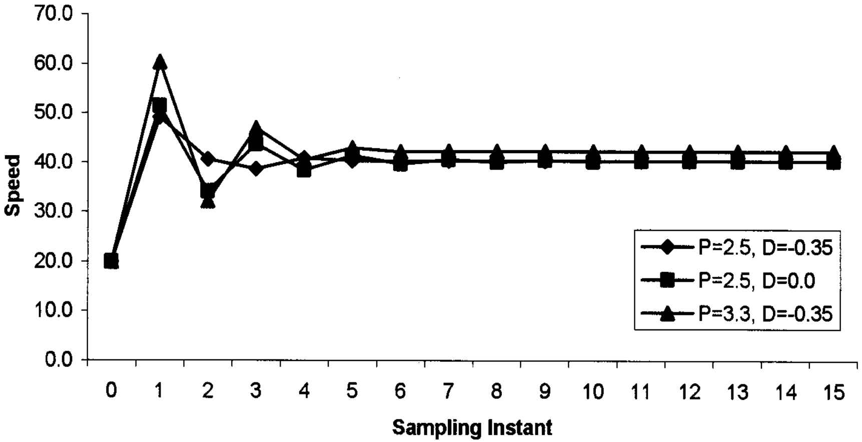
- Need to keep track of error derivative
- E.g. Cruise controller example
  - $v_{t+1} = 0.7v_t+0.5u_t-w_t$
  - Let $u_t = P * e_t + D * (e_t-e_{t-1})$, $e_t=r_t-v_t$
  - $v_{t+1}=0.7v_t+0.5*(P*(r_t-v_t)+D*((r_t-v_t)-(r_{t-1}-v_{t-1})))-w_t$
  - $v_{t+1}=(0.7-0.5*(P+D))*v_t+0.5D*v_{t-1}+0.5*(P+D)*r_t-0.5D*r_{t-1}-w_t$
  - Assume reference input and distribance are constant, the steady-state speed is
    - $V_{ss}=(0.5P/(1-0.7+0.5P)) * r$
    - Does not depend on D!!!
- P can be set for best tracking and disturbance control
- Then D set to control oscillation/overshoot/rate of convergence

