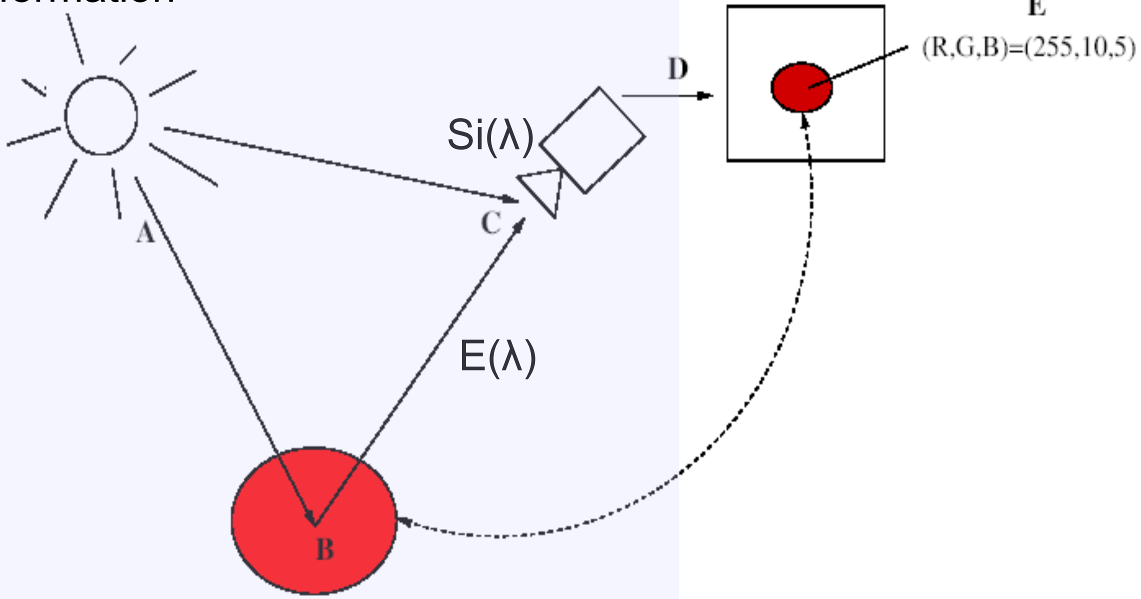
# Colorimetry and Colorimetric color models

# RGB color model

Image formation



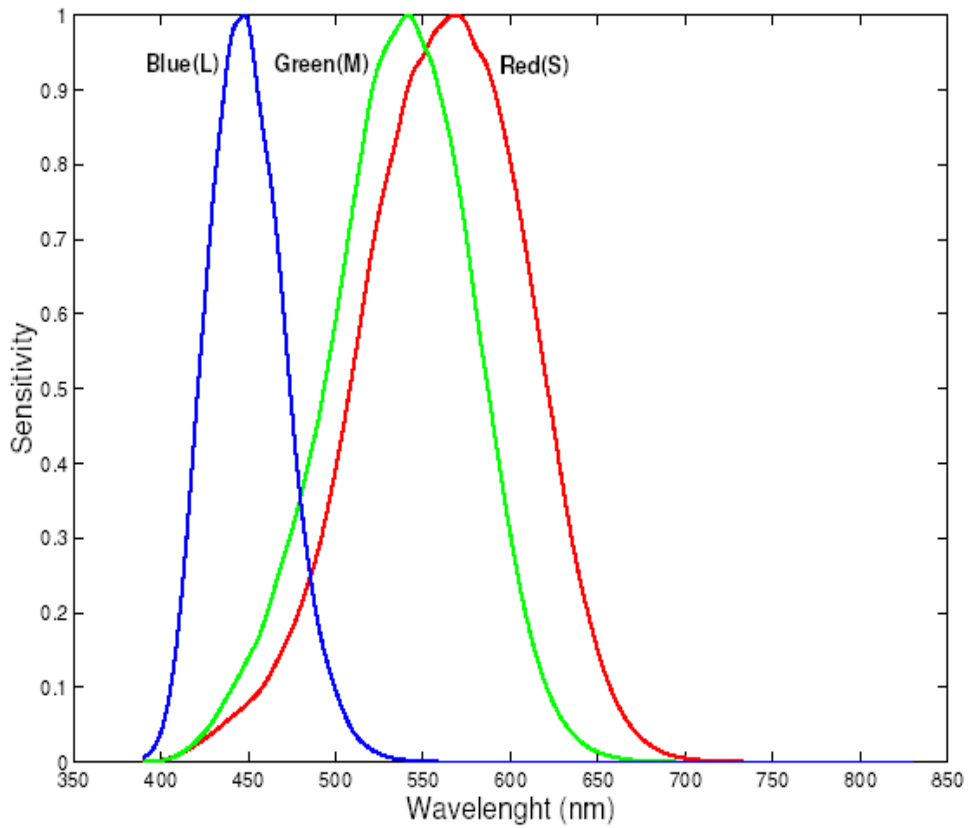

$$C_i = \int_{\lambda} E(\lambda) S_i(\lambda) d\lambda$$

$S_i(\lambda)$: sensitivity of the $i^{th}$ sensor

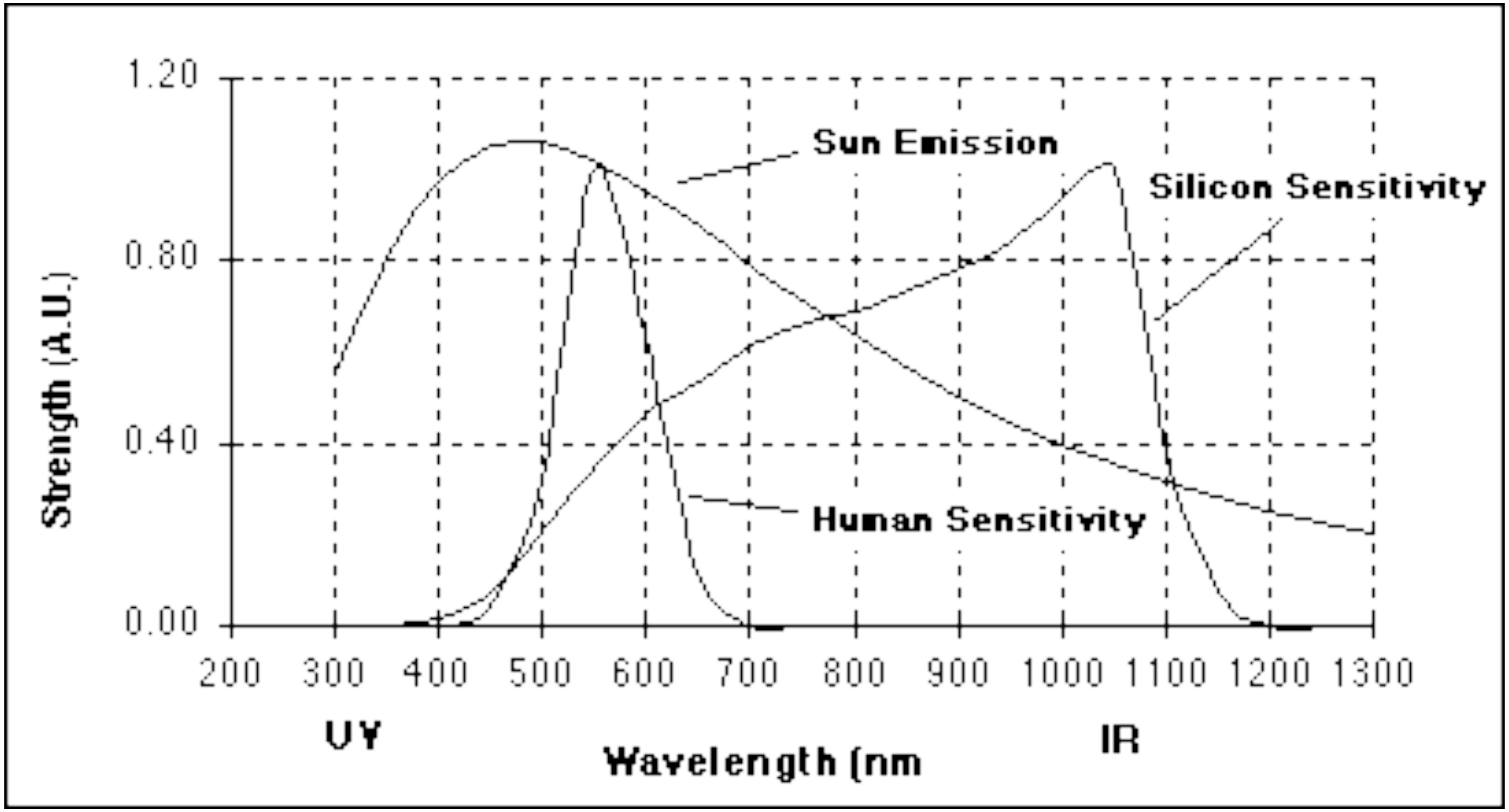
$E(\lambda)$: Spectral Power Distribution (SPD) of the diffused light

# Spectral sensitivities

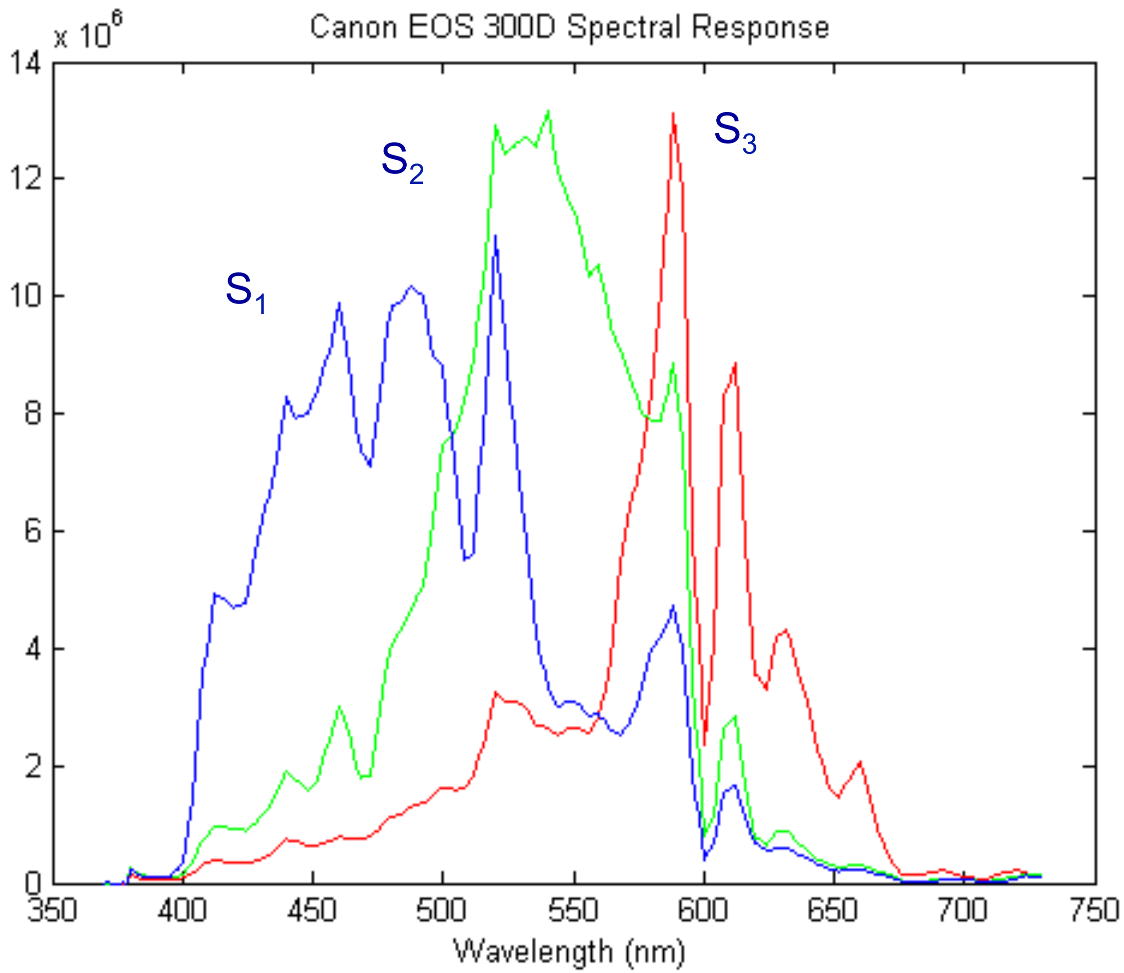
Target: (normalized) spectral sensitivities of the eye



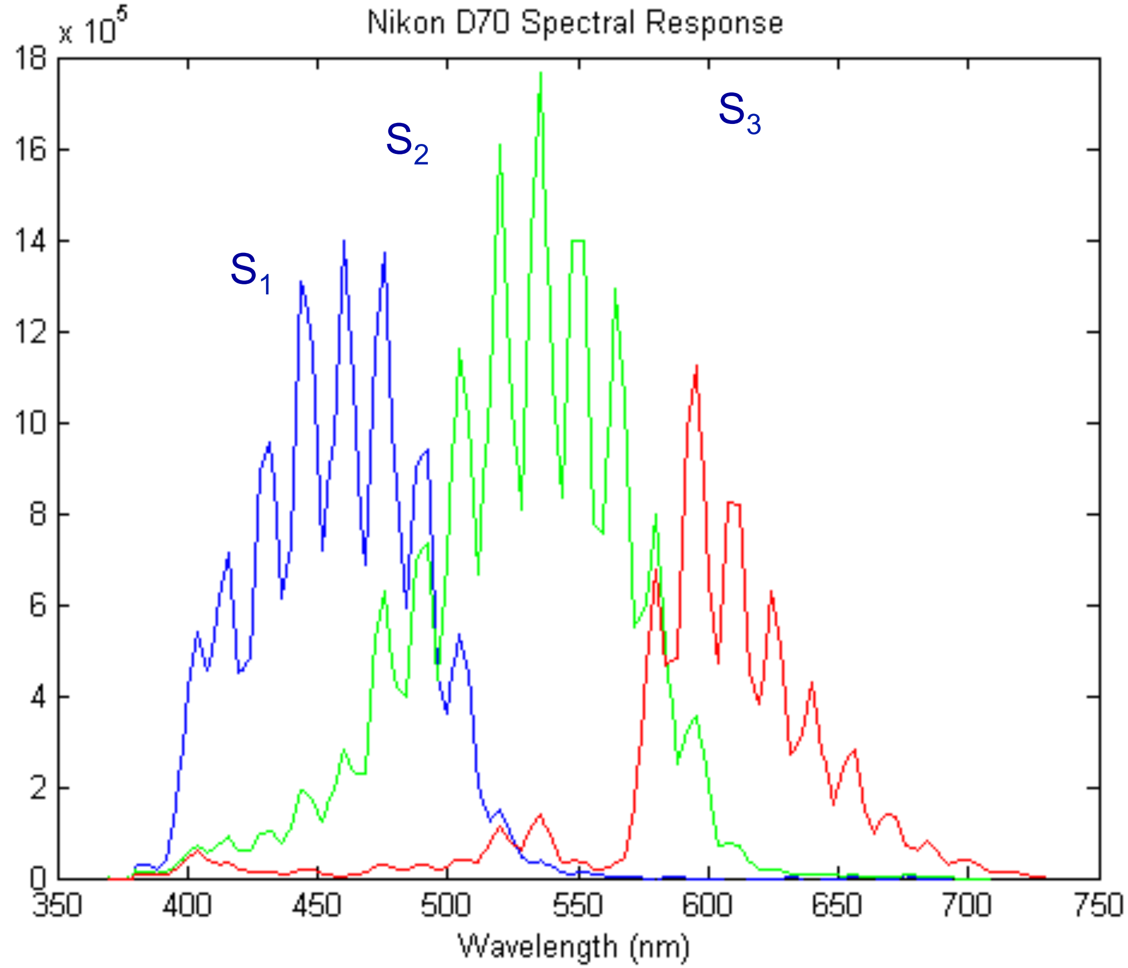
# Broad range sensitivity



# Sensor sensitivity: Ex. 1



# Spectral sensitivity: Ex. 2



# RGB model

$$C_i = \int_\lambda P(\lambda) S_i(\lambda)\, d\lambda$$

$P(\lambda)$: PSD (Power Spectral Density of the incident light)

