Sampling

From continuous to discrete time

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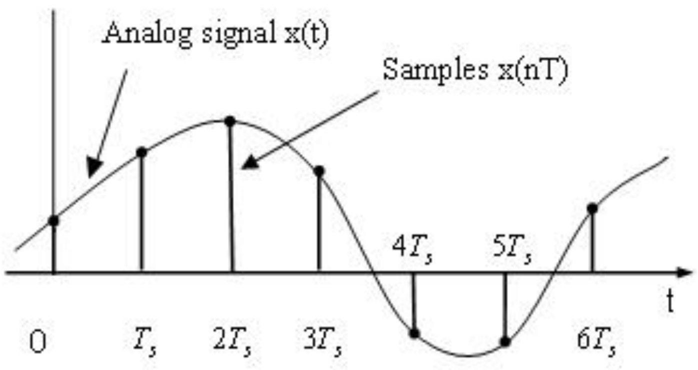
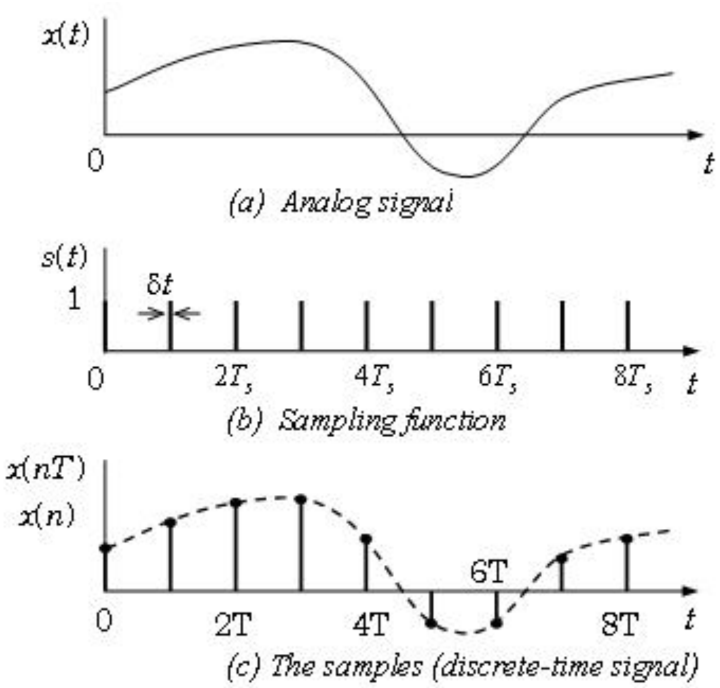
From continuous to discrete time

- Sampling is the process that consists in taking a continuous time signal and transforming it into a discrete time signal



How to obtain it?

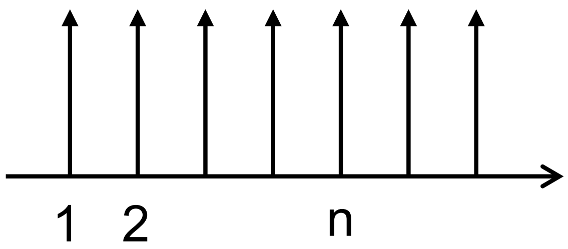
- The main actors are
  - The impulse train
  - The convolution operator



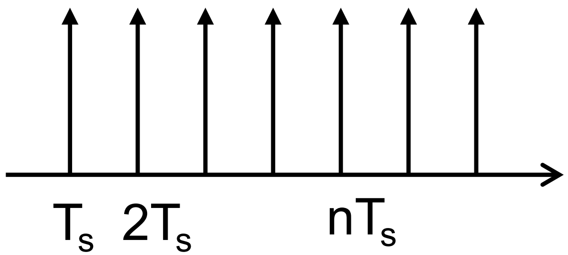
The sampled signal can be seen as the product of the CT signal times the impulse train

What else do we know?