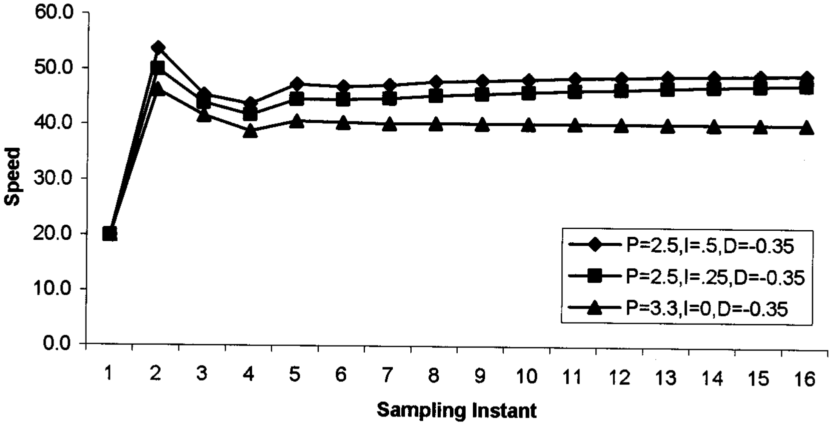
# PD Control Example



# PI Control

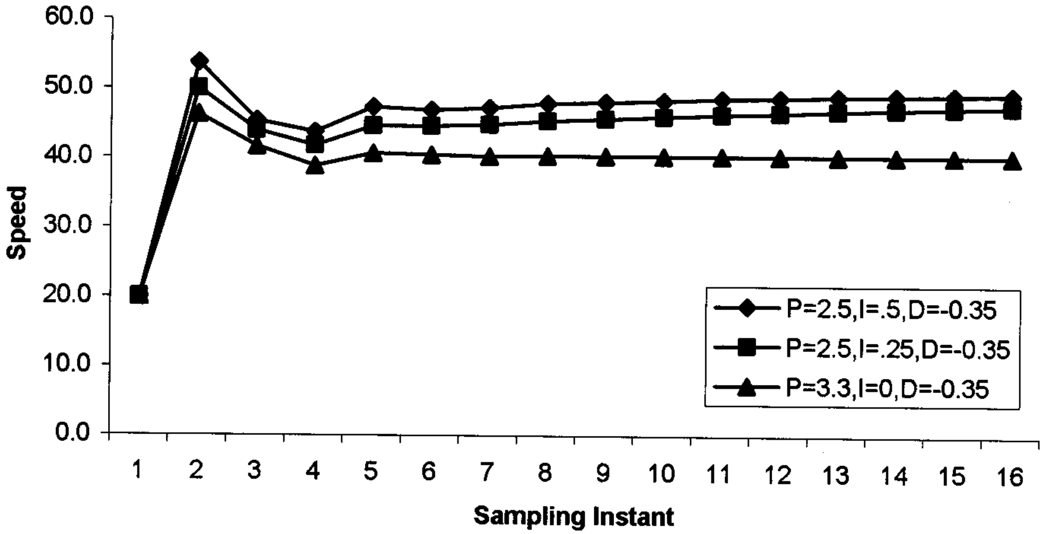
- Proportional plus integral control
  - $u_t = P*e_t + I*(e_0 + e_1 + \ldots + e_t)$
- Sum up error over time
  - Ensure reaching desired output, eventually
  - $v_{ss}$ will not be reached until $e_{ss}=0$
- Use P to control disturbance
- Use I to ensure steady state convergence and convergence rate





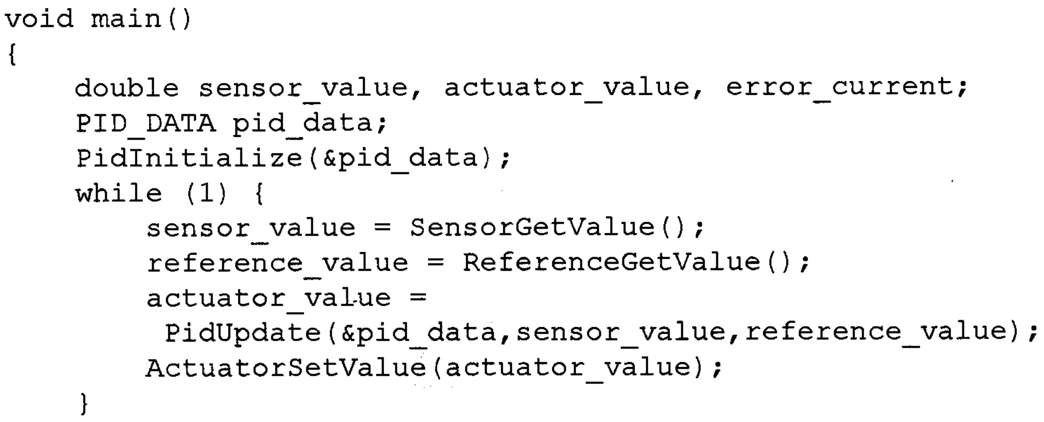
# PID Controller

- Combine Proportional, integral, and derivative control
  - $u_t = P*e_t + I*(e_0 + e_1 + \ldots + e_t) + D*(e_t - e_{t-1})$
- Available off-the shelf