$S_i(\lambda)$: spectral sensitivity of the "red", "green" and "blue" sensors

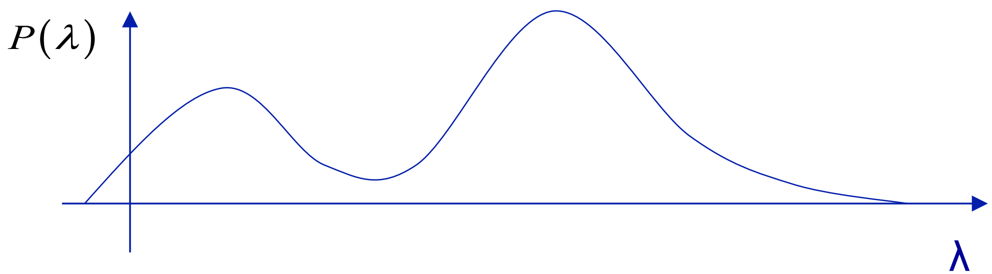
Intensity of the signals recorded by the camera in the three channels

$$R_c = k_1 \int_\lambda P(\lambda) S_1(\lambda)\, d\lambda$$

$$G_c = k_2 \int_\lambda P(\lambda) S_2(\lambda)\, d\lambda$$

$$B_c = k_3 \int_\lambda P(\lambda) S_3(\lambda)\, d\lambda$$

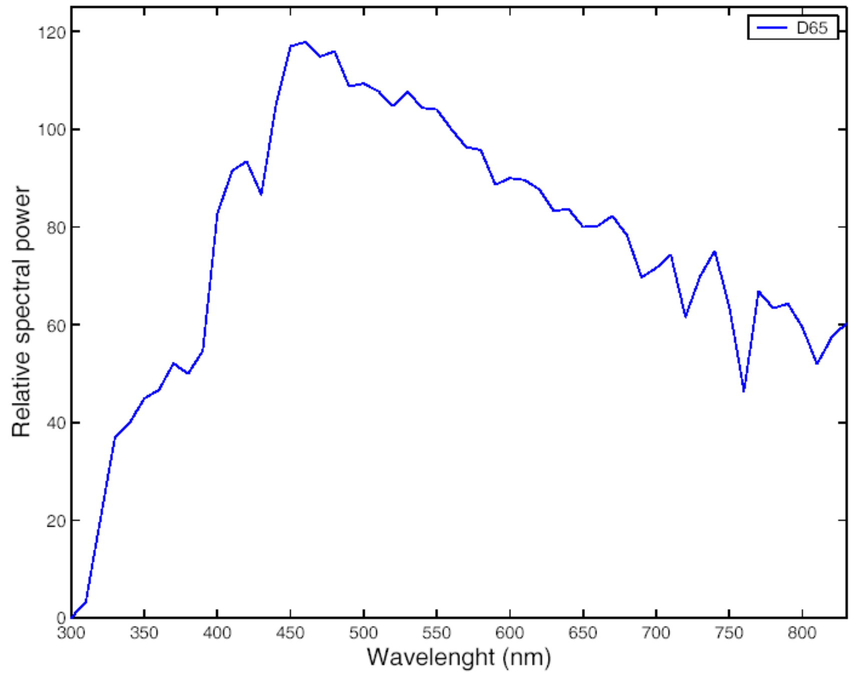
$c$ → relative to the camera




We need a PSD representing the "white" to calculate k1, k2 and k3 such that for that PSD ($P(\lambda)=E(\lambda)$) *$R_c=G_c=B_c=1$ (255)*. This is called the *reference white*

# Reference white

- The reference white is the light source that is chosen to approximate the white light
  - D65, D50



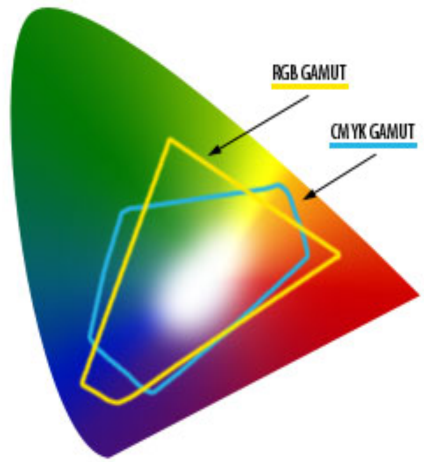
# Reference white

- The reference white, E(λ), will be given the maximum tristimulus values in all channels ($R_c$=$G_c$=$B_c$=255)
- The numerical values of the R,G,B coordinates of a generic PSD P(λ) will depend on the choice of E(λ)
    - We neglect the pedices for easyness of notations

$$
\begin{aligned}
R_{Ec} &= k_1 \int_{\lambda} E(\lambda) S_1(\lambda) d\lambda = 255 \\
G_{Ec} &= k_2 \int_{\lambda} E(\lambda) S_2(\lambda) d\lambda = 255 \rightarrow k_1, k_2, k_3 \\
B_{Ec} &= k_3 \int_{\lambda} E(\lambda) S_3(\lambda) d\lambda = 255
\end{aligned}
$$

# RGB tristimulus values

- The R,G,B coordinates does not have an *absolute* meaning, as their values depend on
  - The spectral sensitivity of the sensors that are used in the capture device
  - The reference white
- Thus, R,G,B values of the *same physical stimulus* (image) acquired with *different cameras* are different, in general
- Gamut: set of colors that is "manageable" by the device
  - Acquisition devices: set of colors that are represented by the device
  - → gamut mapping



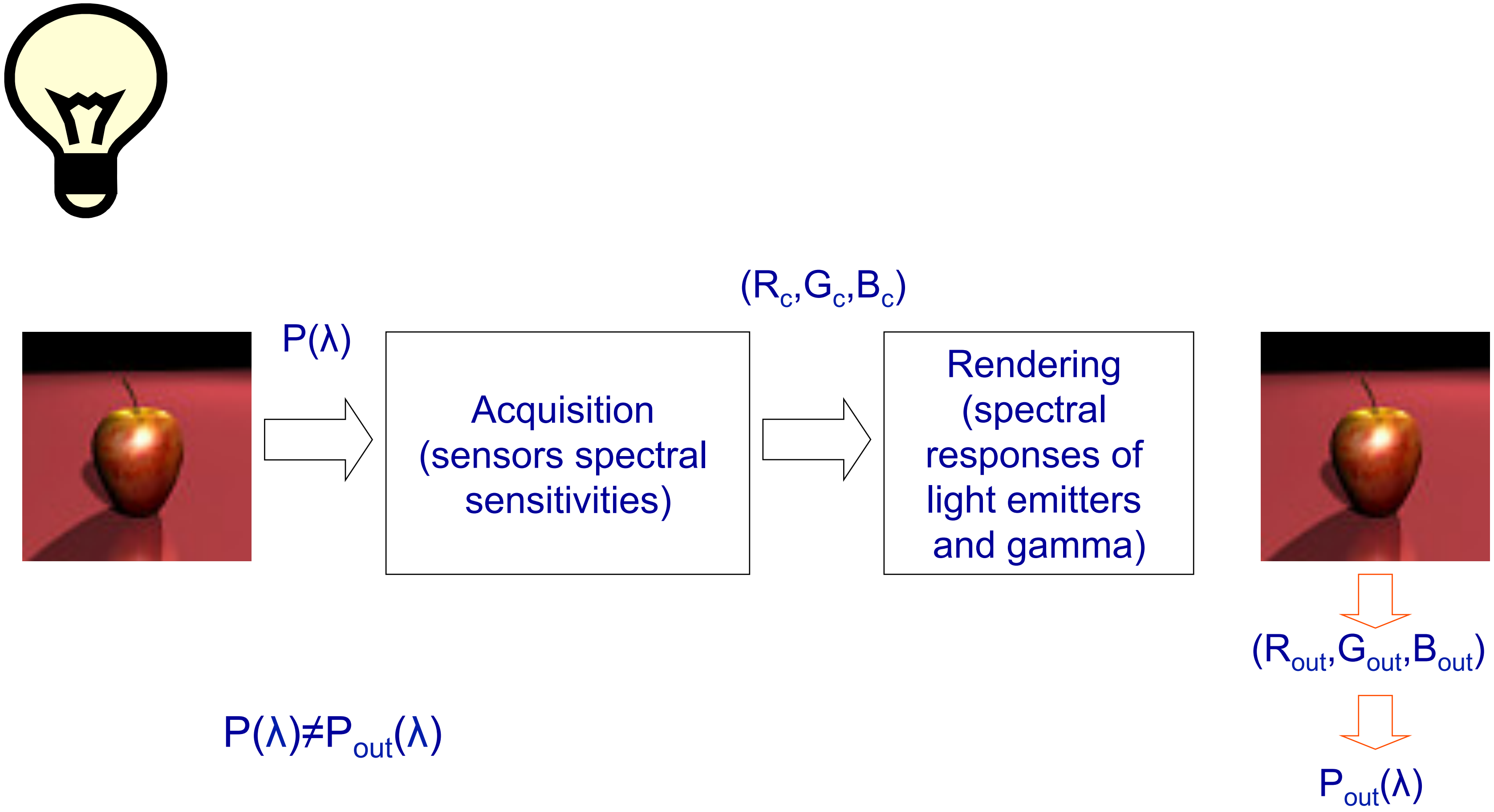
# RGB model

- Similar considerations apply to rendering devices: the rendering of a color with given tristimulus coordinares (R,G,B) will depend on
  - The spectral responses of the emitters
    - phosphors for a CRT
    - color filters in a LCD
  - The calibration of the device
    - As for the acquisition devices, the color corresponding to the rendered white must be set
    - To define the entire gamut for a monitor, you only need mark the points on the diagram that represent the colors the monitor actually produces. You can measure these colors with either a colorimeter or a photospectrometer along with software that ensures the monitor is showing 100 percent red for the red measurement, 100 percent green for the green measurement, and 100 percent blue for the blue measurement.
  - The linearity of the monitor transfer function (gamma)

# RGB model: rendering ex.

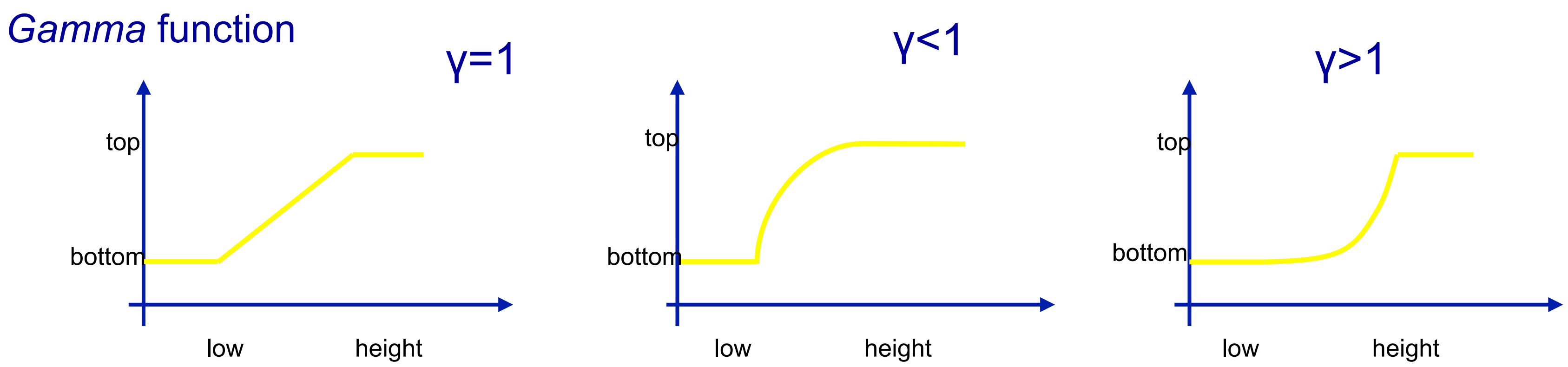
- The RGB values depend on the phosphores
- Different for the different reproduction media (CRT, television displays)
  - Example:
    - Red phosphore: x=0.68, y=0.32
    - Green phosphore: x=0.28, y=0.60
    - Blue phosphore: x=0.15, y=0.07
  - Given the x,y coordinates of the phosohores, the reference white point and the illuminant (D65), the RGB coordinates can be calculated
  - Calibration
    - the R=G=B=100 points must match in appearance with the white color as observed by 10 deg observer under the D65 illuminant
    - The brightness of the three phosphores is non linear with the RGB values. A suitable correction factor must be applied (Gamma correction)

# RGB model



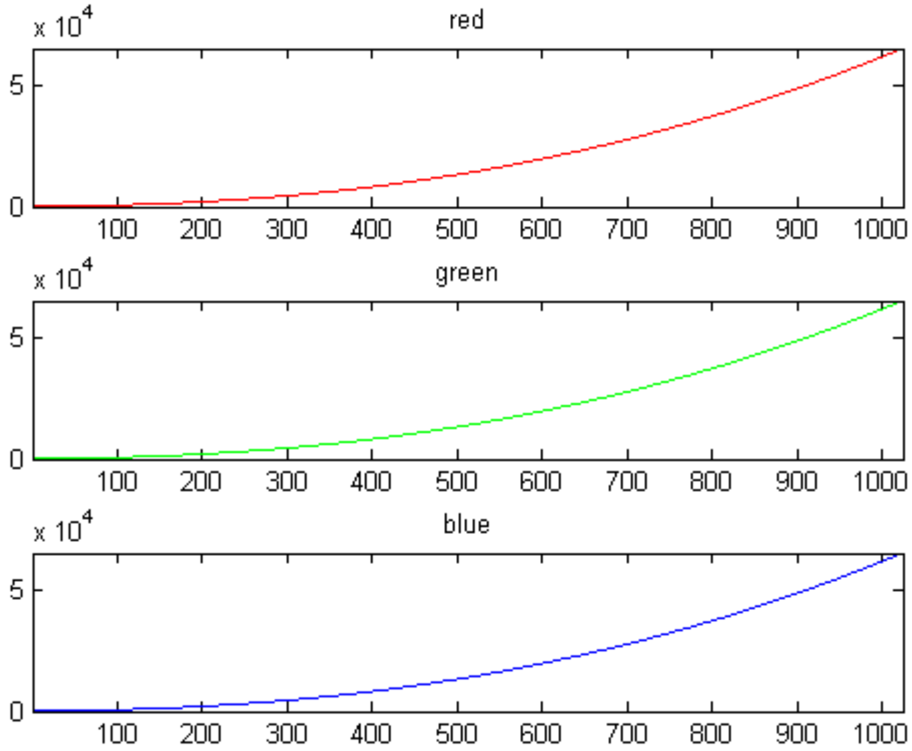
$P(\lambda)$
Acquisition (sensors spectral sensitivities)
$(R_c,G_c,B_c)$
Rendering (spectral responses of light emitters and gamma)
$(R_{out},G_{out},B_{out})$
$P_{out}(\lambda)$
$P(\lambda) \neq P_{out}(\lambda)$

# Gamma function




- Typical CRT monitors: gamma=2.2
- The non-linearity of the monitor can be compensated by non-uniform scaling of the RGB coordinates at input (*RGB linearization*)
- This led to the definition of the sRGB color model

# sRGB



x 10^4
red
5
0
100 200 300 400 500 600 700 800 900 1000
x 10^4
green
5
0
100 200 300 400 500 600 700 800 900 1000
x 10^4
blue
5
0
100 200 300 400 500 600 700 800 900 1000

# CIE-RGB

Colorimetric standard observer

# RGB standard observer

- Spectral sensitivities for the human eye have been measured in reference conditions by a very large number of observers
- Performed by the CIE (Commission Intérnationale d' Eclairage) standardization committee
- Such curves are called Color Matching Functions (CMFs) after the type of experiment
- The so-derived tristimulus values
  - Are not device dependent
  - Are still relative as they depend on (1) the choice of the red, green and blue monochromatic primaries that were used (2) the reference white and (3) the experimental conditions

# Wavelength encoding

- Scotopic matching experiment → Scotopic luminosity function V' (λ)
  - Characterizes vision at low illumination conditions
  - Rod responses
  - One primary light and one test light
  - The intensity of the light beam is the parameter

- Photopic color matching experiment→ Color matching finctions (CMF), photopic luminosity function V(λ)
  - Characterizes vision under high illumination conditions
  - Cones responses
  - Three primary lights and one test light
  - The intensities of each primary lights are the parameters

