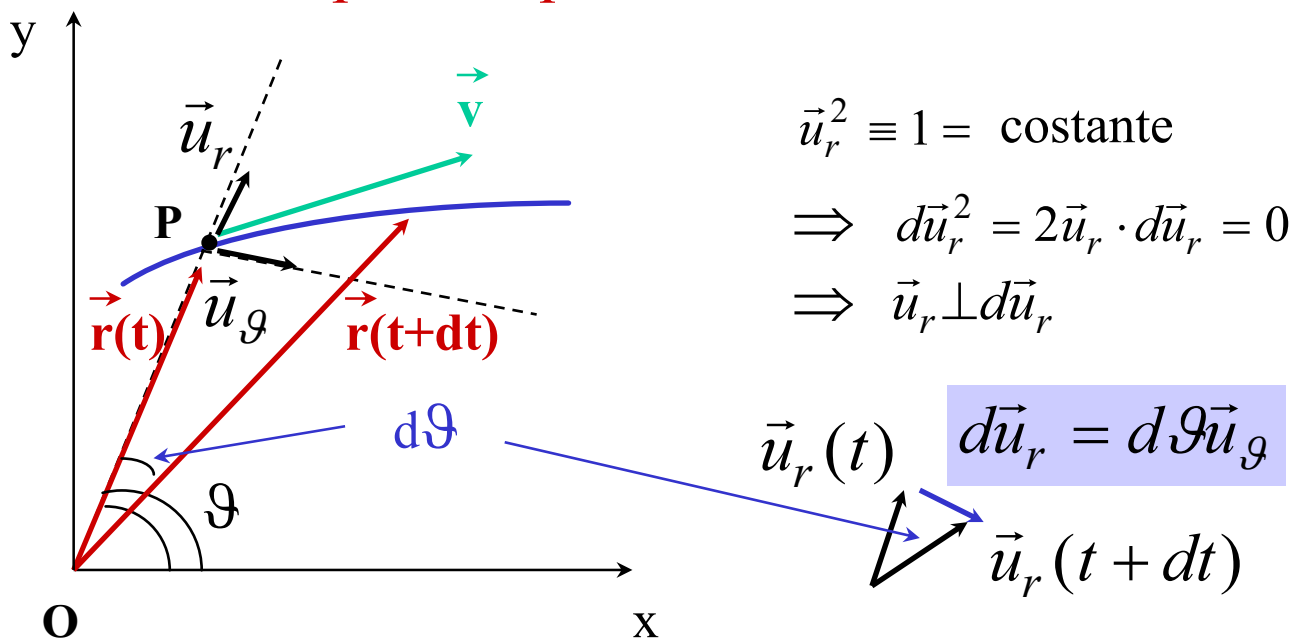


Componenti polari della velocità:



$$\vec{r}(t) = r(t)\vec{u}_r(t) \quad \frac{d}{dt} \vec{u}_r = \frac{d\theta}{dt} \vec{u}_g$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dr(t)}{dt} \vec{u}_r + r(t) \frac{d\vec{u}_r(t)}{dt}$$

$$\Rightarrow \vec{v}(t) = \frac{dr(t)}{dt} \vec{u}_r + r(t) \frac{d\theta(t)}{dt} \vec{u}_g$$



“velocità radiale”

“velocità trasversa”

$$\vec{v}(t) = \left(\frac{dr(t)}{dt}, r(t) \frac{d\theta(t)}{dt} \right) = \left(\frac{dx(t)}{dt}, \frac{dy(t)}{dt} \right)$$

componenti polari

componenti cartesiane

Componenti polari dell'accelerazione

$$\vec{a} \equiv \frac{d\vec{v}(t)}{dt} = \frac{d}{dt} \left(\frac{dr}{dt} \vec{u}_r + r \frac{d\vartheta}{dt} \vec{u}_\vartheta \right) =$$

$$= \frac{d^2 r}{dt^2} \vec{u}_r + \frac{dr}{dt} \frac{d\vec{u}_r}{dt} + \frac{dr}{dt} \frac{d\vartheta}{dt} \vec{u}_\vartheta + r \frac{d^2 \vartheta}{dt^2} \vec{u}_\vartheta + r \frac{d\vartheta}{dt} \frac{d\vec{u}_\vartheta}{dt}$$

$$\begin{array}{c} \vdots \\ \downarrow \\ = \frac{d\vartheta}{dt} \vec{u}_\vartheta \end{array}$$

$$\begin{array}{c} \swarrow \\ = -\frac{d\vartheta}{dt} \vec{u}_r \end{array}$$

$$= 2 \frac{dr}{dt} \frac{d\vartheta}{dt} \vec{u}_\vartheta$$

$$\Rightarrow \boxed{\vec{a} = \left(\frac{d^2 r}{dt^2} - r \left(\frac{d\vartheta}{dt} \right)^2 \right) \vec{u}_r + \left(2 \frac{dr}{dt} \frac{d\vartheta}{dt} + r \frac{d^2 \vartheta}{dt^2} \right) \vec{u}_\vartheta}$$

$$\vec{a}_r$$

$$\vec{a}_\vartheta$$

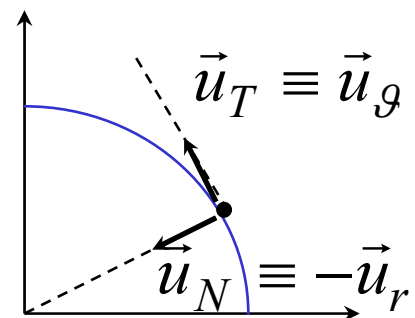
“accelerazione radiale”