# Software Coding

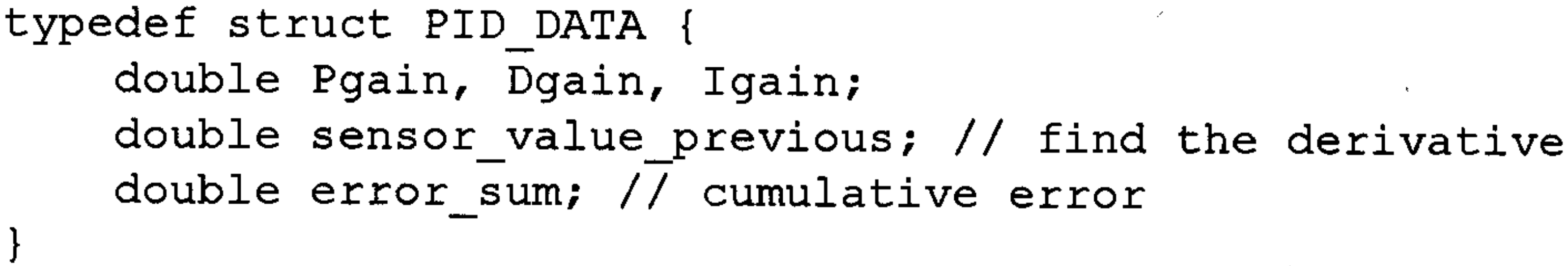
- Main function loops forever, during each iteration
  - Read plant output sensor
    - May require A2D
  - Read current desired reference input
  - Call PidUpdate, to determine actuator value
  - Set actuator value
    - May require D2A



```
void main()
{
    double sensor_value, actuator_value, error_current;
    PID_DATA pid_data;
    PidInitialize(&pid_data);
    while (1) {
        sensor_value = SensorGetValue();
        reference_value = ReferenceGetValue();
        actuator_value =
         PidUpdate(&pid_data,sensor_value,reference_value);
        ActuatorSetValue(actuator_value);
    }
```

# Software Coding (continue)

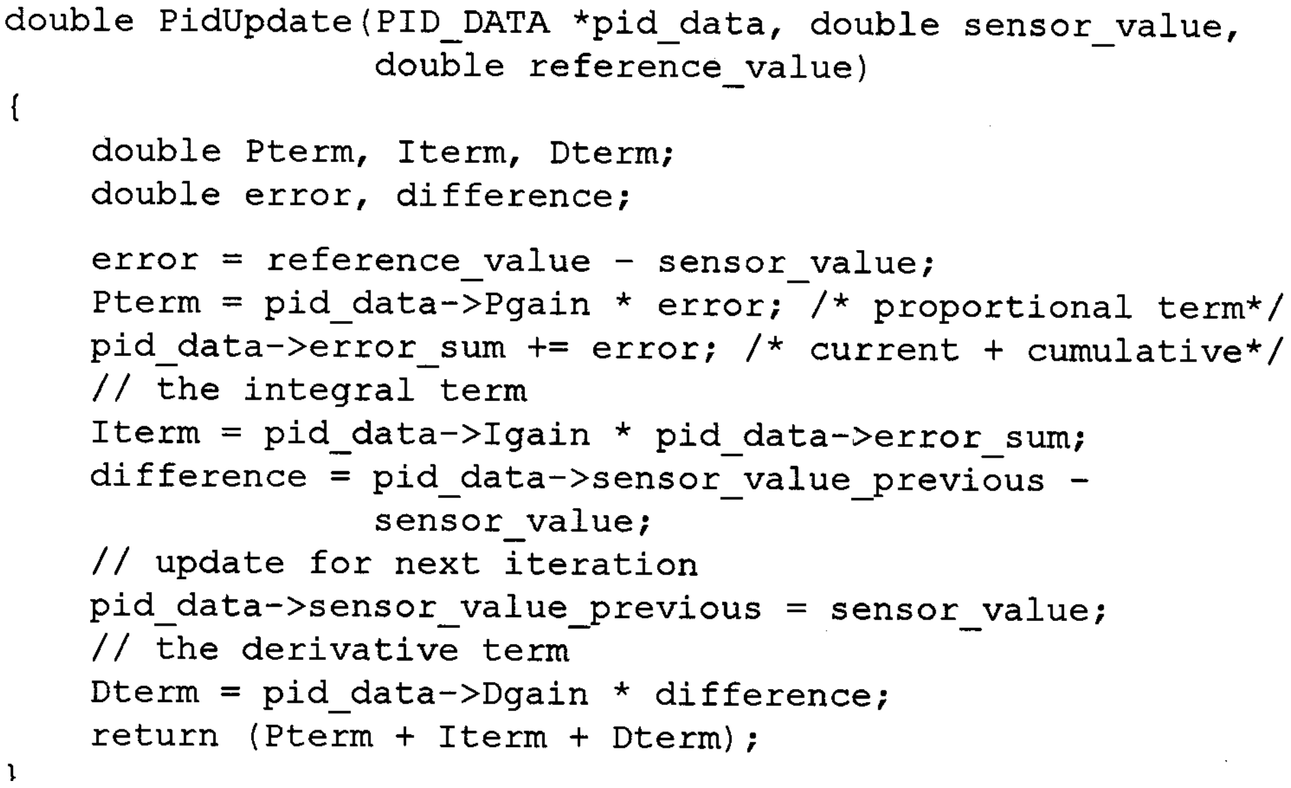
- Pgain, Dgain, Igain are constants
- sensor_value_previous
  - For D control
- error_sum
  - For I control



```
typedef struct PID_DATA {
    double Pgain, Dgain, Igain;
    double sensor_value_previous; // find the derivative
    double error_sum; // cumulative error
}
```

