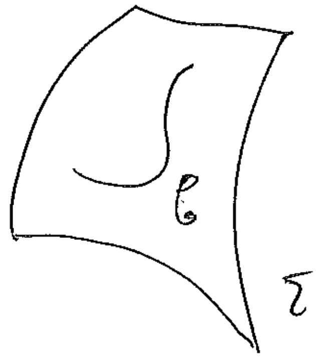
Addendum alla lezione X

Ancora sul trasporto parallelo e la curvatura geodetica



Sia $\mathscr{C}: \; r = r(s)$

calcoliamo $\dfrac{\nabla}{ds}\underset{\underset{\underline{t}}{\parallel}}{\underline{r}'}$ in generale

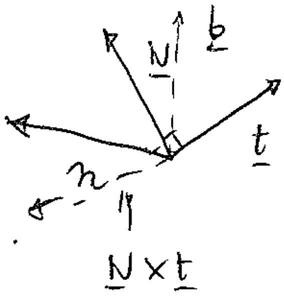
$$\frac{\nabla}{ds}\underline{r}' = \underline{r}'' - \langle \underline{r}'', \underline{N}\rangle \underline{N} = k\,\underline{n} - \langle k\,\underline{n}, N\rangle \underline{N}$$

$$= k\,\underline{n} - k_n \underline{N}$$

curv. normale

piano tangente a Σ in un pto di 𝒞: generato da

$\underline{t}$ e $\underline{N}\times\underline{t}$



$$\frac{\nabla}{ds}\underline{r}' = c.l.(\underline{t}, \underline{N}\times\underline{t})$$

ora $\underline{n} = \langle \underline{N}\times\underline{t}, \underline{n}\rangle \underline{N}\times\underline{t} + \langle \underline{N}, \underline{n}\rangle \underline{N}$

$$k\,\underline{n} = k\langle \underline{N}\times\underline{t}, \underline{n}\rangle \underline{N}\times\underline{t} + k_n \underline{N}$$

$$\Rightarrow \frac{\nabla}{ds}\underline{r}' = k\langle \underline{N}\times\underline{t}, n\rangle \underline{N}\times\underline{t}$$

$$= k\langle \underbrace{\underline{t}\times\underline{n}}_{\underline{b}}, \underline{N}\rangle \underline{N}\times\underline{t}$$

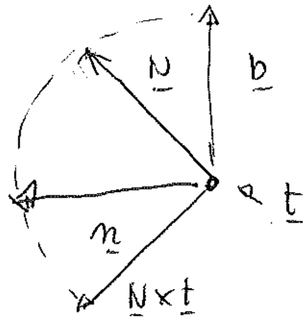
$$= k_g \cdot \underline{N}\times\underline{t}$$

$$\boxed{\frac{\nabla}{ds}\underline{r}' = k_g \cdot \underline{N}\times\underline{t}}$$

(è sul piano tangente ed è $\perp \underline{t}$)

ma è chiaro a priori che $\dfrac{\nabla}{ds}\underline{r}' = \xi\, \underline{N}\times\underline{t}$

Riassumendo:



☆ il piano tangente e il piano osculatore (in un pto) appartengono allo stesso fascio, di asse dato dalla retta tangente a C in P

$$\underline{r}'' = k\,\underline{n} =$$

$$= k \, \underbrace{\langle \underline{n}, \underline{N}\times\underline{t}\rangle}_{\langle \underline{N}, \underbrace{\underline{t}\times\underline{n}}_{\underline{b}}\rangle} \underline{N}\times\underline{t} \;+\; k\langle \underline{n}, \underline{N}\rangle\, \underline{N}$$

$$= \underbrace{k_g \cdot \underline{N}\times\underline{t}}_{\in T_P\Sigma} + \underbrace{k_n \underline{N}}_{\perp T_P\Sigma}$$

$$\Rightarrow \quad k^2 = k_g^2 + k_n^2 \qquad \text{che già conosciamo}$$

e di nuovo $\quad \dfrac{\nabla \underline{r}'}{ds} = P_P\, \underline{r}'' = k_g\, \underline{N}\times\underline{t}$