# Brightness matching

Wavelength encoding

# The photometric principle

- Basic postulate

  ***Whatever the visual stimulus, fixed in all respects, of one patch, and whatever the fixed relative spectral distribution of the stimulus on the second patch, a brightness match can always be achieved by varying the absolute value of the second stimulus***

- Basic laws of brightness matching
  - Symmetry
    - If A matches B then B matches A
  - Transitivity
    - If A matches B and B matches C than A matches C
  - Proportionality
    - If A matches B then kA matches kB
  - Additivity
    - If A matches B and C matches D than (A+C) matches (B+D)

# Brightness match

- Definition
  - *Similar uniform light patches, producing visual stimuli defined by* $\{P_\lambda d\lambda\}$ *and* $\{P'_\lambda d\lambda\}$*, respectively, are in brightness match for the standard photopic observer if*

$$\int_\lambda P_\lambda V(\lambda) d\lambda = \int_\lambda P'_\lambda V(\lambda) d\lambda$$

    - For brightness matches, the photopic luminous flux entering the eye per unit solid angle must be the same for the two patches

# Matching experiments

- Scotopic matching experiment (brightness matching)
  - Low illumination conditions
  - Rod responses
  - One primary light and one test light
  - The *intensity* of the primary light beam is the parameter
- Measures the scotopic spectral sensitivity function V' (λ)

- Photopic color matching experiment
  - High illumination conditions
  - Brightness matching
    - Rods
  - Color matching
    - Cones
    - Three primary lights and one test light
    - The intensities of each primary lights are the parameters
- Measures the photopic spectral sensitivity function V(λ)
- Measures the Color Matching Functions (CMFs)

# Brightness matching

Necessary and sufficient condition for a brightness match between two stimuli of radiant power distribution $\{P_\lambda d\lambda\}$ and $\{P'_\lambda d\lambda\}$, respectively

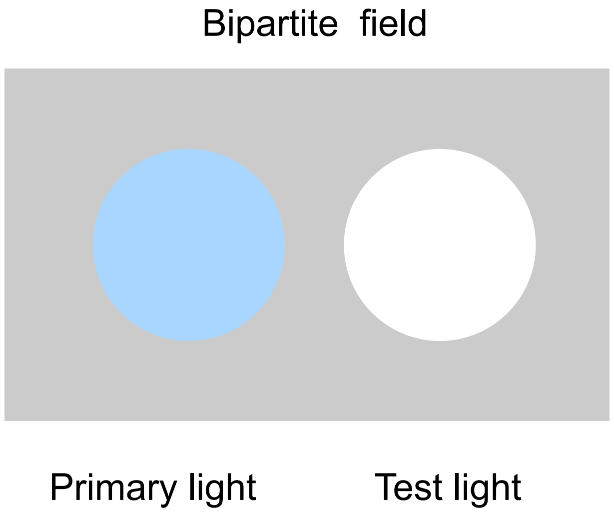
$$\int_\lambda P_\lambda \beta(\lambda) d\lambda = \int_\lambda P'_\lambda \beta(\lambda) d\lambda$$

Where $\beta(\lambda)$ is a fixed function characterizing the brightness-matching process depending on

- the spectral radiance power distributions (relative or absolute) of the matching stimuli
- the observational conditions
  - field size, eccentricity of the field of view, state of adaptation as modified by previous or surrounding stimuli
- *Quantum efficiency* of the human visual system

- *Ideal photometric observer*
  - *defined by the CIE by the specification of two fixed functions*
- *Scotopic* matching $\beta(\lambda) \rightarrow V'(\lambda)$

# *Scotopic* brightness matching




The **primary** light has a fixed relative spectral distribution and only the *intensity* can vary

The **test** light can have any spectral distribution. It is common to use an equal-energy spectral light

Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable

# Scotopic spectral sensitivity function

$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

**r** : system vector (transfer function)

**t** : spectral distribution of the test light

e : response of the observer

Assuming that the system is linear (homogeneity and superposition hold), the system vector can be measured by feeding it with $n_\lambda$ monochromatic lights.

It is common to choose an **equal-energy spectral** light (*reference white*) as test light.

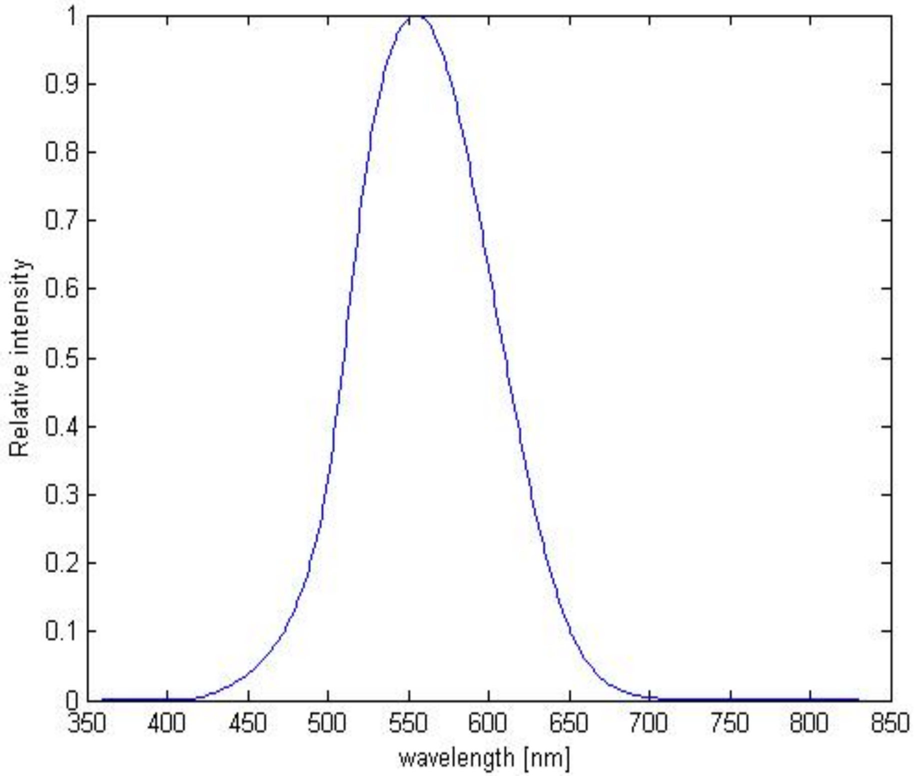
$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = r_1$$

Each monochromatic light will determine one entry of the system vector, resulting in the *Scotopic Spectral Sensitivity function*.

# Scotopic spectral sensitivity function

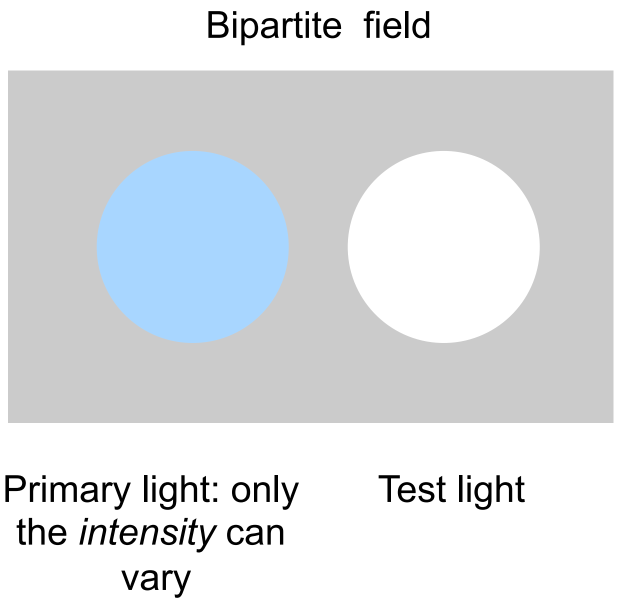
- In scotopic conditions, the eye is sensible only to *relative intensities* of the two lights. The spectral distribution is immaterial (in low illumination conditions, color is not "perceivable").
  - Physiological interpretation: the rhodopsin absorption coefficient depends on the wavelength, but the response is the same for any wavelength. Once a photon is absorbed, the information about its wavelength is lost. Hence, **the appearance of the stimulus is independent of its spectrum**.
  - **The shape of V\`(λ) reflects the dependence of the absorption coefficients from the wavelength.**
- In order to measure the spectral sensitivity at each wavelength a set of equal energy spectral (monochromatic lights) are used as test lights
- Relative intensities are recorded ($I_{REF}/I_{test}=I_{REF}$ since $I_{test}$=const.=1) and the normalized (values between zero and one)
  - Prior to normalization, due to the linearity of the system, the system vector is unique up to a scale factor.
- *V\`(λ) was adopted by CIE in 1951 in a field of 10 degrees (Crawford 1949) and eccentrically with more than 5 degrees (Wald 1945) with complete darkness adaptation.*

# Scotopic spectral sensitivity function V' (λ)




Data available at http://cvision.ucsd.edu/cie.htm

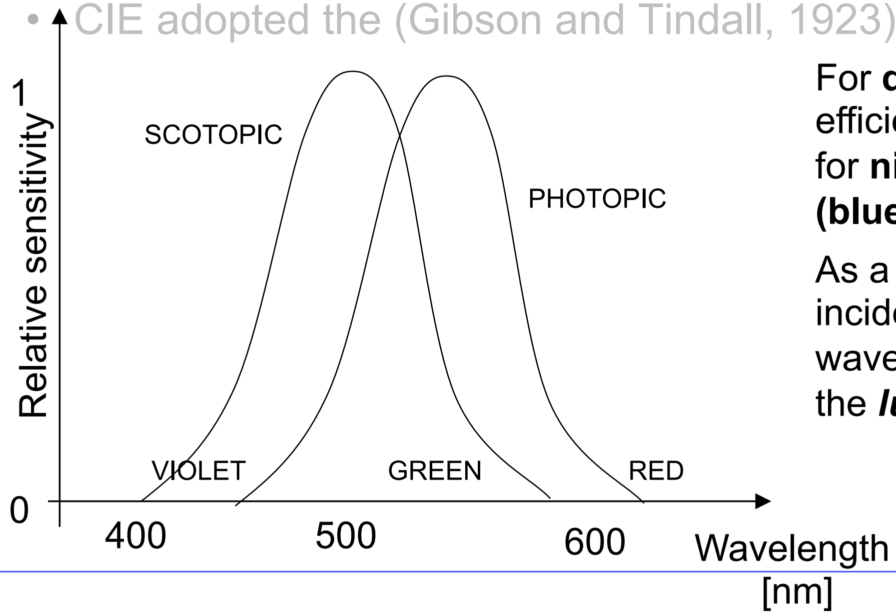
# Photopic brightness matching




Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable, following a ad-hoc paradigm

# Photopic curve V(λ)

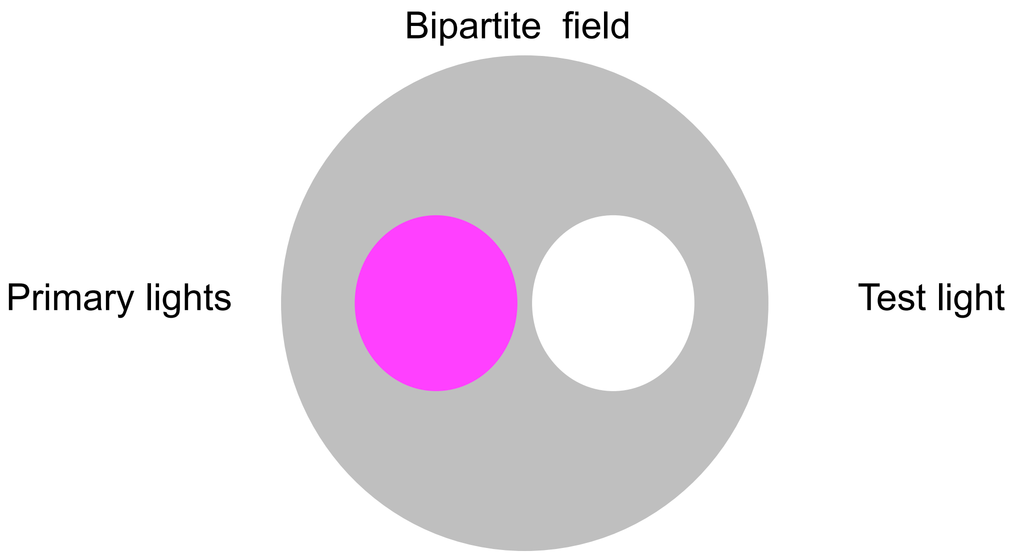
- High illumination levels
- Different paradigms
  - The direct comparison of the brightness leads to unreliable results due to the difference in color
    - Flickering method (Coblentz and Emerson, 1918)
    - Step-by-step method of heterochromatic fotometry (Hyde et al. 1918)
    - CIE adopted the (Gibson and Tindall, 1923)




For **daylight** vision the maximum efficiency is at **555 nm (yellow)** while for **night** vision it shifts to **505 nm (blue)**

As a measure of the sensitivity of the eye to incident monochromatic light at each wavelength, it corresponds to a measure of the ***luminous efficacy*** of the eyes.

# Color matching

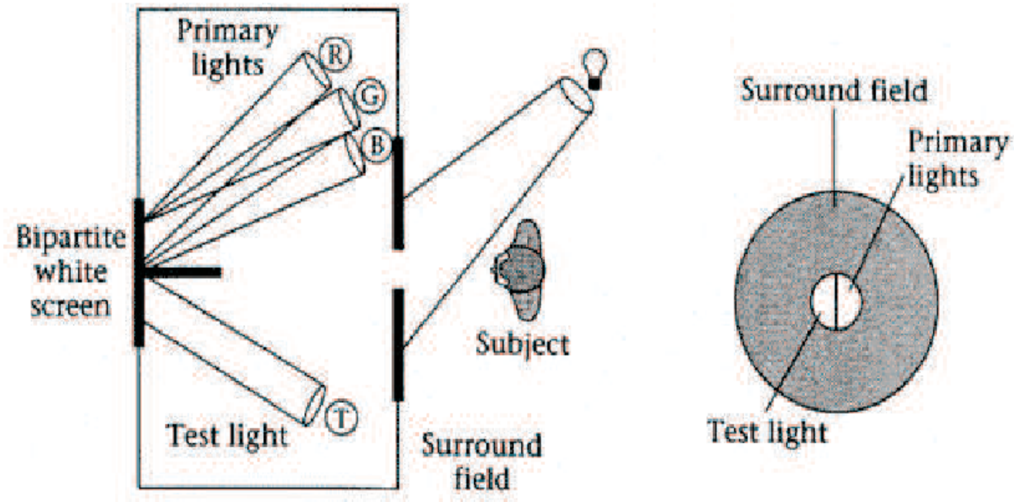


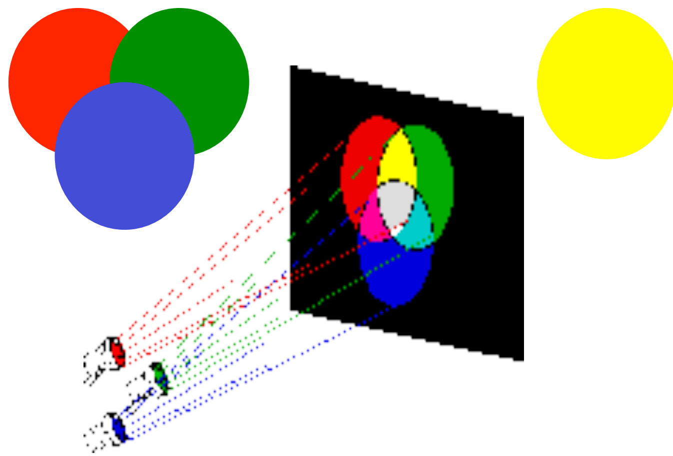

There are **three primary lights** with fixed relative spectral distribution and only the intensity can vary. These are chosen to be **monochromatic**

The test light can have any spectral distribution. It is common to chose a equal energy light and decompose it into the monochromatic components for testing the entire set of wavelengths.