# Computation

- $u_t=P*e_t+I*(e_0+e_1+\ldots+e_t)+D*(e_t-e_{t-1})$



```
double PidUpdate(PID_DATA *pid_data, double sensor_value,
                 double reference_value)
{
    double Pterm, Iterm, Dterm;
    double error, difference;

    error = reference_value - sensor_value;
    Pterm = pid_data->Pgain * error; /* proportional term*/
    pid_data->error_sum += error; /* current + cumulative*/
    // the integral term
    Iterm = pid_data->Igain * pid_data->error_sum;
    difference = pid_data->sensor_value_previous -
                 sensor_value;
    // update for next iteration
    pid_data->sensor_value_previous = sensor_value;
    // the derivative term
    Dterm = pid_data->Dgain * difference;
    return (Pterm + Iterm + Dterm);
}
```

# PID tuning

- Analytically deriving P, I, D may not be possible
  - E.g. plant not is not available, or to costly to obtain
- Ad hoc method for getting "reasonable" P, I, D
  - Start with a small P, I=D=0
  - Increase D, until seeing oscillation
    - Reduce D a bit
  - Increase P, until seeing oscillation
    - Reduce D a bit
  - Increase I, until seeing oscillation
- Iterate until can change anything without excessive oscillation

# Practical Issues with Computer-Based Control

- Quantization
- Overflow
- Aliasing
- Computation Delay

# Quantization & Overflow

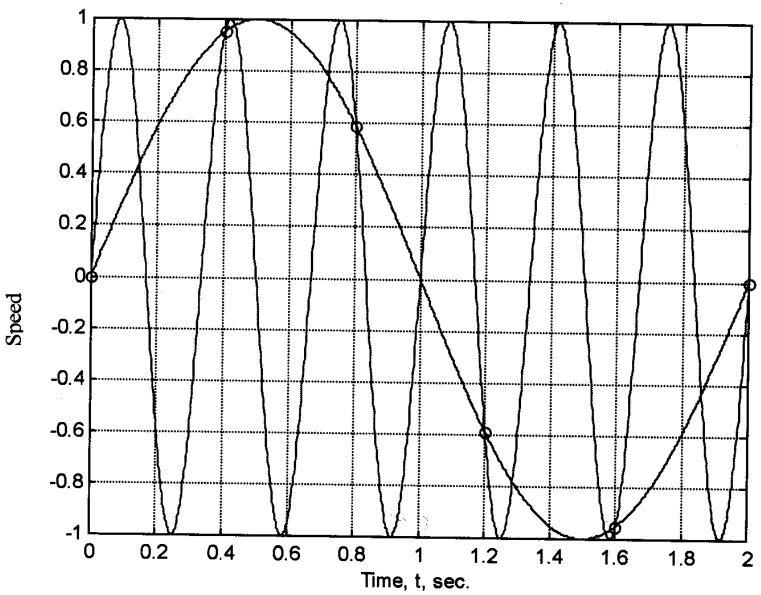
- Quantization
  - Can’t store 0.36 as 4-bit fractional number
  - Can only store 0.75, 0.59, 0.25, 0.00, -0.25, -050,-0.75, -1.00
  - Choose 0.25
    - Result in quantization error of 0.11
- Sources of quantization error
  - Operations, e.g. 0.50*0.25=0.125
    - Can use more bits until input/output to the environment/memory
  - A2D converters

