Quivers, algebras and representations
Exercises

Throughout, Q will denote a quiver and $\mathbb{K}$ an algebraically closed field.

Exercise 1. *Let* $Q = \bullet \rightrightarrows \bullet$ *be the Kronecker quiver. Show that there is an isomorphism of algebras*

$$\mathbb{K}Q \cong \begin{pmatrix} \mathbb{K} & 0 \\ \mathbb{K}^2 & \mathbb{K} \end{pmatrix}$$

where we view $\mathbb{K}^2$ *as a* $\mathbb{K}$-$\mathbb{K}$*-bimodule in the obvious way.*

Exercise 2. *Show that, in general, the set* $\{e_i \mid i \in Q_0\}$ *of lazy paths in* Q *is not the unique complete set of primitive orthogonal idempotents for* $\mathbb{K}Q$. *Hint: Consider for example the quiver* $Q = \bullet \longrightarrow \bullet$.

Exercise 3. *A* $\mathbb{K}$*-algebra* A *is called connected if* 0 *and* 1 *are the only idempotents that lie in the center of* A. *Show that the path algebra* $\mathbb{K}Q$ *is connected if and only if the quiver* Q *is connected.*

Exercise 4. *Let* $Q = \alpha \circlearrowleft \bullet \circlearrowright \beta$. *Decide if the following ideals are admissible:*

- $I_1 = \langle \beta^2, \alpha^3, \beta\alpha\beta \rangle$.
- $I_2 = \langle \alpha\beta - \beta\alpha, \beta^2, \alpha^2 \rangle$.

Exercise 5. *Let* $Q = \bullet \circlearrowright$. *Show that* $rad(\mathbb{C}Q) = 0$. *In particular,* $rad(\mathbb{C}Q)$ *is not given by the arrow ideal of* $\mathbb{C}Q$.

Exercise 6. *Give an example of a finite dimensional* $\mathbb{K}$*-algebra* A *with* $rad(A)^{1000} \neq 0$ *and* $rad(A)^{1001} = 0$.

Exercise 7. *Let* $\Phi : \mathbb{K}Q_A \to A$ *be the surjective algebra homomorphism constructed in the proof of the main theorem in §3. Show that* $ker(\Phi) \subseteq R_{Q_A}^2$, *where* R_{Q_A} *denotes the arrow ideal of* $\mathbb{K}Q_A$.

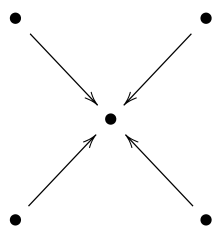
Exercise 8. *Write the following two* $\mathbb{K}$*-algebras as bound path algebras:*

- $A_1 = \begin{pmatrix} \mathbb{K} & 0 & 0 & 0 \\ \mathbb{K} & \mathbb{K} & 0 & 0 \\ 0 & 0 & \mathbb{K} & 0 \\ \mathbb{K}^3 & \mathbb{K}^3 & \mathbb{K} & \mathbb{K} \end{pmatrix}$.

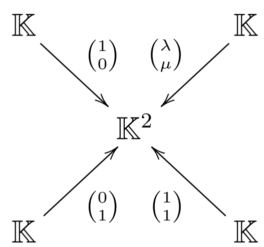
- $A_2 = B/J$ *where* $B = \{\begin{pmatrix} a & 0 & 0 \\ c & b & 0 \\ e & d & a \end{pmatrix} \mid a, b, c, d, e \in \mathbb{K}\}$ *and* J *is given by* $\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e & 0 & 0 \end{pmatrix} \mid e \in \mathbb{K}\}$.

Exercise 9. *Let* $Q = \bullet \longrightarrow \bullet \longrightarrow \bullet$ *. Describe all (finite dimensional) indecomposable representations of* Q *(up to isomorphism) and all possible morphisms between them.*

Exercise 10. *Consider the quiver*

$Q =$ 

Show that there are infinitely many pairwise non-isomorphic indecomposable representations of Q*. Hint: Consider representations of the form*



for $\lambda, \mu \in \mathbb{K}$ *and* $\mu \neq 0$.