**Task**: Adjust the intensities of the primary lights so that the primary and test lights appear indistinguishable

# Color matching



# Measuring the CMFs

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

**R** : system matrix (transfer function). Each line represents the *Color Matching Function* (CMF) for the corresponding primary light

**t** : spectral distribution of the test light

**e** : response of the observer

Assuming a equal-energy test light

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix} = t \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

since we are measuring relative intensities we can choose $t=1$

# Color Matching Functions (CMFs)

Assuming that the symmetry, transitivity and homogeneity hold (*Grassmann's laws of additive color mixtures*), the system matrix can be measured by feeding it with $n_\lambda$ monochromatic lights

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} r_1^1 \\ r_1^2 \\ r_1^3 \end{bmatrix}$$

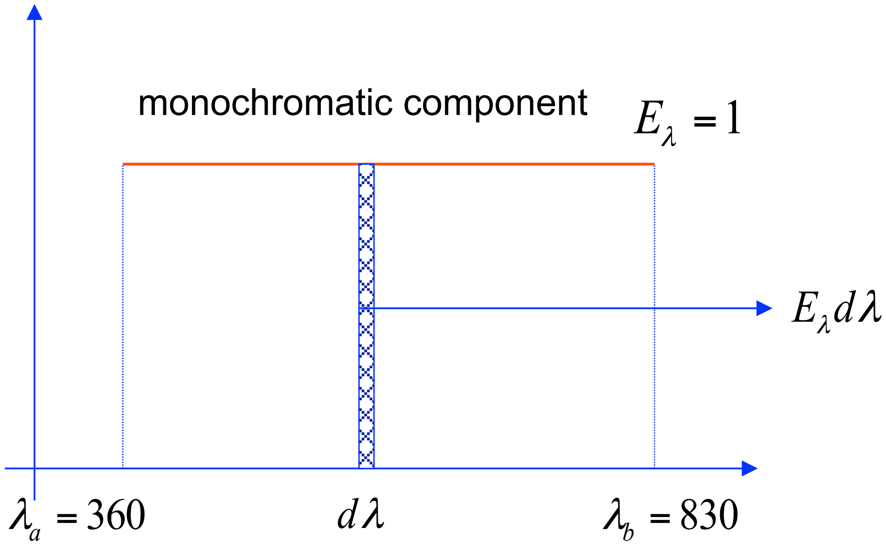
The response to each monochromatic light will determine one *column* of the system matrix, so one entry of each CMF.

It can be shown that the system matrix is not unique. Using **different sets of primaries** leads to different CMFs. Though, different sets of CMFs are related by a **linear transformation**

→ **Need to choose one set of primaries**

# Color Matching Functions

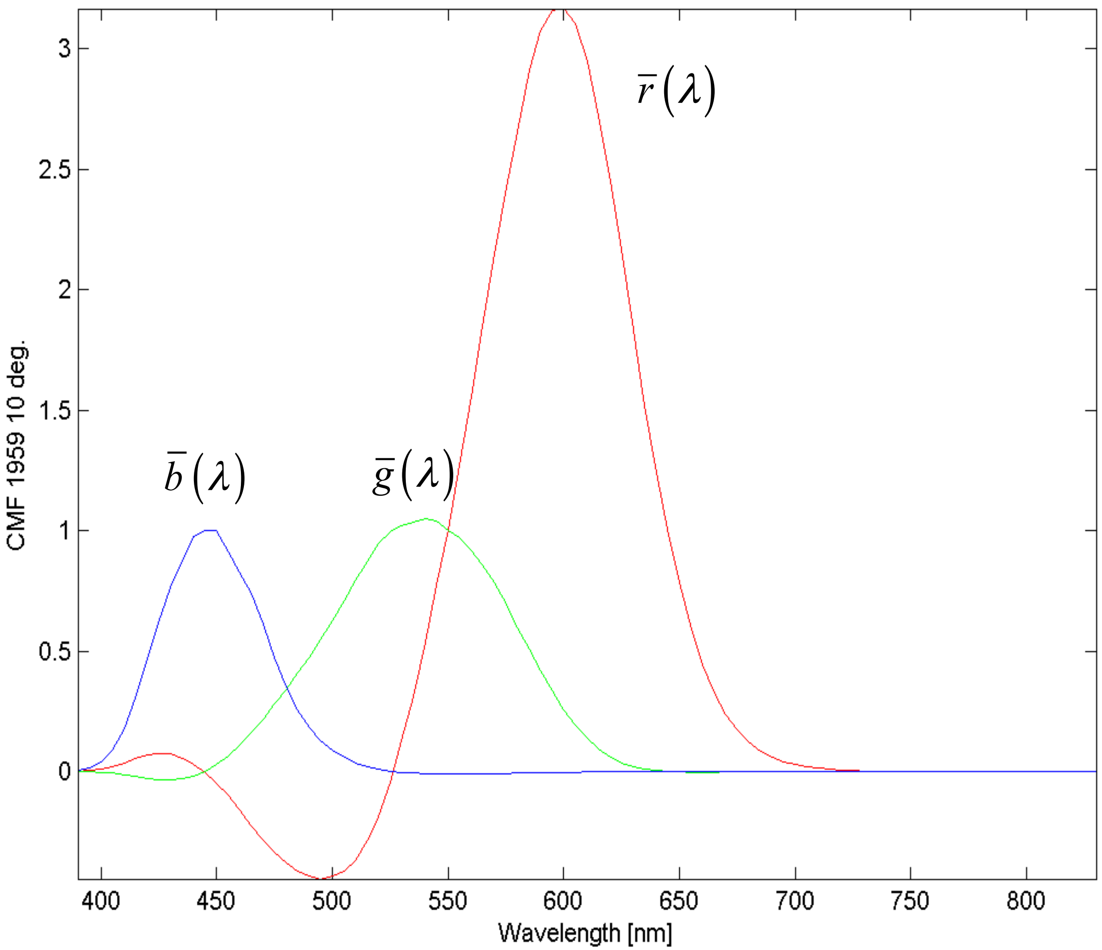
- In other words, the CMFs are the *spectral* tristimulus values of the *equal energy* stimulus E (*reference white*)




$$E_\lambda = \bar{r}(\lambda)R + \bar{g}(\lambda)G + \bar{b}(\lambda)B$$
$$\lambda_R = 700nm$$
$$\lambda_G = 546.1nm$$
$$\lambda_B = 435.8nm$$

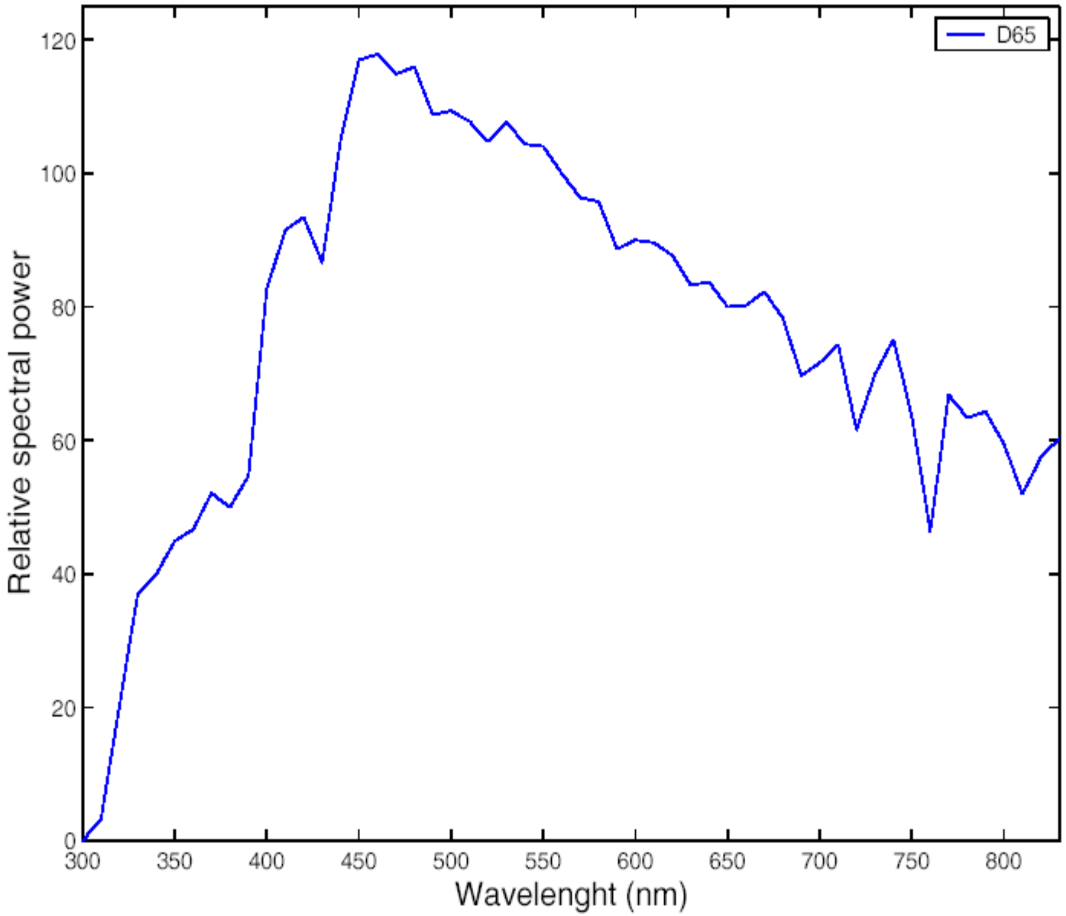
$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ are called color matching functions

# CMFs



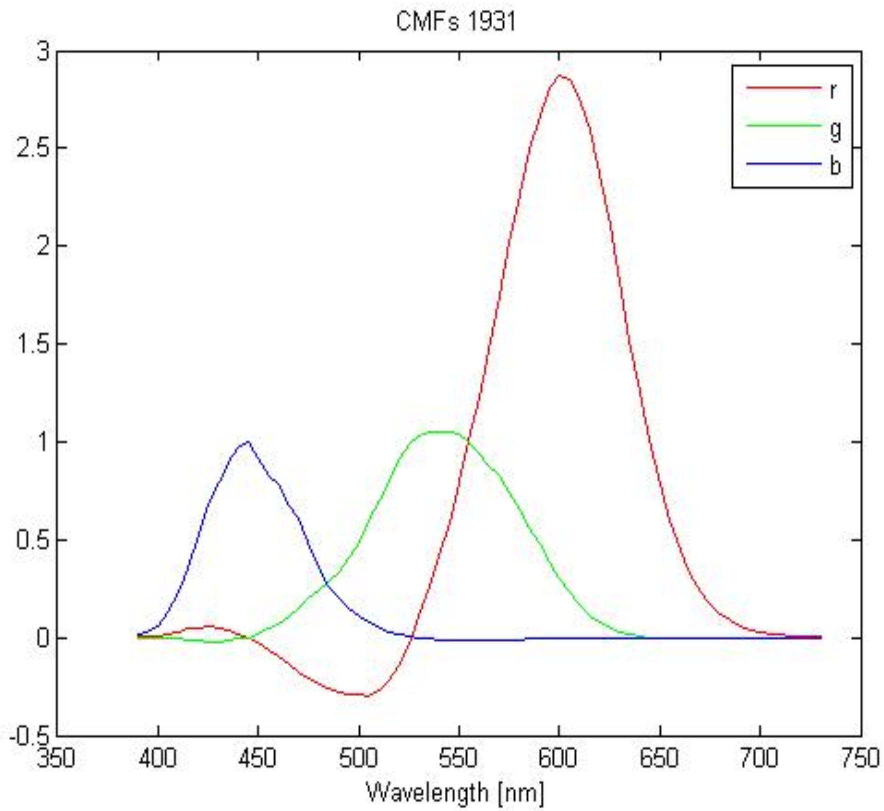
$\bar{r}(\lambda)$
$\bar{b}(\lambda)$
$\bar{g}(\lambda)$
3
2.5
2
1.5
1
0.5
0
CMF 1959 10 deg.
400
450
500
550
600
650
700
750
800
Wavelength [nm]

# D65



D65
Relative spectral power
120
100
80
60
40
20
0
300
350
400
450
500
550
600
650
700
750
800
Wavelenght (nm)

# CMF rgb 1931



CMFs 1931
3
2.5
2
1.5
1
0.5
0
-0.5
350
400
450
500
550
600
650
700
750
Wavelength [nm]
r
g
b

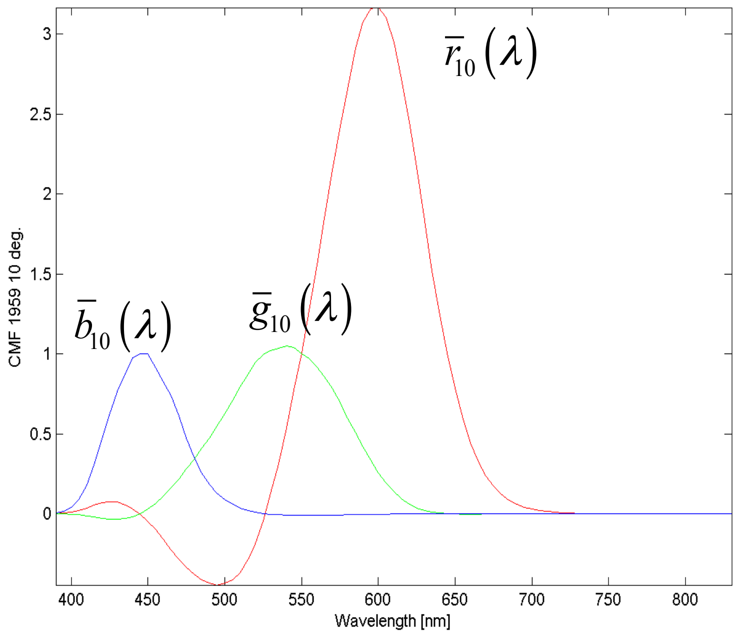
# Stiles and Burch 10deg (1959)

Primary lights: monochromatic

$\lambda_R$=645.16 nm
$\lambda_G$= 526.32 nm
$\lambda_B$= 444.44 nm

$\bar{r}_{10}(\lambda), \bar{g}_{10}(\lambda), \bar{b}_{10}(\lambda)$ CMFs

$$t(\lambda) = R \cdot \bar{r}_{10}(\lambda) + G \cdot \bar{g}_{10}(\lambda) + B \cdot \bar{b}_{10}(\lambda)$$




***t*** : monochromatic test light

(R,G,B) : ***tristimulus values*** of ***t***

A **10 degrees bipartite field** was used

Negative values for the tristimulus value mean that the corresponding primary was added to the test light in order to match the color appearance.

This outlines that not every test color can be matched by an additive mixture of the three primaries.

The presence of negative values could be impractical, so another color coordinate system was chosen as the reference by the *Commisione Internationale d'Eclairage (CIE)* in 1931.

# Cone photopigments and CMF

- How well do the spectral sensitivities of the cone photopigments predict performance on the photopic color matching experiment?

