Nov. 8, 2012

II. Exercises Representation Theory

4. Exercise: Let $K = \mathbb{C}$, $\lambda \in \mathbb{C}$ and $n \in \mathbb{N}$. Denote

$$J_n(\lambda) = \begin{pmatrix} \lambda & & & & \\ 1 & \lambda & & & \\ & 1 & \lambda & & \\ & & \ddots & \ddots & \\ & & & 1 & \lambda \end{pmatrix} \in \mathrm{M}_n(\mathbb{C}).$$

For the representation $V = (\mathbb{C}^n, J_n(\lambda))$ of the one-loop quiver compute the endomorphism ring $\mathrm{End}(V)$. Conclude that V is indecomposable, and that it is simple only for $n = 1$.
10 P.

5. Exercise: Let $K = \mathbb{R}$. Show that the representation of the one-loop quiver $V = (\mathbb{R}^2, A)$ with $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \in \mathrm{M}_2(\mathbb{R})$ is simple with $\mathrm{End}(V) \simeq \mathbb{C}$. 10 P.

6. Exercise: Let Q be a quiver with two connectedness components Q' and Q''. That is, Q' and Q'' are quivers such that $Q_0 = Q'_0 \coprod Q''_0$ and $Q_1 = Q'_1 \coprod Q''_1$ (disjoint unions). Show that every indecomposable representation V of Q coincides either with its restriction to Q' or with its restriction to Q''. ("Restriction" defined in the natural way.)
10 P.

Submission of solutions: In the lecture on Nov. 20, 2012.