A example of nonuniqueness.

Fix a real number p with $1 < p < 2$. Let

$$f(t,x) = p|x|^{\frac{p-1}{p}} \quad \text{for } (t,x) \in \mathbf{R} \times \mathbf{R}.$$

Note that f is continuous but that f is *not* regular.

Consider the **initial value problem**

$$\text{(IVP)} \qquad x'(t) = f(t, x(t)),\ t \in \mathbf{R}, \quad x(0) = 0.$$

(So an IVP is an ODE together with an IC.)

Suppose $0 < a < \infty$ and let

$$x_a(t) = \begin{cases} 0 & \text{if } t \le a, \\ (t-a)^p & \text{if } a < t. \end{cases}$$

Then

$$x_a'(t) = \begin{cases} 0 = f(t, x_a(t)) & \text{if } t \le a, \\ p(t-a)^{p-1} = p\left((t-a)^p\right)^{\frac{p-1}{p}} = f(t, x_a(t)) & \text{if } a < t. \end{cases}$$

That is,

$$x_a'(t) = f(t, x_a(t)),\ t \in \mathbf{R} \quad \text{and} \quad x_a(0) = 0.$$