Biological measurements

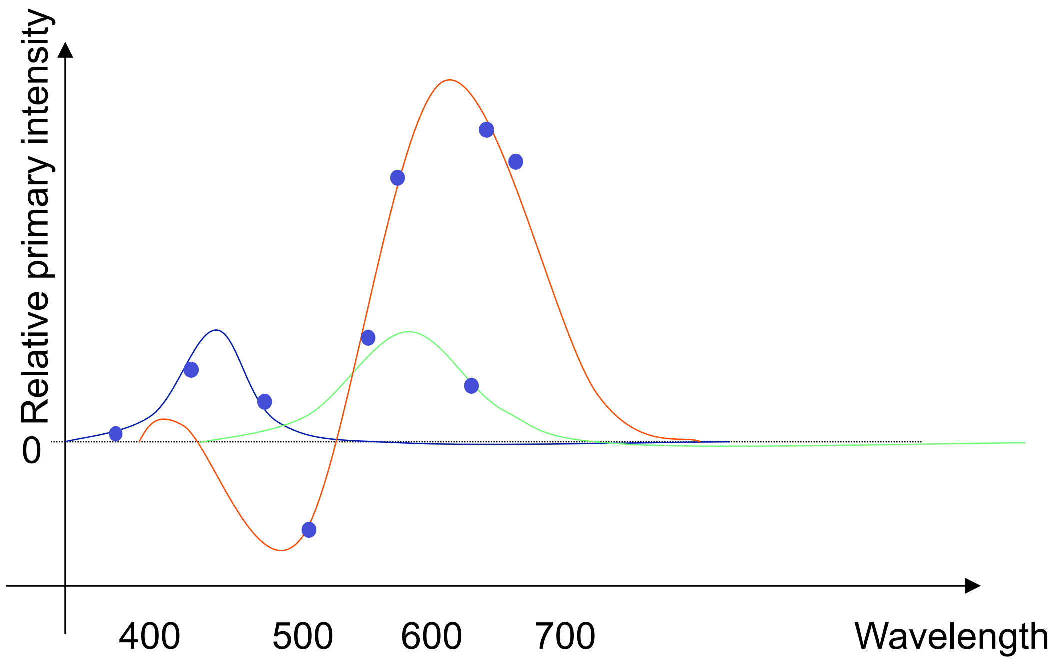
$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \text{Spectral sensitivity of L photopigments} \\ \text{Spectral sensitivity of M photopigments} \\ \text{Spectral sensitivity of S photopigments} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

Psychophyiscal measurements

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \text{CMF of primary 1} \\ \text{CMF of primary 2} \\ \text{CMF of primary 3} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

- There should be a linear transformation that maps the cone absorption curves to the system matrix of the color matching experiment
- *Linking hypothesis*

# Cone photopigments and CMFs




From the agreement between these two datasets one can conclude that the photopigment spectral responsivities provide a satisfactory biological basis to explain the photopic color matching experiments

# Tristimulus values for complex stimuli

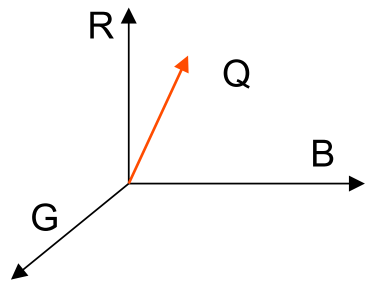
- Color stimuli are represented by vectors in a three-dimensional space, called the *tristimulus* space
  - Let Q be an arbitrary **monochromatic** color stimulus and **R,G and B the fixed primary stimuli** chosen for the color matching experiment

$$Q = R_Q\vec{R} + G_Q\vec{G} + B_Q\vec{B}$$

  - $R_Q, G_Q, B_Q$ : *tristimulus values* of Q
  - The scalar multipliers $R_Q, G_Q, B_Q$ are measured in terms of the *assigned respective units of the corresponding primaries*
  - It is customary to choose these units such that when additively mixed yield a complete color match with a specified *achromatic* stimulus, usually one with an *equal-energy spectrum* on a wavelength basis

The units of these primaries was chosen in the radiant power ratio of 72.1:1.4:1.0, which places the chromaticity coordinates of the equal energy stimulus E at the center of the $(r,g)$ chromaticity diagram

→ $R_W=G_W=B_W=1$ for the reference white

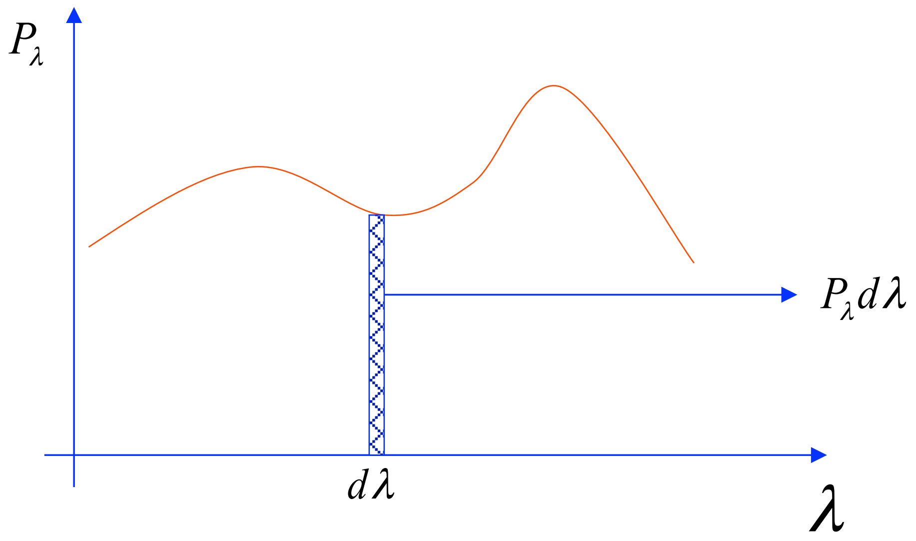


# Complex stimuli

- A given complex stimuli Q with spectral power density (SPD) $\{P_\lambda d\lambda\}_Q$ can be seen as an *additive mixture* of a set of monochromatic stimuli $Q_i$ with SPD $\{P_\lambda \, d\lambda\}_{Qi}$
  - For each monochromatic stimulus

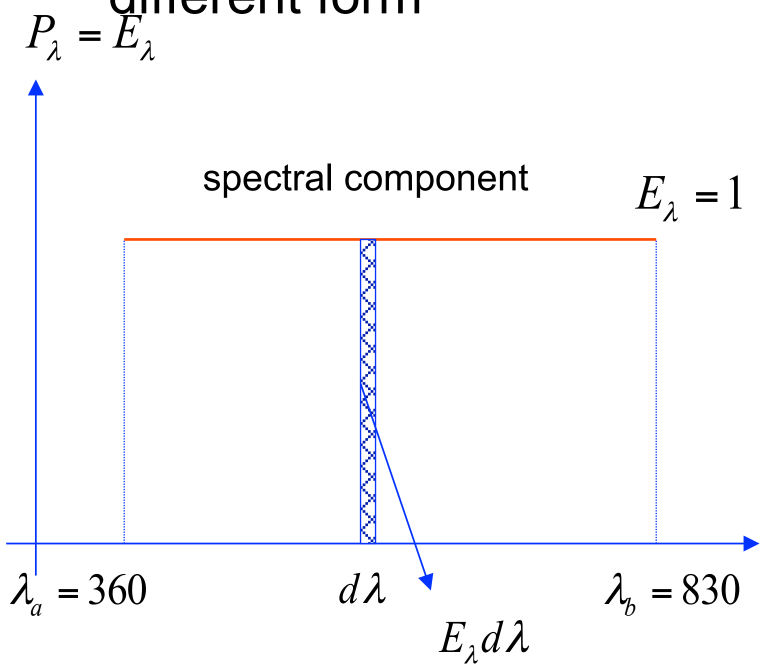
$$\vec{P}_\lambda = R_\lambda \vec{R} + G_\lambda \vec{G} + B_\lambda \vec{B}$$

$R_\lambda, G_\lambda, B_\lambda$ spectral tristimulus values



# Special case: reference white

- The *reference white* is used to express the complex spectrum in a different form




$$\vec{E}_\lambda = \bar{r}(\lambda)\vec{R} + \bar{g}(\lambda)\vec{G} + \bar{b}(\lambda)\vec{B} \Rightarrow$$
$$\int_\lambda \vec{E}_\lambda \, d\lambda = \int_\lambda \left(\bar{r}(\lambda)\vec{R} + \bar{g}(\lambda)\vec{G} + \bar{b}(\lambda)\vec{B}\right) d\lambda =$$
$$\int_\lambda \left(\bar{r}(\lambda)\vec{R}\right) d\lambda + \int_\lambda \left(+\bar{g}(\lambda)\vec{G}\right) d\lambda + \int_\lambda \left(\bar{b}(\lambda)\vec{B}\right) d\lambda =$$
$$\int_\lambda \bar{r}(\lambda) d\lambda \times \vec{R} + \int_\lambda \bar{g}(\lambda) d\lambda \times \vec{G} + \int_\lambda \bar{b}(\lambda) d\lambda \times \vec{B}$$

Normalization conditions

$$E_R = \int_{-\infty}^{+\infty} \bar{r}(\lambda) \, d\lambda = 1$$
$$E_G = \int_{-\infty}^{+\infty} \bar{g}(\lambda) \, d\lambda = 1$$
$$E_B = \int_{-\infty}^{+\infty} \bar{b}(\lambda) \, d\lambda = 1$$

Then

$$\vec{E} = 1\vec{R} + 1\vec{G} + 1\vec{B}$$

The Color Matching Functions are normalized such that the tristimulus values of the reference white are equal.

# Tristimulus values of a complex stimulus

$$Q_\lambda = \left(P_\lambda d\lambda\right) E_\lambda = \left(P_\lambda d\lambda\right) \bar{r}\left(\lambda\right) \vec{R} + \left(P_\lambda d\lambda\right) \bar{g}\left(\lambda\right) \vec{G} + \left(P_\lambda d\lambda\right) \bar{b}\left(\lambda\right) \vec{B} \rightarrow$$

$$R_Q = \int_\lambda \left(P_\lambda d\lambda\right) \bar{r}\left(\lambda\right) = \int_\lambda P_\lambda \bar{r}\left(\lambda\right) d\lambda$$

$$G_Q = \int_\lambda \left(P_\lambda d\lambda\right) \bar{g}\left(\lambda\right) = \int_\lambda P_\lambda \bar{g}\left(\lambda\right) d\lambda$$

$$B_Q = \int_\lambda \left(P_\lambda d\lambda\right) \bar{b}\left(\lambda\right) = \int_\lambda P_\lambda \bar{b}\left(\lambda\right) d\lambda$$

Metameric stimulidifferent SPD, same color appearance

$$R_Q = \int P_\lambda^1 \bar{r}(\lambda) d\lambda = \int P_\lambda^2 \bar{r}(\lambda) d\lambda$$

$$G_Q = \int P_\lambda^1 \bar{g}(\lambda) d\lambda = \int P_\lambda^2 \bar{g}(\lambda) d\lambda$$

$$B_Q = \int P_\lambda^1 \bar{b}(\lambda) d\lambda = \int P_\lambda^2 \bar{b}(\lambda) d\lambda$$

# Chromatic coordinates

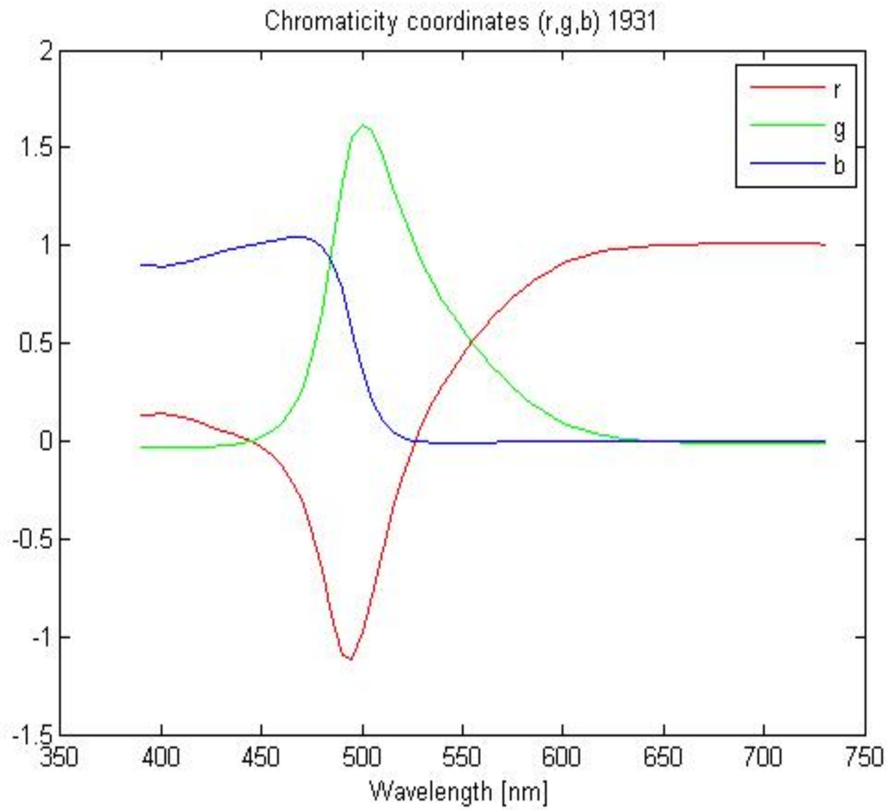
- Spectral chromaticity coordinates

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$r(\lambda) + g(\lambda) + b(\lambda) = 1$$

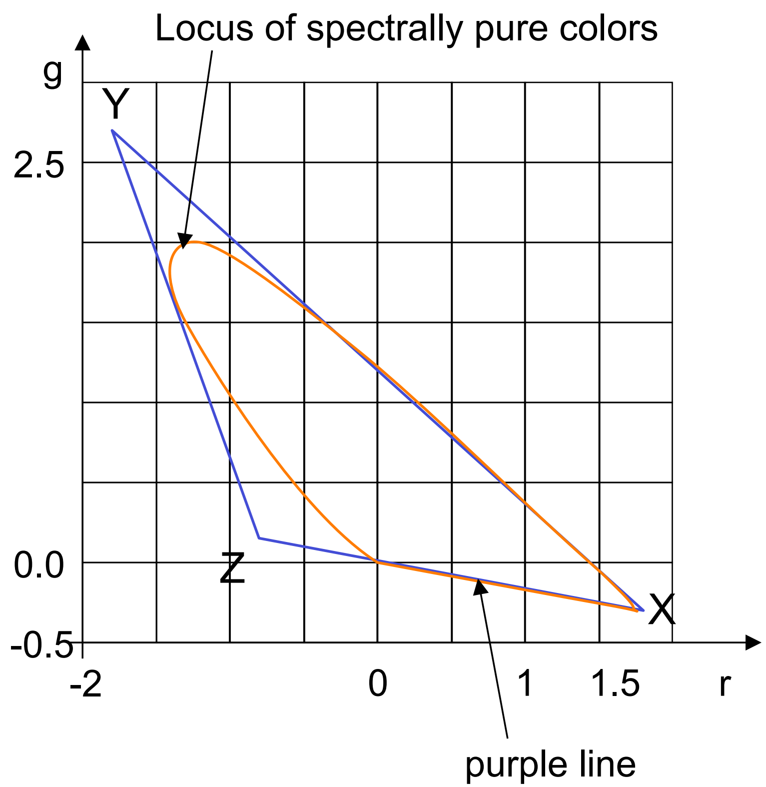


# (r,g) chromaticity diagram

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)}$$




$r,g,b$ : chromaticity coordinates

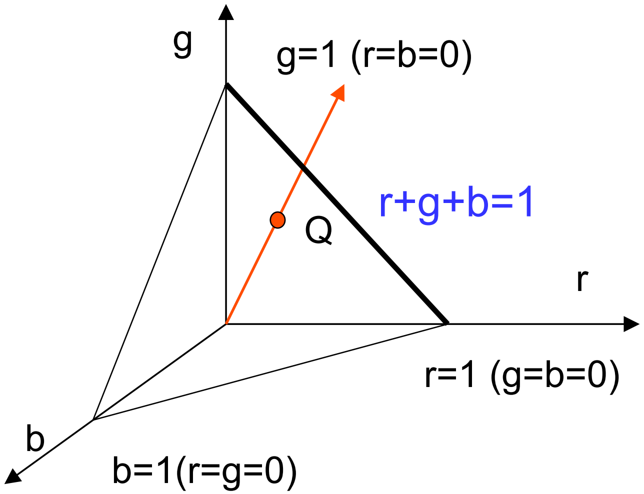
$\bar{r},\bar{g},\bar{b}$ color matching functions (tristimulus values of the reference white)

# Chromaticity coordinates

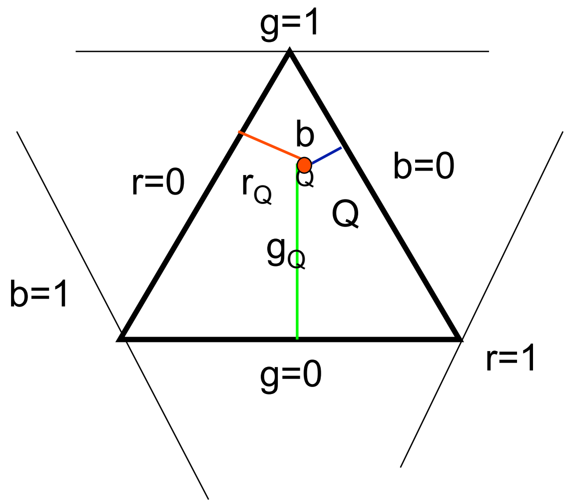
*Chromaticity coordinates*

$$r = \frac{R}{R+G+B}$$
$$g = \frac{G}{R+G+B} \quad \Rightarrow \quad r+g+b=1$$
$$b = \frac{B}{R+G+B}$$

R,G,B: tristimulus value of the generic color




Maxwell color triangle




(r,g) specify the *hue and saturation* of the color while the information about the luminance is lost

*A step in colorimetry…*

# CIE 1931 Standard Observer

- In colorimetric practice, the main objective is to obtain results valid for the group of normal trichromats. To this end, the color matching properties of an *ideal trichromatic observer* are defined by specifying three independent functions of λ which are identified with the ideal observer CMFs.
- The CIE 1931 SO also embodies the additivity law for brightness (V(λ) photopic luminous efficiency function)
  - For an observer who makes brightness matches that *conform to the additivity law for brightness*, and who also *makes color matches* that are trichromatic in the stronger sense, it can be shown that *V(λ) is a combination of the CMFs*, provided all the pairs of metameric stimuli are also in brightness match.