- The product in signal domain corresponds to the convolution in Fourier domain
- The DTFT of a discrete time signals is periodic of period 2π
- So far, we have considered sample indices that were integer numbers which amounts to assume unitary spacing among samples



while for sampled signals the spacing among samples is T_s



Stratching (T_s>1) or shrinking (T_s<1) of the time axis

What do we need

- F-transform of the impulse train but we know that
  - The impulse train is a periodic function of period T_s

$$S_{T_s}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

  - Thus its transform is

$$S_{T_s}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \rightarrow S_{T_s}(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \delta(\omega - n\omega_s)$$

  - Thus it is a scaled version of the train of delta where the spacing between deltas is $1/T_s$

Sampling

- Exploiting the convolution theore

$$f_s(t) = f(nT_s) = f(t)S_{T_s}(t) = \sum_n f(nT_s)\delta(t - nT_s)$$

$$F_s(\omega) = f(\omega) \star S_{T_s}(\omega) = \frac{1}{T_s}\sum_n F(\omega - n\omega_s)$$

Sampling periodizes the spectrum

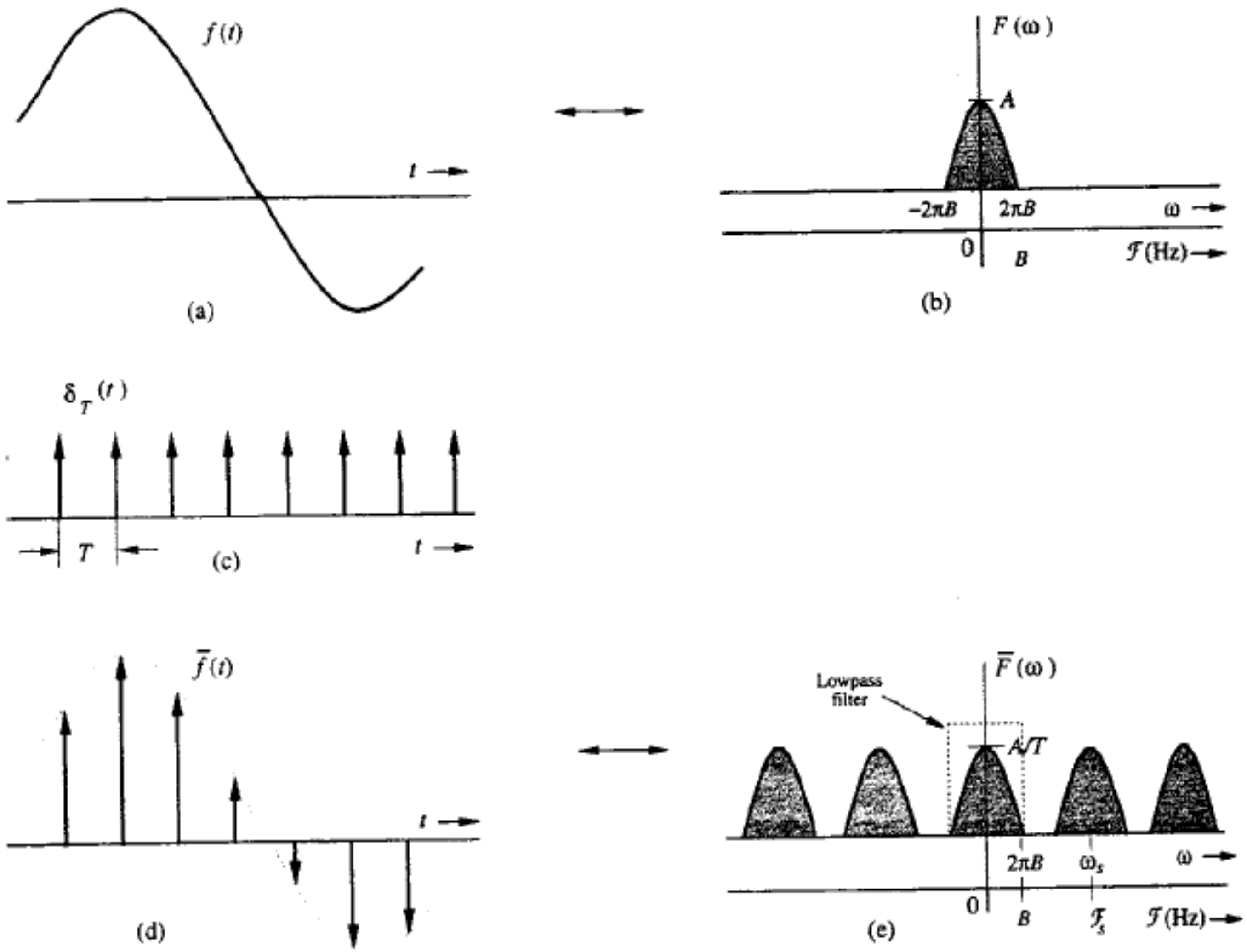


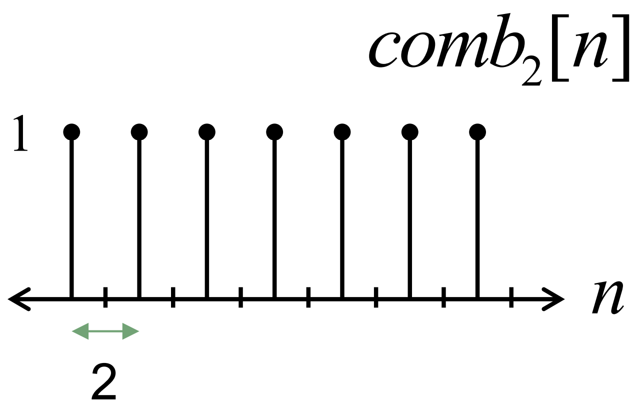
Fig. 5.1 Sampled signal and its Fourier spectrum.

Discrete impulse train

- Define a *comb* function (impulse train) as follows

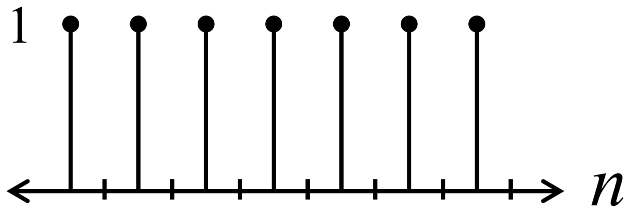
$$comb_N[n] = \sum_{l=-\infty}^{\infty} \delta[n - lN]$$

