ONE DAY – YOUNG RESEARCHER SEMINARS: MATHS, APPLICATIONS & MODELS

Polo Universitario Santa Marta
Università degli Studi di Verona

July 8th, 2022

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ABSTRACT BOOKLET

https://mathseminarsverona.wordpress.com/

OBJECTIVES AND THEMES

Multiagent system is a reference framework to model complex real-world systems describing interacting agents. Examples are ubiquitous in applications arising from economics, social and life sciences.
The techniques used in the analysis range from mathematical analysis, probability, dynamical systems, control theory, game theory, statistical mechanics, and requires refined tools of numerical analysis and simulation.
The aim of the workshop is to gather young scientists working in the broad field of multiagent system study, developing synergies between different point of views and techniques.

ORGANIZERS

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SPONSORS

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UNIVERSITÀ di VERONA
Dipartimento di INFORMATICA

PRIN 2017
Numerical methods for hyperbolic and kinetic equations with applications to engineering and social sciences

VENUE

The workshop will take place at

Polo Universitario Santa Marta
Via Cantarane 24, 37129 Verona

Room: SMT 03

To open the map in Google Maps click on it.

PROGRAM

FRIDAY 8TH, JULY

09:15	**OPENING**
09:25	**PEDRO ACEVES-SANCHEZ** *Emergence of vascular networks*
10:00	**MICHELE ALEANDRI** *Invariant measure of diffusion processes on metric graphs: a combinatorial approach*
10:35	**FABIO CASSINI** *Mean field optimal control problems: an exponential integrators approach*
11:10	COFFEE BREAK
11:40	**RICCARDO MOLINAROLO** *On the wave equation on moving domains: regularity, energy balance and application to dynamic debonding*
12:15	**ELISA IACOMINI** *Filtering methods for coupled inverse problems*
12:50	**LUNCH**
14:30	**GIANMARCO BET** *Detecting anomalies in geometric networks*
15:05	**ELISA CALZOLA** *A second order Lagrange-Galerkin scheme for a class of Fokker-Planck equations and applications to MFGs*
15:40	**FEDERICA FERRARESE** *Collective motion of birds: swarming dynamics with transient leadership*
16:15	COFFEE BREAK
16:45	**DAVIDE MURARI** *Learning Hamiltonians of constrained mechanical systems*
17:20	**CHIARA SEGALA** *Moment-driven predictive control of mean-field collective dynamics*
17:55	**ROSSANA CAPUANI** *Introduction to constrained mean field games equilibria*
18 : 30 18 : 40	**CLOSING**

ABSTRACTS

PEDRO ACEVES-SANCHEZ

University of California, Los Angeles

EMERGENCE OF VASCULAR NETWORKS

The emergence of vascular networks is a long-standing problem that has been the subject of intense research in the past decades. One of the main reasons being the widespread applications that it has in tissue regeneration, wound healing, cancer treatment, etc. The mechanisms involved in the formation of vascular networks are complex and despite the vast amount of research devoted to it, there are still, many mechanisms involved that are poorly understood. Our aim is to bring insight into the study of vascular networks by defining heuristic rules, as simple as possible, and to simulate them numerically to test their relevance in the vascularization process. We will introduce a hybrid agent-based/continuum model coupling blood flow, oxygen flow, capillary network dynamics and tissue dynamics. And we will show a few simulations exhibiting the capability of our model to capture the main features of vascular networks.

MICHELE ALEANDRI

Università degli Studi di Roma "La Sapienza"

INVARIANT MEASURE OF DIFFUSION PROCESSES ON METRIC GRAPHS: A COMBINATORIAL APPROACH

We take an irreducible diffusion process on a metric graph and we give a combinatorial representation of its unique invariant measure. The result is a generalization to a continuous setting of the classical Markov chain tree theorem. In particular, we prove that the invariant measure has an atomic component on the vertices and an absolutely continuous part on the edges. The corresponding density at x can be represented by a normalized superposition of the weights associated to metric arborescences oriented toward the point x, where a metric arborescence is a metric tree oriented towards its root. The weight of each oriented metric arborescence is the product of three factors: the exponential of integrals of the form $\int \frac{b_e}{\sigma_e}$, where b_e is the drift and σ_e is the diffusion coefficient, along the oriented edge e; a coefficient for each node determined by the local orientation of the arborescence around the node; the inverse of the diffusion coefficient at x. The metric arborescences are obtained by cutting the original metric graph along some edges.
This is a joint work with M. Colangeli and D. Gabrielli from University of L'Aquila.