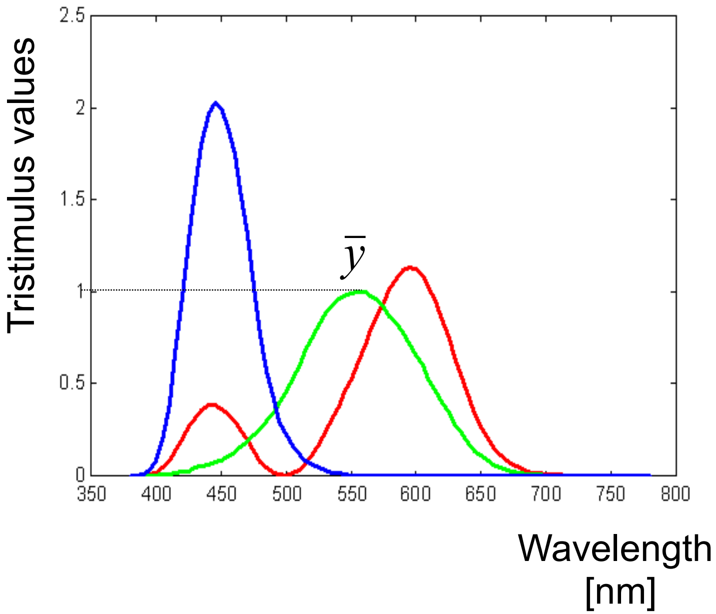
    For such an observer, it is possible to select from the infinitely many equivalent sets of CMFs one set for which one of the three CMFs, usually taken to be the $\bar{y}$ central one

    coincides with V(λ).

    **In this way, the CIE 1931 SO combines both color matching and heterochromatic brightness matching properties in a single quantitative scheme**.

# CIE 1931 Standard Colorimetric Observer

- Standard system for color representation: X,Y,Z tristimulus coordinate system $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$
- Color matching functions




Features

- λ=380 to 780 nm, Δλ=5nm
- Measured at 2 degrees
- Always non negative
- $\bar{y}$ is a rough approximation of the *brightness* of monochromatic lights of equal size and duration (*Standard photopic luminosity function V(λ)*)
- They cannot be measured by color matching experiments
- Quite inaccurate at low wavelengths

Improvements

- In 1959 a new set of CIE XYZ coordinates was derived based on the CMFs measured by Stiles&Burch at 10 degrees (CIE 1964 Supplementary Standard Colorimetric Observer).

# Guidelines for the derivation of CIE 1931 SO

- Projective transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

$(r_x, g_x, b_x)$ : coordinates of (1,0,0) as measured in the {r,g,b} system

....

  – Need to determine the matrix A of the transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \dots & \\ & & a_{3,3} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \dots & \\ & & a_{3,3} \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1}$$

  – This is accomplished by imposing some conditions

# Guidelines for the derivation of CIE 1931 SO

1. The function $\bar{y}(\lambda)$ must be equal to the luminosity function of the eye V(λ)

   $$\bar{y}(\lambda) = V(\lambda)$$

   this sets a relation among 3 coefficients

   $$a_{11} + a_{12} + a_{13} = const.$$

2. The constant spectrum of white, E(λ)=1, should have equal tristimulus values

$$\sum_{i=1}^{N} \bar{x}(\lambda_i) = \sum_{i=1}^{N} \bar{y}(\lambda_i) = \sum_{i=1}^{N} \bar{z}(\lambda_i)$$

$$\sum_{i=1}^{N} \bar{x}(\lambda_i) = a_{11} \sum_{i=1}^{N} \bar{r}(\lambda_i) + a_{12} \sum_{i=1}^{N} \bar{g}(\lambda_i) + a_{13} \sum_{i=1}^{N} \bar{b}(\lambda_i)$$

but

$$\sum_{i=1}^{N} \bar{r}(\lambda_i) = \sum_{i=1}^{N} \bar{g}(\lambda_i) = \sum_{i=1}^{N} \bar{b}(\lambda_i) = S$$

thus

$$\sum_{i=1}^{N} \bar{x}(\lambda_i) = (a_{11} + a_{12} + a_{13}) S$$

.....

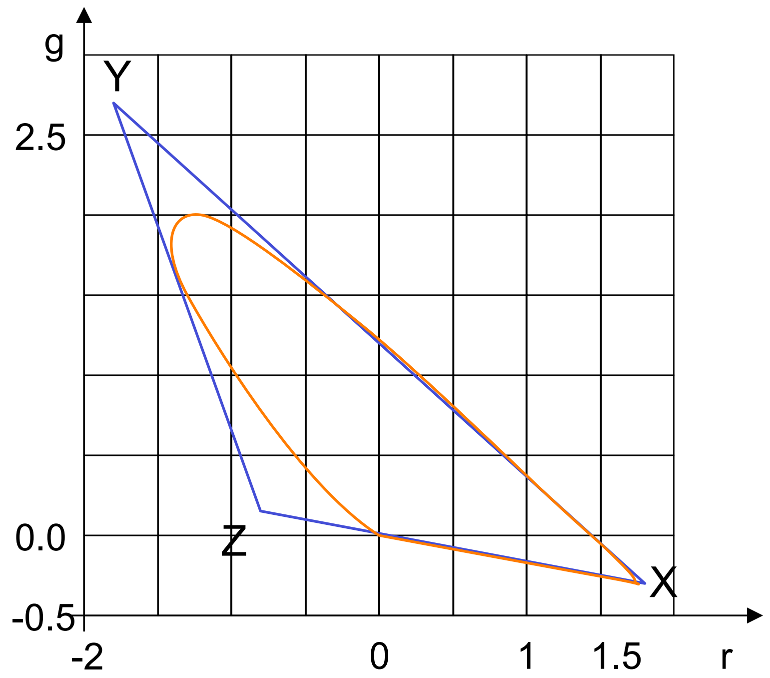
$$\Rightarrow$$

$$(a_{11} + a_{12} + a_{13}) = \\ = (a_{21} + a_{22} + a_{23}) = \\ = (a_{31} + a_{32} + a_{33}) = \sigma$$

[Ref: Color vision and colorimetry, D. Malacara]

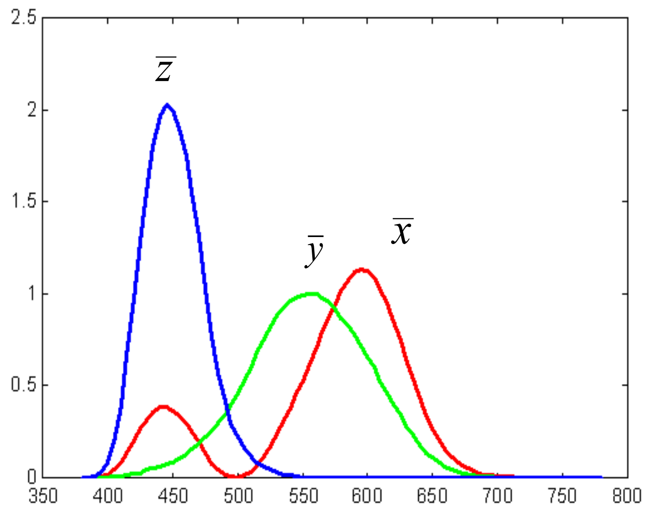
# Guidelines for the derivation of CIE 1931 SO

3. The line joining X and Y be tangent to the curve on the red side
   - In this way, a linear combination of X and Y is sufficient to describe those colors without any Z
   - This introduces other conditions on $a_{31}$, $a_{32}$, $a_{33}$ and sets their values
4. No values of $\bar{x}(\lambda)$ is negative
   - This adds a condition relating a11, a12 and a13, whose sum must be equal to known constant σ. This leaves one degree of freedom that is used to set the area of the XYZ triangle at its minimum



# From rgb to xyz

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} 0.41846 & -0.1586 & -0.08283 \\ -0.09117 & 0.25243 & 0.01571 \\ 0.00092 & -0.00255 & 0.17860 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



# rgb2xyz

- Chromaticity coordinates

$$x = \frac{0.49r + 0.31g + 0.2b}{0.66697r + 1.1324g + 1.20063b}$$

$$y = \frac{0.17697r + 0.81240g + 0.01063b}{0.66697r + 1.1324g + 1.20063b}$$

$$z = \frac{0.0r + 0.01g + 0.99b}{0.66697r + 1.1324g + 1.20063b}$$

- Tristimulus values

$$X = \frac{x}{y}V \quad Y = V \quad Z = \frac{z}{y}V$$

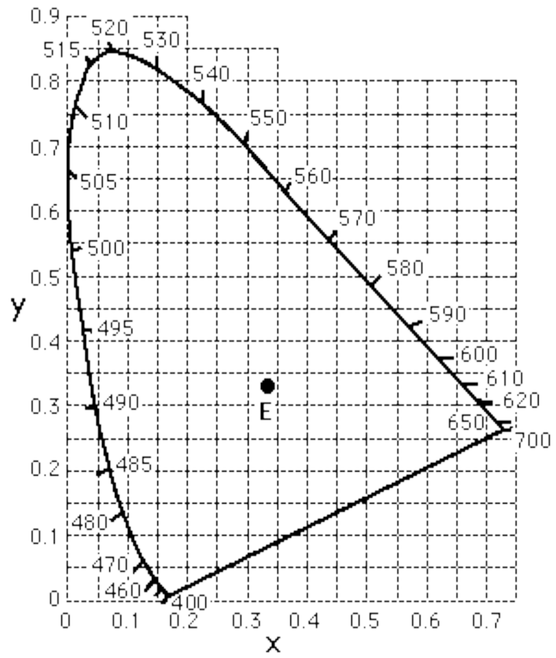
- CMF

$$\bar{x}(\lambda) = \frac{x(\lambda)}{y(\lambda)}V(\lambda)$$

$$\bar{y}(\lambda) = V(\lambda)$$

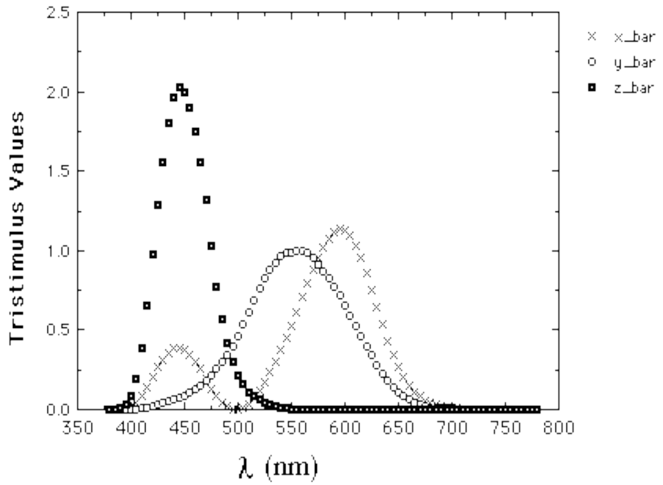
$$\bar{z}(\lambda) = \frac{z(\lambda)}{y(\lambda)}V(\lambda)$$

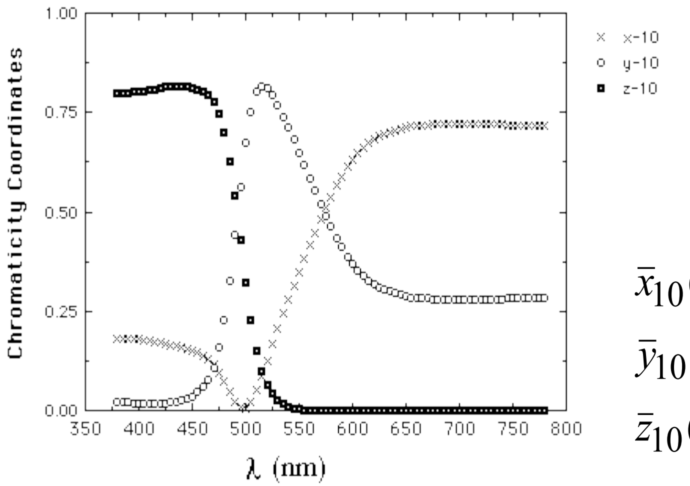
(x,y) chromaticity diagram




$$x_E = y_E = \frac{1}{3}$$

# CIE 1964 SO




Features

- 10 degrees field
- Extended set of wavelengths (390 to 830 nm)
- r,g,b CMFs obtained directly from the observations
  - Measures of the radiant power of each monochromatic test stimulus
- High illumination intensity
  - To minimize rods intrusion
- Data extrapolated at 1nm resolution

$$\bar{x}_{10}(\lambda) = 0.341080\bar{r}_{10}(\lambda) + 0.189145\bar{g}_{10}(\lambda) + 0.387529\bar{b}_{10}(\lambda)$$

$$\bar{y}_{10}(\lambda) = 0.139058\bar{r}_{10}(\lambda) + 0.837460\bar{g}_{10}(\lambda) + 0.073316\bar{b}_{10}(\lambda)$$

$$\bar{z}_{10}(\lambda) = 0.0\bar{r}_{10}(\lambda) + 0.039553\bar{g}_{10}(\lambda) + 2.026200\bar{b}_{10}(\lambda)$$