“accelerazione trasversa”

In un moto circolare ($r = \text{costante}$) :

$$a_r = -r \left(\frac{d\vartheta}{dt} \right)^2 = -r\omega^2 \equiv -a_N$$

$$a_\vartheta = r \frac{d^2 \vartheta}{dt^2} = r \frac{d\omega}{dt} = r\alpha \equiv a_T$$



Moto circolare uniforme:

coordinata curvilinea

velocità con modulo costante:

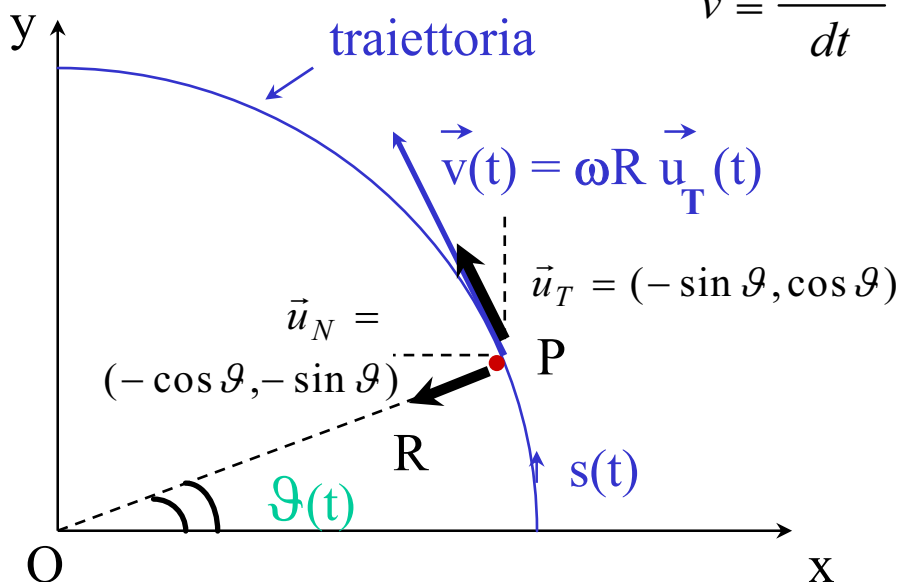
$$s(t) = R\vartheta(t)$$

$$v \equiv \frac{ds(t)}{dt} = R \frac{d\vartheta(t)}{dt} = \omega R$$

“velocità angolare”

$$\omega \equiv \frac{d\vartheta(t)}{dt}$$

$$\vartheta(t) = \vartheta_0 + \omega t$$



$$\begin{aligned} x(t) &= R \cos \vartheta(t) & \Rightarrow & \quad v_x(t) = \frac{dx(t)}{dt} = -R \sin \vartheta(t) \frac{d\vartheta}{dt} \equiv -R\omega \sin \vartheta(t) \\ y(t) &= R \sin \vartheta(t) & \Rightarrow & \quad v_y(t) = \frac{dy(t)}{dt} = R \cos \vartheta(t) \frac{d\vartheta}{dt} \equiv R\omega \cos \vartheta(t) \end{aligned}$$

$$\Rightarrow \quad \vec{v}(t) = (v_x(t), v_y(t)) = R\omega (-\sin \vartheta(t), \cos \vartheta(t))$$

$$\Rightarrow \quad \boxed{\vec{v}(t) = R\omega \vec{u}_T(t) = v \vec{u}_T(t)}$$

\vec{u}_T

$$a_x(t) = \frac{dv_x(t)}{dt} = -R\omega \cos \vartheta(t) \frac{d\vartheta}{dt} = -R\omega^2 \cos \vartheta(t)$$

$$a_y(t) = \frac{dv_y(t)}{dt} = -R\omega \sin \vartheta(t) \frac{d\vartheta}{dt} = -R\omega^2 \sin \vartheta(t)$$

$$\Rightarrow \quad \vec{a}(t) = (a_x(t), a_y(t)) = R\omega^2 (-\cos \vartheta(t), -\sin \vartheta(t))$$

$$\Rightarrow \quad \boxed{\vec{a}(t) = R\omega^2 \vec{u}_N(t) = \frac{v^2}{R} \vec{u}_N(t)}$$

\vec{u}_N