where *M* and *N* are integers



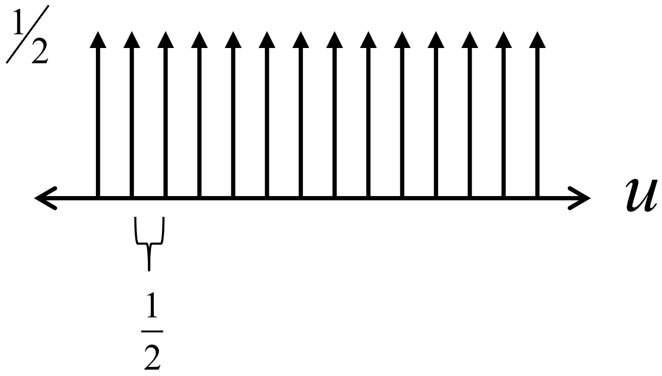
Impulse Train

$comb_2[n]$



$\Longleftrightarrow$

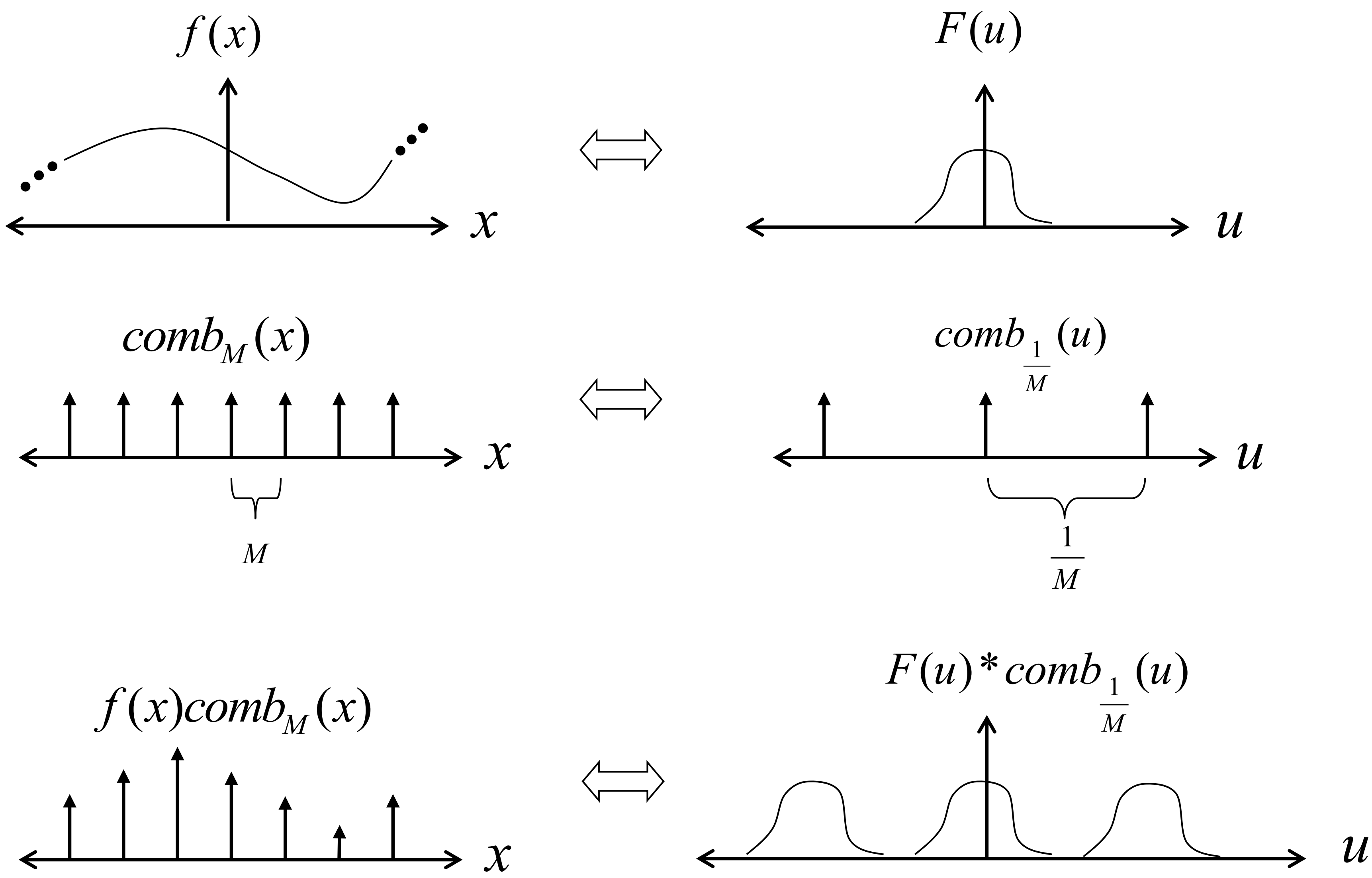
$\frac{1}{2} comb_{\frac{1}{2}}(u)$



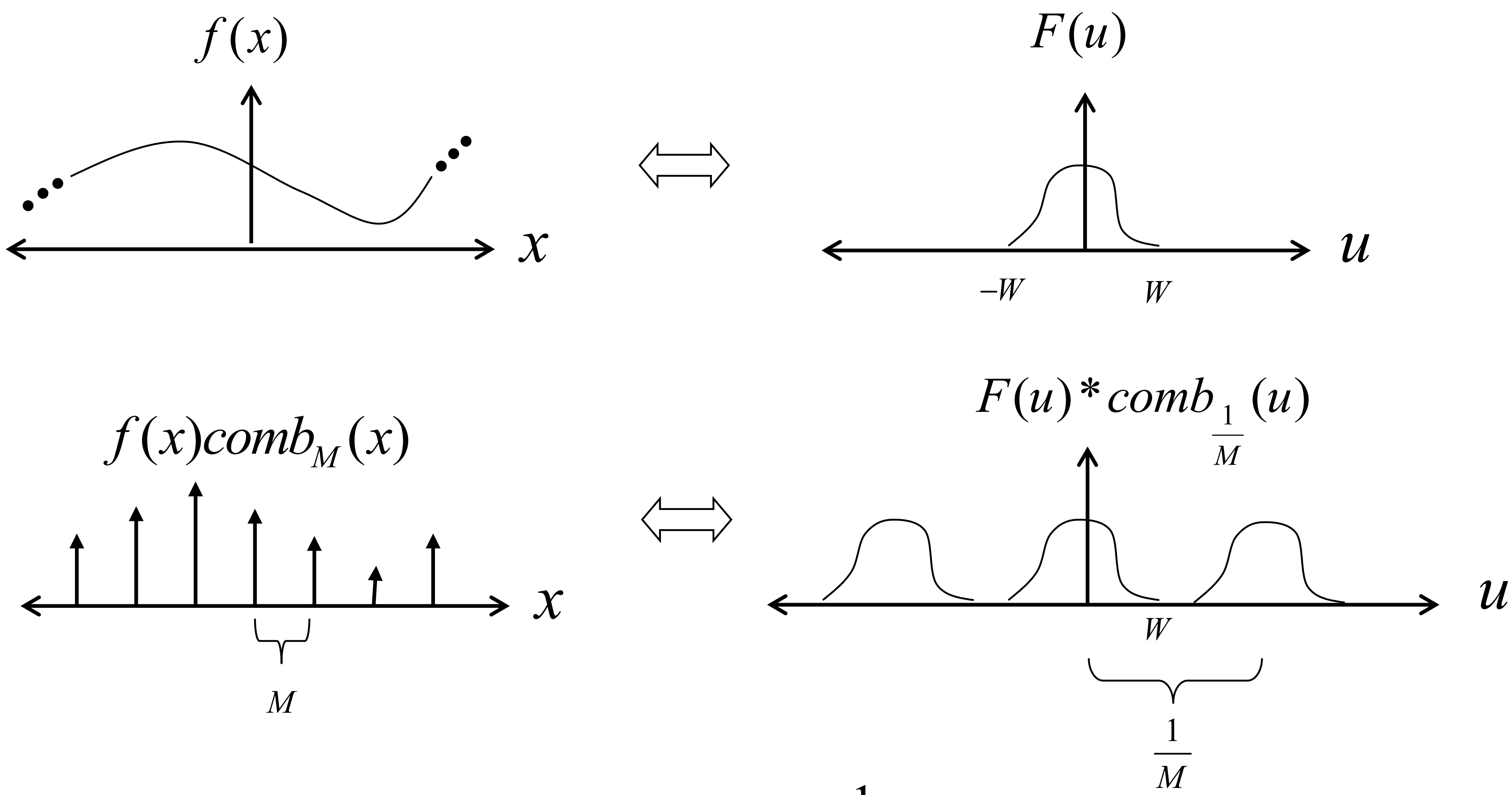