# CIE Chromaticity Coordinates

- (X,Y,Z) tristimulus values

$$X = \int P_\lambda \bar{x}(\lambda) d\lambda$$

$$Y = \int P_\lambda \bar{y}(\lambda) d\lambda$$

$$Z = \int P_\lambda \bar{z}(\lambda) d\lambda$$

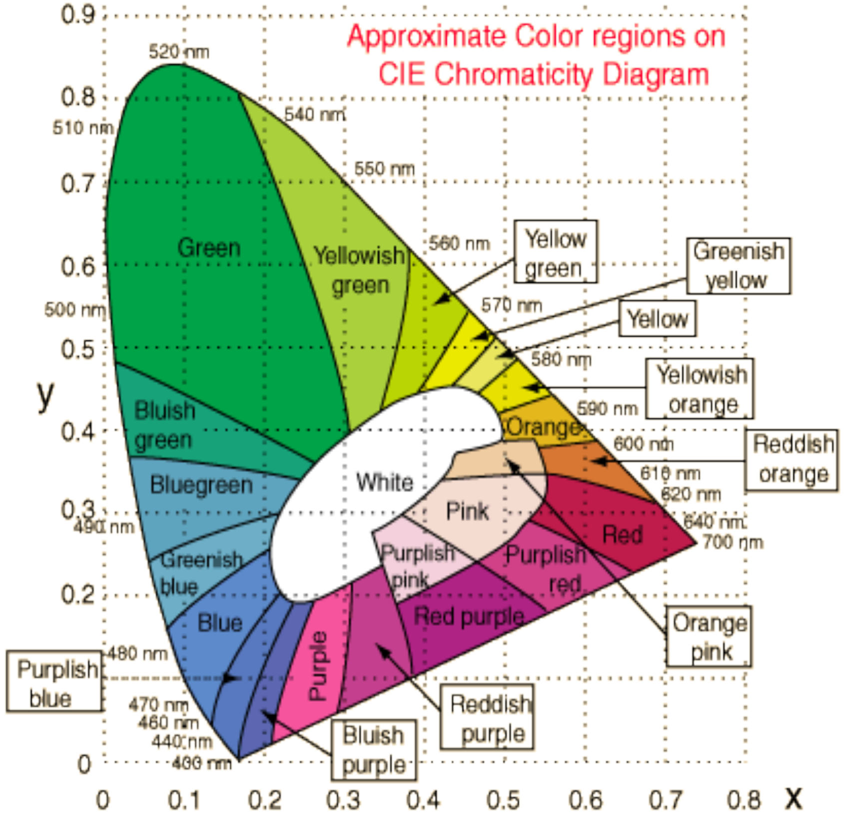
- Chromaticity coordinates

$$x(\lambda) = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

$$y(\lambda) = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

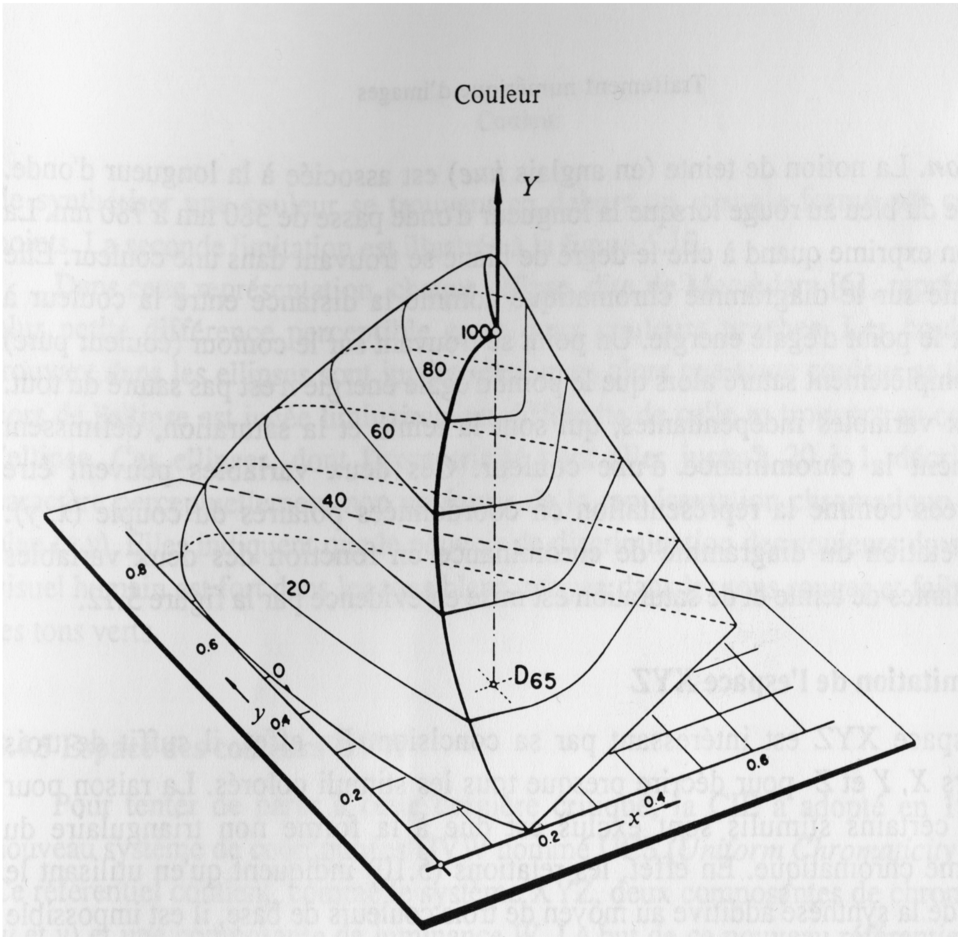
$$z(\lambda) = \frac{\bar{z}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

$$x(\lambda) + y(\lambda) + z(\lambda) = 1$$

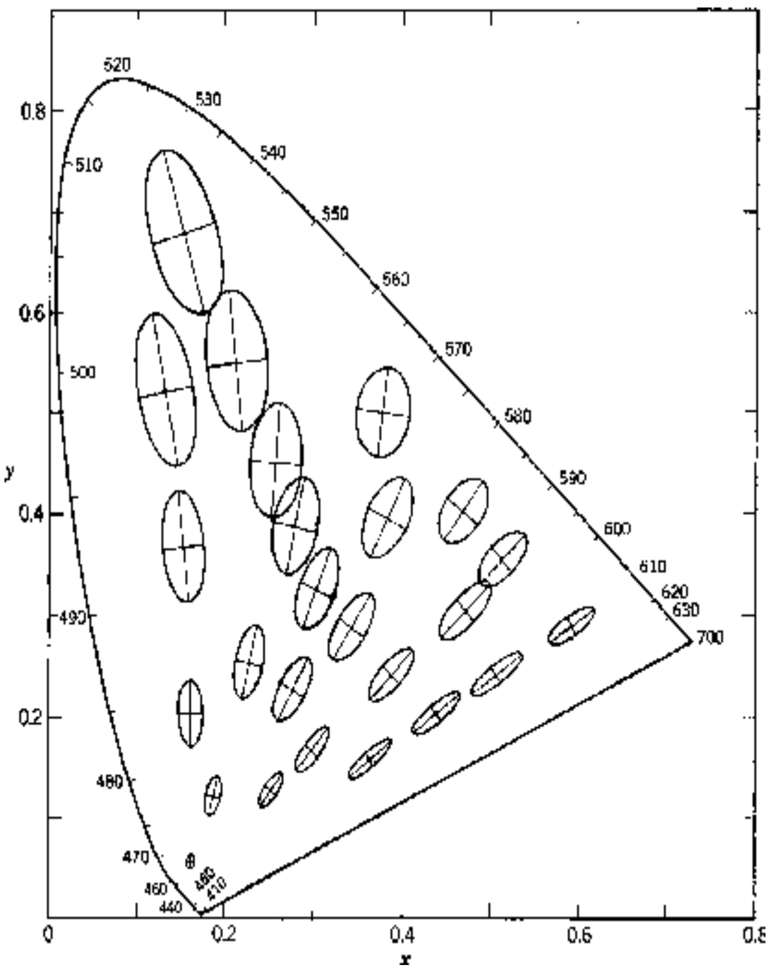



x - y chromaticity diagram

# xyY



# Mac Adams' ellipses




The ellipses represent a **constant perceptual color stimulus**, at a constant luminance, at various positions and in various directions, in the x,y diagram.

The areas of the ellipses vary greatly.

This means that the XYZ colorspace (as the RGB color space) is *not perceptually uniform*.

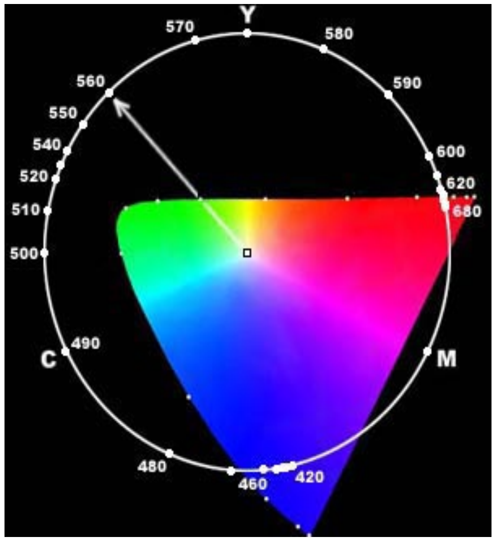
To avoid this nonuniformity, CIE recommended a new CIE 1964 UCS (Uniform-Chromaticity Scale) diagram, to be used with constant luminance levels.

In perceptually uniform colorspaces, the size of the MacAdams' ellipses are more uniform and the eccentricity is lower.

# Perceptually uniform Colorspaces

- CIE 1960 Luv colorspace
  - reversible transformation

$$u = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$
$$v = \frac{6Y}{X + 15Y + 3Z} = \frac{6x}{-2x + 12y + 3}$$




- CIE 1976 L*u*v* (CIELUV)

$$u' = u$$
$$v' = 1.5v$$

L*: *perceived* lightness

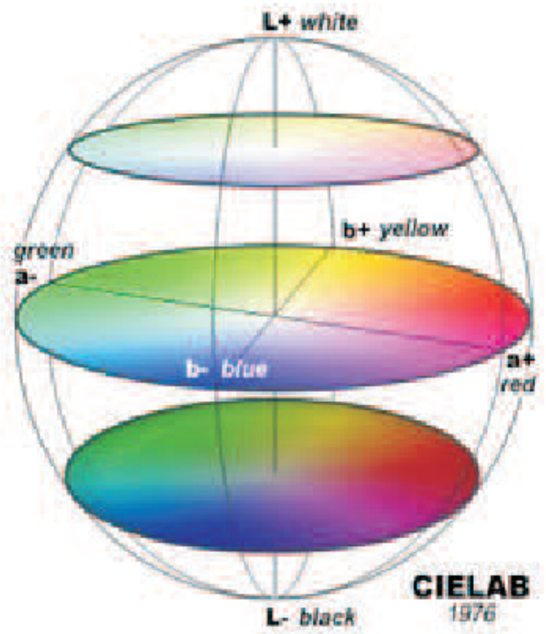
$$L^* = 116\left(\frac{Y}{Y_n}\right)^{1/3} - 16$$
$$u^* = 13L^*(u' - u'_n)$$
$$v^* = 13L^*(v' - v'_n)$$

$$u' = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2 + 12y + 3}$$
$$v' = \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3}$$

$u'_n, v'_n$ : reference white

# Perceptually uniform CM

CIE 1976 L*a*b* (CIELAB)




For: $\frac{Y}{Y_n}, \frac{X}{X_n}, \frac{Z}{Z_n} \geq 0.01$   $X_n$, $Y_n$, $Z_n$ : *reference white*

$$\begin{aligned} L^* &= 116(Y/Y_n)^{1/3} - 16 \\ a^* &= 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}] \\ b^* &= 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}] \end{aligned}$$

otherwise

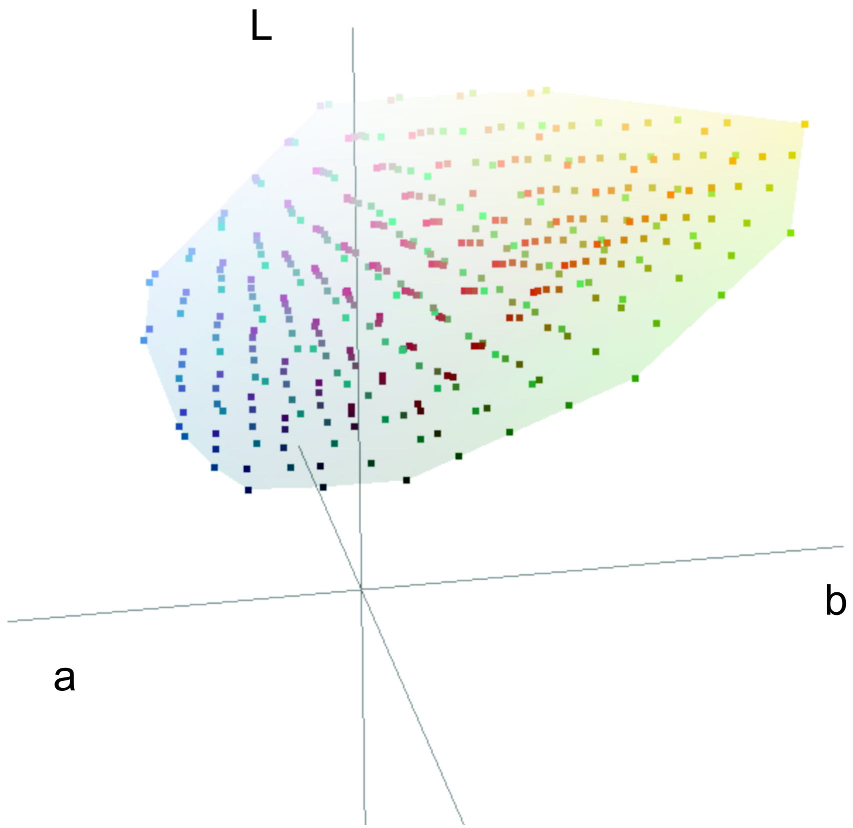
$$\begin{aligned} L^* &= 116\left[f\left(\frac{Y}{Y_n}\right) - \frac{16}{116}\right] \\ a^* &= 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \\ b^* &= 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right] \end{aligned}$$

$$f\left(\frac{Y}{Y_n}\right) = \begin{cases} \left(\frac{Y}{Y_n}\right)^{1/3} & \text{for } \frac{Y}{Y_n} > 0.008856 \\ 7.787\frac{Y}{Y_n} + \frac{16}{116} & \text{for } \frac{Y}{Y_n} \leq 0.008856 \end{cases}$$

Tristimulus values for a *nominally white* object-color stimulus. Usually, it corresponds to the spectral radiance power of one of the CIE standard illuminants (as D65 or A), reflected into the observer's eye by a perfect reflecting diffuser. Under these conditions, $X_n$, $Y_n$, $Z_n$ are the tristimulus values of the standard illuminant with $Y_n$=100.

Hint: the diffused light depends on **both** the **physical properties** of the surface and the **illuminant**

# Color representation in Lab



# Perceptual correlates

- Color difference formula

$$\Delta E^*_{u,v} = \left[ \left( \Delta L^* \right)^2 + \left( \Delta u^* \right)^2 + \left( \Delta v^* \right)^2 \right]^{1/2}$$

- Perceptual correlates

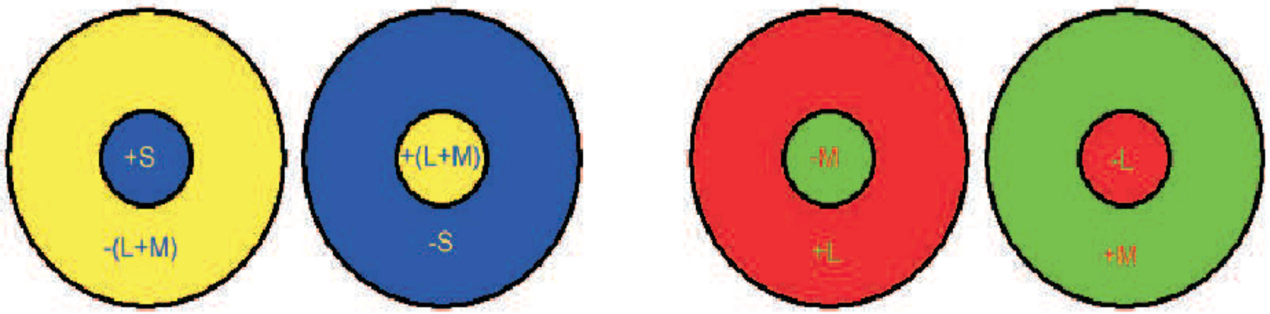
$L^*$: lightness

$C^*_{u,v} = \left[ (u^*)^2 + (v^*)^2 \right]^{1/2}$ : chroma

$s^*_{u,v} = \dfrac{C^*_{u,v}}{L^*}$ : saturation

# Opponent color models

- Underlying model: *opponent channels*

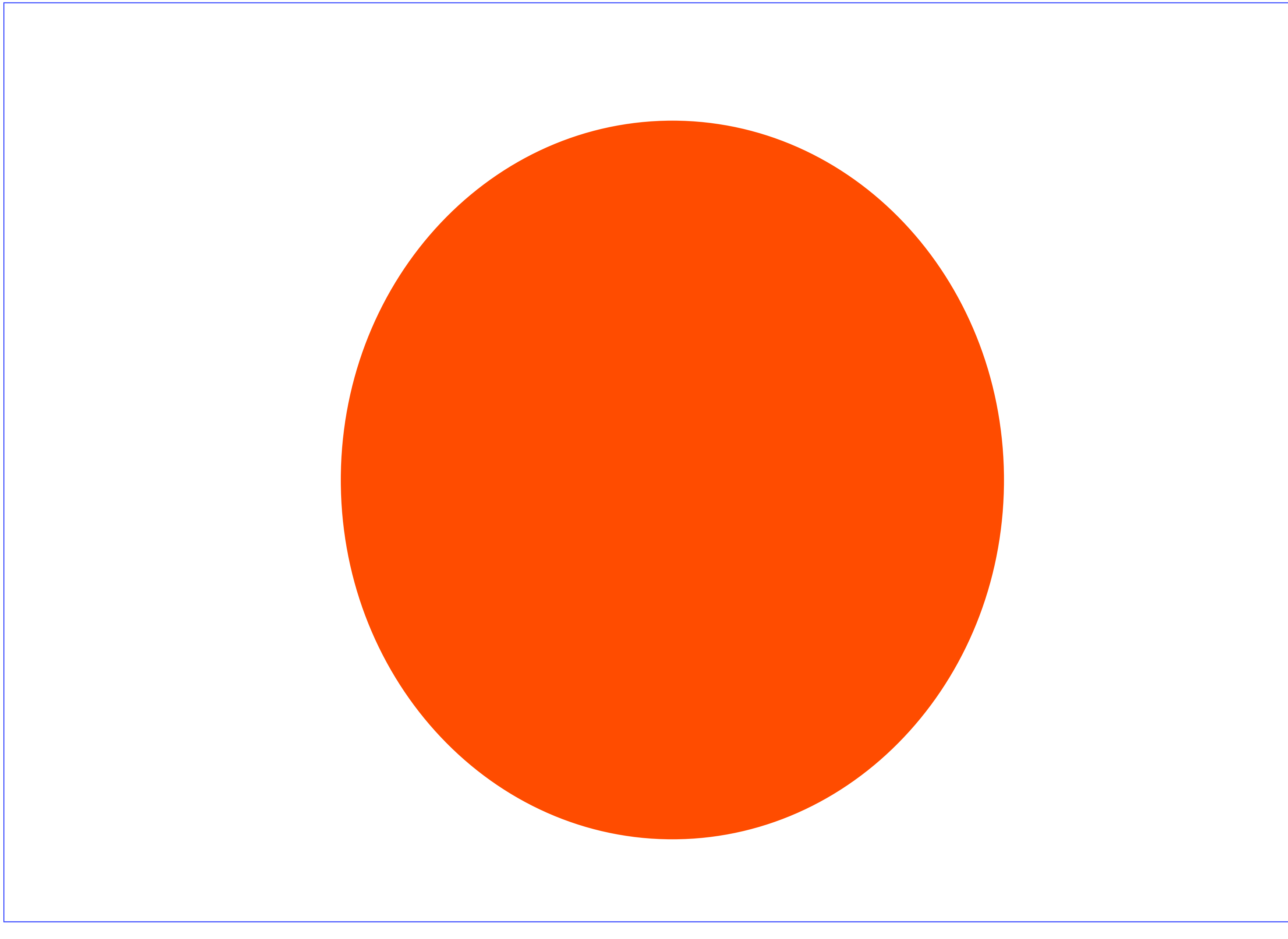



(a) (b)

Example of typical center-surround antagonistic receptive fields: (a) on-center yellow-blue receptive fields; (b) on-center red-green receptive fields.

