Riassunto

☆ Calcolo del differenziale di

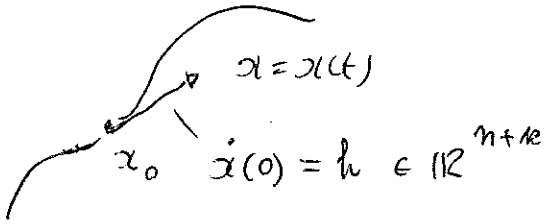
$$f : \mathbb{R}^{n+k} \longrightarrow \mathbb{R}^{k} \qquad \text{(liscia)}$$

in x_0

$$(x_1, \dots x_{n+k}) \longmapsto (y_1 \dots y_k)$$
$$y_i = f_i(x)$$

Sia $\begin{cases} x = x(t) \\ x(0) = x_0 \end{cases}$ $t \in I$ (liscia) Curva in $\mathbb{R}^{n+k}$

0 I



$$F(t) := f(x(t)) \qquad \text{è:}$$

$$\frac{d}{dt} F(t) \Big|_{t=0} = (f_*)_{x_0}(h)$$

der. funz. composte

$$(f_*)_{x_0} h = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_{n+k}} \\ \vdots & & & \vdots \\ \frac{\partial f_k}{\partial x_1} & \frac{\partial f_k}{\partial x_2} & \cdots & \frac{\partial f_k}{\partial x_{n+k}} \end{pmatrix} \begin{pmatrix} h_1 \\ \vdots \\ h_{n+k} \end{pmatrix}$$

matrice jacobiana $\qquad h$

f sommersiva:

$\boxed{f_* \text{ suriettivo}} \iff r(f_*) = \dim \operatorname{Im}(f_*) = r(\quad) = k$

spazio tangente a $f^{-1}(0)$: $\operatorname{Ker} f_*$

"vettori velocità" in x_0 delle curve su $f^{-1}(0)$

$$\dim \operatorname{Ker} f_* = \nu(f_*) = n + k - k \quad (N+R) = n$$

$$n = \dim f^{-1}(0)$$

Come ci si arriva ?

Sia $x = x(t)$ tale che

$$x(t) \in f^{-1}(0) \quad \forall t$$

sicché $F(t) = f(x(t)) \equiv 0 \quad \forall t.$

allora $\dot{x}(0) = h \in \operatorname{Ker} f_* \; : \; (f_*)_{x_0} h = \underset{\mathbb{R}^k}{\underset{\cap}{0}}$