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Title of the Talk: Path dependent partial differential equation with applications in Mathematical Finance

Abstract: We present a probabilistic method in order to represent the viscosity solution of the path dependent PDE:

$$\begin{cases} \partial_t u(t,\phi) + \mathcal{L}u(t,\phi) + f(t,\phi,u(t,\phi),\partial_x u\,(t,\phi)\,\sigma(t,\phi),(u(\cdot,\phi))_t), \\ \hfill t\in[0,T),\ \phi\in\mathbb{A}, \\ u(T,\phi)=h(\phi),\ \phi\in\mathbb{A}, \end{cases}$$

where $\mathbb{A} := \mathcal{C}([0,T];\mathbb{R}^d)$, $(u(\cdot,\phi))_t := (u(s,\phi))_{s\in[t-\delta,t]}$ and operator $\mathcal{L}v(t,\phi) := \frac{1}{2}\mathrm{Tr}\big[\sigma(t,\phi)\sigma^*(t,\phi)\partial^2_{xx}v(t,\phi)\big] + \langle b(t,\phi),\partial_x v(t,\phi)\rangle$, for $v\in\mathcal{C}^{0,2}([0,T]\times\mathbb{A};\mathbb{R}^d)$.

The viscosity solution is a function $u:[0,T]\times\mathbb{A}\to\mathbb{R}$ defined by $u\,(t,\phi) := Y^{t,\phi}\,(t)$, where $Y^{t,\phi}$ is the unique solution of the backward stochastic differential equation with time–delayed generators:

$$\begin{cases} -dY^{t,\phi}\,(s) = F\big(s,Y^{t,\phi}\,(s)\,,Z^{t,\phi}\,(s)\,,Y^{t,\phi}_s\big)ds - Z^{t,\phi}\,(s)\,dW\,(s)\,,\ t\le s\le T, \\ Y^{t,\phi}\,(T) = g\big(X^{t,\phi}_{(T)}\big), \end{cases}$$

with $Y^{t,\phi}_s := (Y^{t,\phi}(r))_{r\in[s-\delta,s]}$ and $X^{t,\phi}_{(T)} := \big(X^{t,\phi}\,(r)\big)_{r\in[0,T]}$ (is the solution of a forward delay SDE).

Applications to the mathematical finance are also envisaged.