Because of the fact that the L, M and S cones have different spectral sensitivities, are in different numbers and have different spatial distributions across the retina, the respective receptive fields have quite different properties.

Experimental evidence: color after-image, non existence of colors like "greenish-read" or "yellowish-blue"



# Color images

- Different approaches
  - An edge is present iif there is a gradient in the luminance
  - An edge exists if there is a gradient in any of the tristimulus components
  - “Total gradient” above a predefined threshold

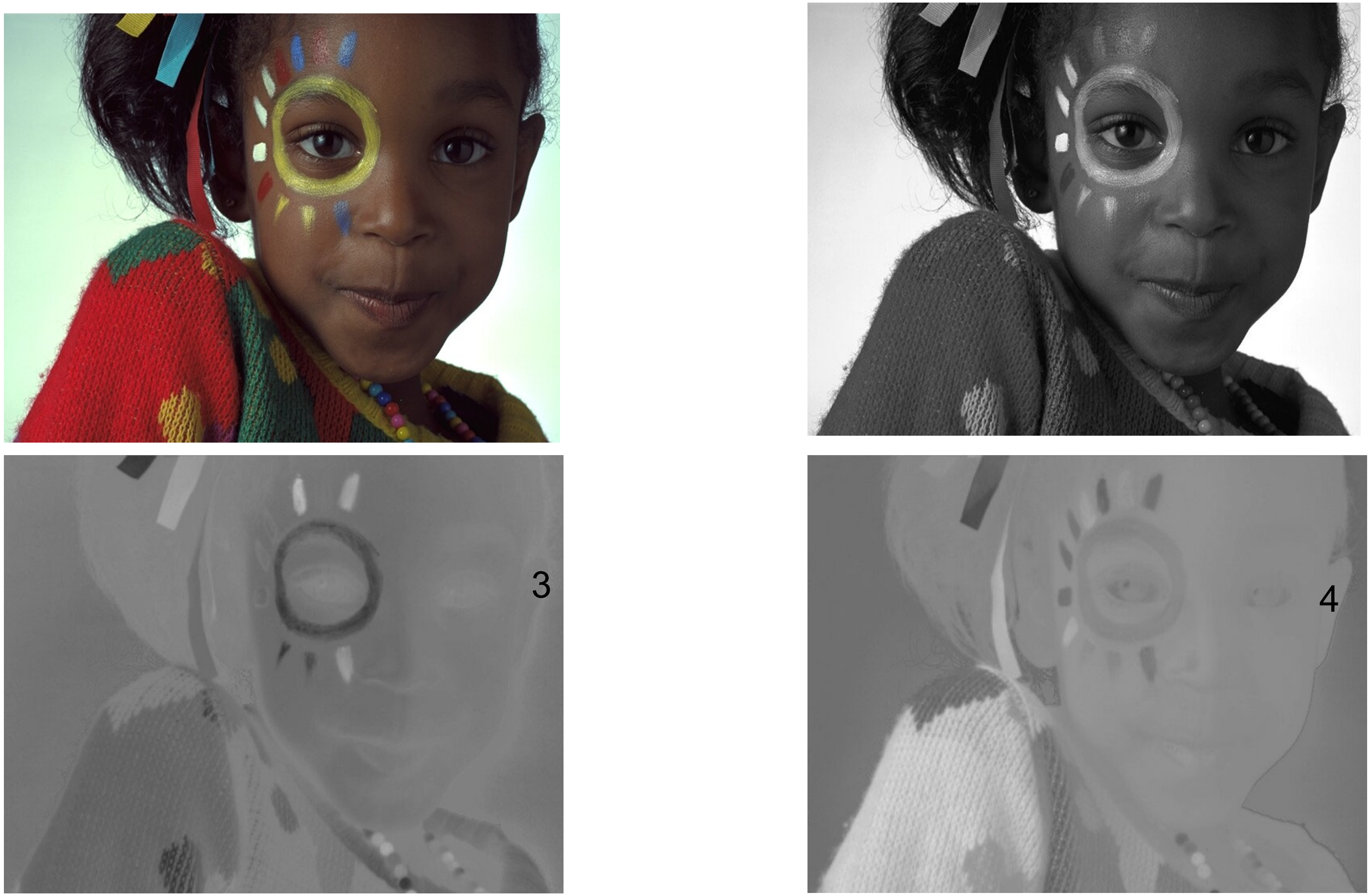
$$G(j,k) = G_1(j,k) + G_2(j,k) + G_3(j,k)$$

  - “Vector sum gradient” above a predefined threshold

$$G(j,k) = \left\{ \left|G_1(j,k)\right|^2 + \left|G_2(j,k)\right|^2 + \left|G_3(j,k)\right|^2 \right\}^{1/2}$$

$G_i(j,k)$ : i-th linear or non-linear tristimulus value

# Opponent Colors

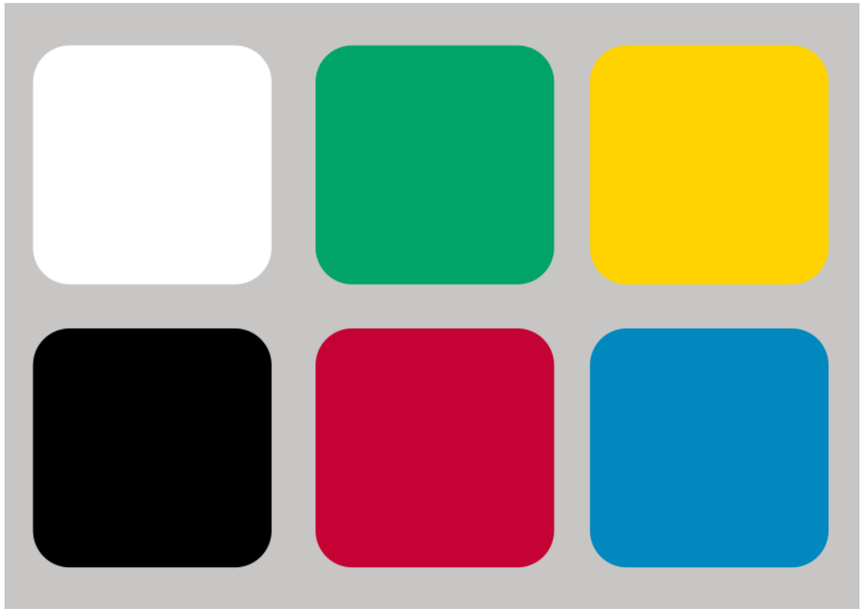



R
G
B


L
a
b

# Opposite channels model

- Encode color images taking human perception into account
- RGB -> luminance + 2 chrominances
- Going from Y’ (physical entity) to Y implies a non linear operation



# YUV Color model

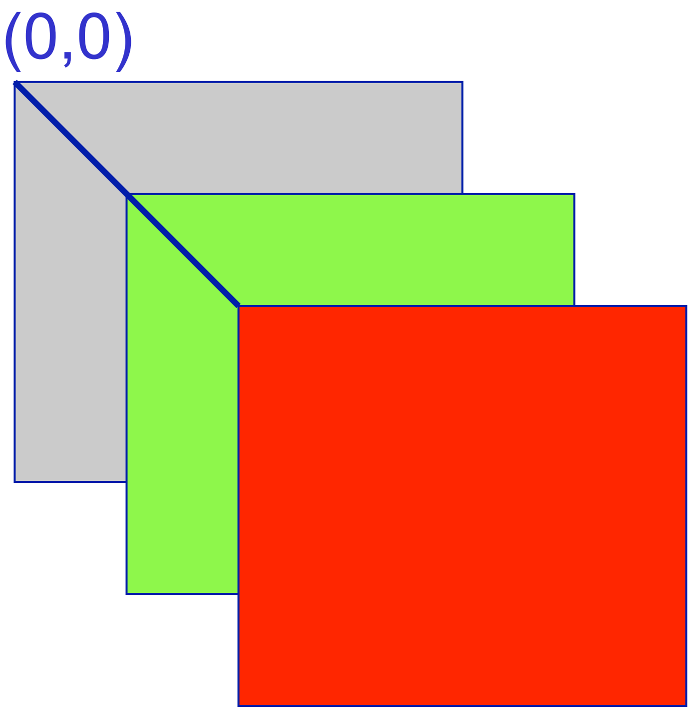
- YUV color model "imitates" human vision.
  - Implementation of the opposed channel model, also called luminance / chrominance color spaces
- Historically, YUV color space was developed to provide compatibility between color and black /white analog television systems.
    - YUV color image information transmitted in the TV signal allowed proper reproducing an image contents at the both types of TV receivers, at the color TV sets as well as at the black / white TV sets.
- PAL TV standard
  - YCbCr similar, used in JPEG and MPEG
    - YCbCr color space is defined in the ITU-R BT.601-5 [1] and ITU-R BT.709-5 [2] standards of ITU (International Telecommunication Union).
  - YIQ (similar) used in NTSC

[1] RECOMMENDATION ITU-R BT.601-5, 1982-1995; [2] RECOMMENDATION ITU-R BT.709-5, 1990-2002.

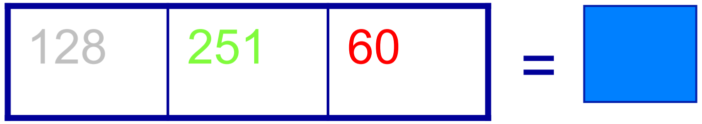
# YUV color model

- Color channels
  - Y: luminance
  - UV (Cb, Cr): chrominance. These are often downsampled exploiting the lowers cutting frequency and sensitivity of the human visual system with respect to the luminance component
- Conversion formulas from/to RGB are available in the literature and implemented in Matlab

# YUV repprentation




A single pixel consists of three components.
Each pixel is a Vector / Array.




Pixel-Vector in the computer memory

Final pixel in the image

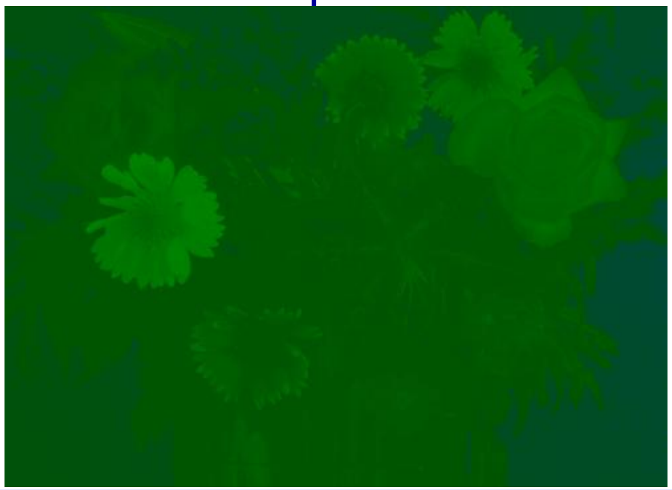
Same Caution as before applies here!

# YUV example

Original Image

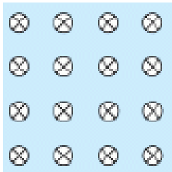
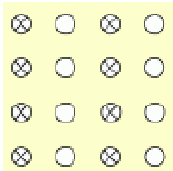
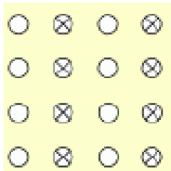
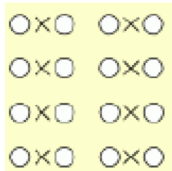
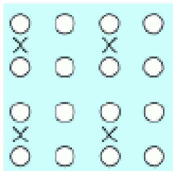
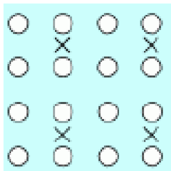
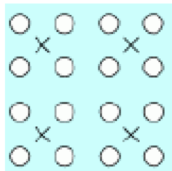
Y-Component

U-Component



V-Component

# YUV possible subsampling patterns

| Sub sampling ratio | Sub sampling pattern | | | | Color component size | | |
|---|---|---|---|---|---|---|---|
| | Uniform | Co site Even | Co site Odd | Centered | Luma Y | Chroma Cb | Chroma Cr |
| 4:4:4 |  | | | | 1 | 1 | 1 |
| 4:2:2 | |  |  |  | 1 | 1/2 | 1/2 |
| 4:2:0 | |  |  |  | 1 | 1/4 | 1/4 |

Designation of used symbols are the following:

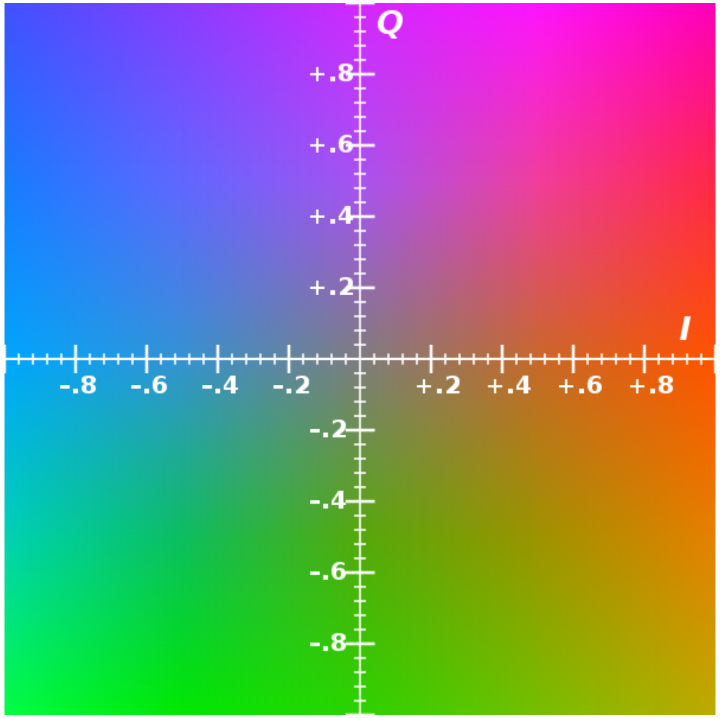
○ - position of luma sample only

× - position of 2 chroma samples only

⊗ - positions of luma and 2 chroma samples are co sited.

# YIQ model

- NTSC (National Television Color System)
- Y is the luminance, meaning that light intensity is nonlinearly encoded based on gamma corrected RGB primaries




The YIQ color space at Y=0.5. Note that the I and Q chroma coordinates are scaled up to 1.0

# YIQ

- Chromaticity is represented by I and Q
  - in phase and in quadrature components
- RGB2YIQ

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.528 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

# Colorimetric color models

- CIE-RGB
- CIE-XYZ
- CIELAB
- CIELUV

# Summary

- References
  - B. Wandell, "Foundations of visions"
  - Wyszecki&Stiles, "Color science, concepts, methods, quantitative data and formulae", Wiley Classic Library
  - D. Malacara, "Color vision and colorimertry, theory and applications", SPIE Press