Consequences

Sampling (Nyquist) theorem

Sampling

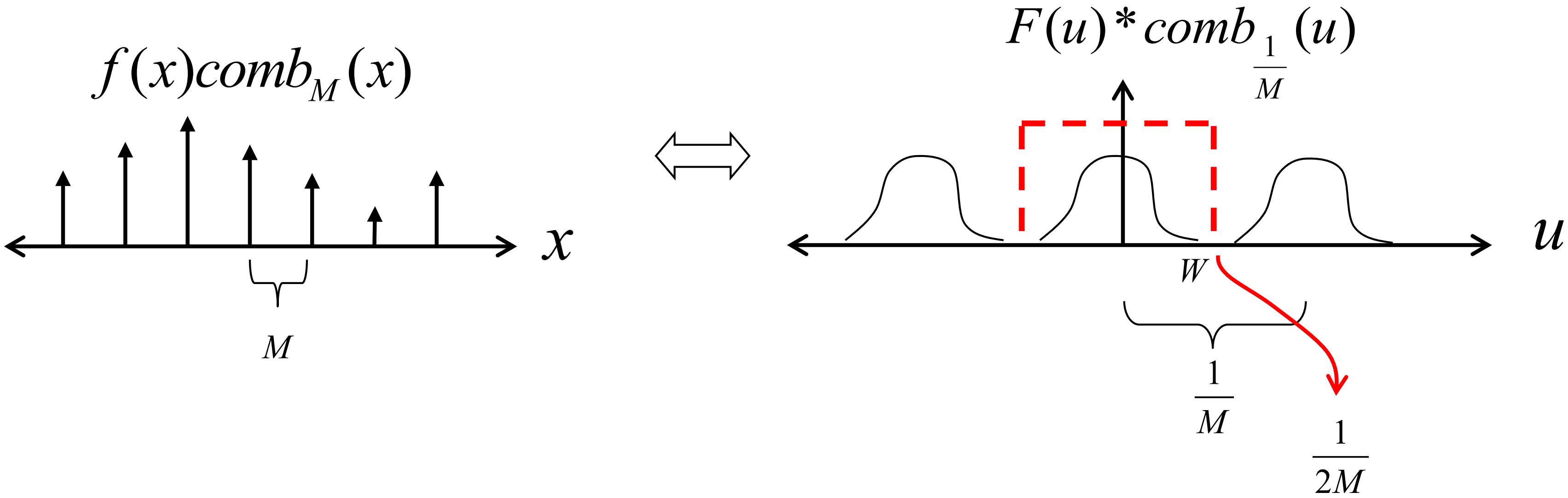

$f(x)$
x
$F(u)$
u
$comb_M(x)$
x
M
$comb_{\frac{1}{M}}(u)$
u
$\frac{1}{M}$
$f(x)comb_M(x)$
x
$F(u) * comb_{\frac{1}{M}}(u)$