GIANMARCO BET

Università degli Studi di Firenze

DETECTING ANOMALIES IN GEOMETRIC NETWORKS

Recently there has been an increasing interest in the development of statistical techniques and algorithms that exploit the structure of large complex-network data to analyze networks more efficiently. For this talk, I will focus on detection problems. In this context, the goal is to detect the presence of some sort of anomaly in the network, and possibly even identify the nodes/edges responsible. Our work is inspired by the problem of detecting so-called botnets. Examples are fake user profiles in a social network or servers infected by a computer virus on the internet. Typically a botnet represents a potentially malicious anomaly in the network, and thus it is of great practical interest to detect its presence and, when detected, to identify the corresponding vertices. Accordingly, numerous empirical studies have analyzed botnet detection problems and techniques. However, theoretical models and algorithmic guarantees are missing so far. We introduce a simplified model for a botnet, and approach the detection problem from a statistical perspective. More precisely, under the null hypothesis we model the network as a sample from a geometric random graph, whereas under the alternative hypothesis there are a few botnet vertices that ignore the underlying geometry and simply connect to other vertices in an independent fashion. We present two statistical tests to detect the presence of these botnets, and we show that they are asymptotically powerful, i.e., they correctly distinguish the null and the alternative with probability tending to one as the number of vertices increases. We also propose a method to identify the botnet vertices. We will argue, using numerical simulations, that our tests perform well for finite networks, even when the underlying graph model is slightly perturbed. Our work is not limited in scope to botnet detection, and in fact is relevant whenever the nature of the anomaly to be detected is a change in the underlying connection criteria.
Based on joint work with Kay Bogerd (TU/e), Rui Pires da Silva Castro (TU/e) and Remco van der Hofstad (TU/e).

ELISA CALZOLA

Università degli Studi di Verona

A SECOND ORDER LAGRANGE-GALERKIN SCHEME FOR A CLASS OF FOKKER-PLANCK EQUATIONS AND APPLICATIONS TO MFGs

(Joint work with Elisabetta Carlini and Francisco J. Silva)

We propose a second order Lagrange-Galerkin method to approximate the solution of a class of linear Fokker-Planck (FP) equations with constant positive diffusion. The scheme is constructed starting from the representation formula for the FP equation. Firstly, we discretize the solution of the stochastic differential equation for the characteristic curves using the Crank-Nicolson method, then we consider symmetric Lagrange interpolation basis functions of odd degree and a suitable quadrature method for the approximation of the integrals in the representation formula. We have proved mass conservation, consistency, L2-stability, and convergence of the exactly-integrated scheme. Finally, we couple this scheme with a second order semi-Lagrangian discretization of the Hamilton-Jacobi-Bellman equation, in order to obtain an approximation method for the solution of Mean Field Games (MFGs). In the end, we conclude our work with some numerical simulations, both in one and two spatial dimensions.

ROSSANA CAPUANI

Università degli Studi della Tuscia

INTRODUCTION TO CONSTRAINED MEAN FIELD GAMES EQUILIBRIA

This talk will address deterministic mean field games for which agents are restricted in a closed domain with smooth boundary. In this case, the existence and uniqueness of Nash equilibria cannot be deduced as for unrestricted state space because, for a large set of initial conditions, the uniqueness of solutions to the minimization problem which is solved by each agent is no longer guaranteed. Therefore we attack the problem by considering a relaxed version of it, for which the existence of equilibria can be proved by set-valued fixed point arguments. By analyzing the regularity and sensitivity with respect to space variables of the relaxed solution, we will show that it satisfies the MFG system in a suitable point-wise sense.

FABIO CASSINI

Università degli Studi di Trento

MEAN FIELD OPTIMAL CONTROL PROBLEMS: AN EXPONENTIAL INTEGRATORS APPROACH

In the last years, many phenomena with applications ranging from biology to social and economical sciences have been succesfully modeled as mean field optimal control problems. In the context of pedestrian dynamics, we refer for instance to [3], while in the context of opinion formation we mention for example [2]. In this talk, we present a model of mean field optimal control type which, for suitable choices of the involved quantities and parameters, takes into account both of the just mentioned dynamics. After deriving the corresponding first order optimality conditions, we end up with a coupled system of forward/backward time-dependent Partial Differential Equations, characterised (in general) by the presence of nonlocal interaction terms. Due to the intrinsic structure of the system, its numerical solution is challenging. After a brief introduction to exponential integrators [4], we proceed by presenting how, in this context, we can employ these schemes in order to efficiently compute the desired approximated solution. We conclude the talk by illustrating extensive numerical experiments, for both opinion and pedestrian dynamics problems, which highlight the effectiveness of the approach.
Joint work with Giacomo Albi (University of Verona), Marco Caliari (University of Verona), and Elisa Calzola (University of Verona).

References

[1] G. Albi, M. Caliari, E. Calzola and F. Cassini, *Mean field optimal control problems: an exponential integrators approach*, In preparation, 2022.