- Overflow
  - Can’t store 0.75+0.50 = 1.25 as 4-bit fractional number

- Solutions:
  - Use fix-point representation/operations carefully
    - Time-consuming
  - Use floating-point co-processor
    - Costly

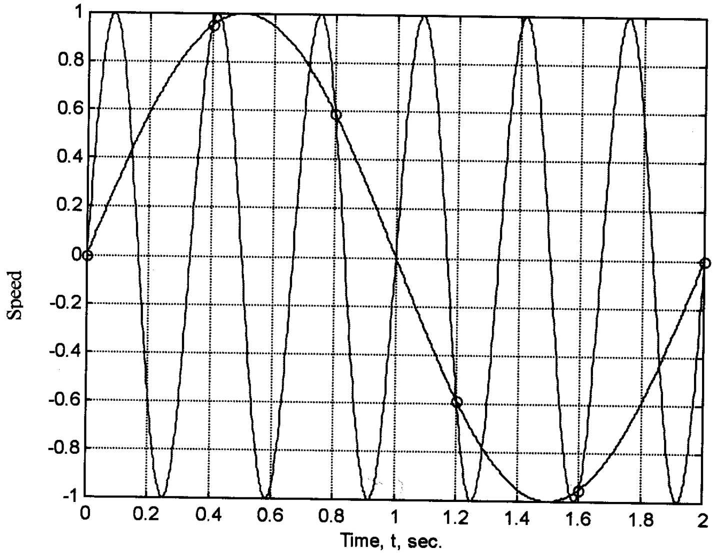
# Aliasing

- Quantization/overflow
  - Due to discrete nature of computer data
- Aliasing
  - Due to discrete nature of sampling



# Aliasing Example

- Sampling at 2.5 Hz, period of 0.4, the following are indistinguishable
  - y(t)=1.0*sin(6πt), frequency 3 Hz
  - y(t)=1.0*sin(πt), frequency of 0.5 Hz
- In fact, with sampling frequency of 2.5 Hz
  - Can only correctly sample signal below Nyquist frequency 2.5/2 = 1.25 Hz



# Computation Delay

- Inherent delay in processing
  - Actuation occurs later than expected
- Need to characterize implementation delay to make sure it is negligible
- Hardware delay is usually easy to characterize
  - Synchronous design
- Software delay is harder to predict
  - Should organize code carefully so delay is predictable and minimized
  - Write software with predictable timing behavior (be like hardware)
    - Time Trigger Architecture
    - Synchronous Software Language

# Benefit of Computer Control

- Cost!!!
  - Expensive to make analog control immune to
    - Age, temperature, manufacturing error
  - Computer control replace complex analog hardware with complex code
- Programmability!!!
  - Computer Control can be “upgraded”
    - Change in control mode, gain, are easy to do
  - Computer Control can be adaptive to change in plant
    - Due to age, temperature, …etc
  - “future-proof”
    - Easily adapt to change in standards,..etc