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FEDERICA FERRARESE

Università degli Studi di Trento

COLLECTIVE MOTION OF BIRDS: SWARMING DYNAMICS WITH TRANSIENT LEADERSHIP

The study of the collective and synchronized behaviour of animals, like bird flocks, fish schools and insect swarms, plays a central role in Mathematical Biology. Here the focus is on a model that describes the collective motion of birds in which spontaneous sudden changes of direction happen without the influence of predators. The main idea is that each bird can be a turn initiator becoming a leader whose influence acts on its nearest neighbours that are supposed to be in the followers status. Once that an agent becomes a leader it initializes a change of direction which is propagated along the whole flock. However, the leaders influence is assumed to be limited in time. Indeed, the interest is on the dynamics of *switching leaders* or *transient leadership*: each agent can change its label in time from leader to follower and vice-versa. The model can also include food sources which are visible only by the agents in the leaders status. Starting from the microscopic model we derive a kinetic description of the agents distribution which combines an update of the positions and velocities based on binary interactions rules with a dynamic change of labels between the followers and leaders status. We show how to solve the problem numerically with a Monte Carlo algorithm to simulate the labels evolution and a Nanbu algorithm to simulate the interactions. To approximate the topological ball, we substitute the classical exhaustive search with a k-nearest neighbour search in order to reduce the computational cost from quadratic to logarithmic. We conclude by presenting different numerical tests to show how the dynamics evolves at both the microscopic and kinetic level.

ELISA IACOMINI

RWTH Aachen University

FILTERING METHODS FOR COUPLED INVERSE PROBLEMS

In many applications is required to determine a parameters set which is suitable for competing models. To this end, we are interested in ensemble methods to solve multi-objective optimization problems. In this talk, the ensemble Kalman Filter method is extended and adapted in order to solve coupled inverse nonlinear problems using a weighted function approach. An analysis of the mean field limit of the ensemble method yields an explicit update formula for the weights. Numerical examples show the improved performance of the proposed method.

RICCARDO MOLINAROLO

Università degli Studi di Firenze

ON THE WAVE EQUATION ON MOVING DOMAINS: REGULARITY, ENERGY BALANCE AND APPLICATION TO DYNAMIC DEBONDING

In this talk we present a notion of weak solution for the wave equation on a time-dependent domain with homogeneous Dirichlet boundary value data and standard initial conditions. Under regularity assumptions of the time evolution of the space domain, existence can be proven via penalization argument (cf. [4]) or via time discretization and approximation argument by cylindrical domains (cf. [1]).
Moreover, in the case of a $C^{1,1}$-evolution, by means of suitable diffeomorphisms, the problem can be recast into an hyperbolic equation on a fixed domain: we prove the equivalence of the notions of weak solutions of the two problems.
Finally, this equivalence allows us to obtain an energy balance, following a double regularization argument in the spirit of [2], and to discuss, by means of a standard Galerkin method, better regularity properties depending on the evolution of the domain and the initial data.
As an application, we give a rigorous definition of dynamic energy release rate density for some problems of debonding, and we formulate a proper notion of solutions for such problems.
This talk is based on a joint work with G. Lazzaroni, F. Riva, and F. Solombrino (cf. [3]).

References

[1] J. Calvo, M. Novaga and G. Orlandi, *Parabolic equations in time-dependent domains*, J. Evol. Equ. 17, 781–804, 2017.

[2] G. Dal Maso and I. Lucardesi, *The wave equation on domains with cracks growing on a prescribed path: existence, uniqueness, and continuous dependence on the data*, Appl. Math. Res. Express. AMRX 1, 184–241, 2017.

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DAVIDE MURARI

NTNU Norwegian University of Science and Technology

LEARNING HAMILTONIANS OF CONSTRAINED MECHANICAL SYSTEMS

Recently, there has been an increasing interest in the modelling and computation of physical systems with neural networks. Hamiltonian systems are an elegant and compact formalism in classical mechanics, where the dynamics is entirely determined by one scalar function, the Hamiltonian. The solution trajectories of Hamiltonian systems are often constrained to evolve on a submanifold of a linear vector space. Thus, after a brief overview of the unconstrained case, we derive a strategy to approximate constrained Hamiltonian vector fields with neural networks. In particular, we show how numerical methods are involved in this approximation task, focusing on the importance of preserving the constrained space of the system during the learning procedure.

CHIARA SEGALA

RWTH Aachen University

MOMENT-DRIVEN PREDICTIVE CONTROL OF MEAN-FIELD COLLECTIVE DYNAMICS

The synthesis of control laws for interacting agent-based dynamics and their mean-field limit is studied. A linearization-based approach is used for the computation of sub-optimal feedback laws obtained from the solution of differential matrix Riccati equations. Quantification of dynamic performance of such control laws leads to theoretical estimates on suitable linearization points of the nonlinear dynamics. Subsequently, the feedback laws are embedded into nonlinear model predictive control framework where the control is updated adaptively in time according to dynamic information on moments of linear mean-field dynamics. The performance and robustness of the proposed methodology is assessed through different numerical experiments in collective dynamics.