u

Sampling



Nyquist theorem: No aliasing if $\dfrac{1}{M} > 2W$

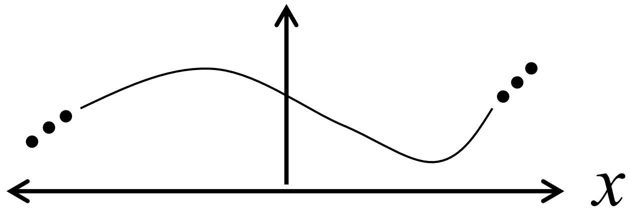
Sampling



If there is no aliasing, the original signal can be recovered from its samples by low-pass filtering.

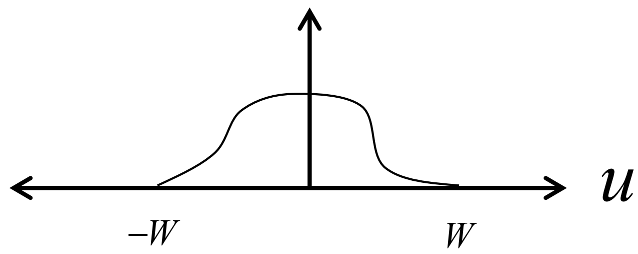
Sampling

$f(x)$

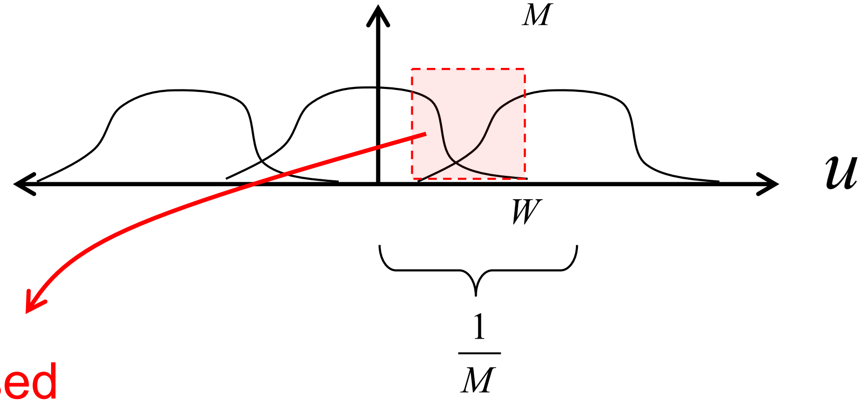


$\Leftrightarrow$

$F(u)$

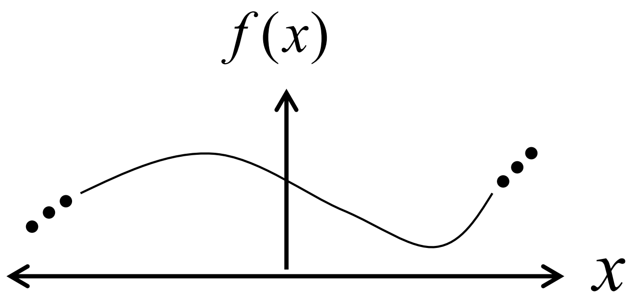


$$f(x)comb_M(x) \Leftrightarrow F(u) * comb_{\frac{1}{M}}(u)$$

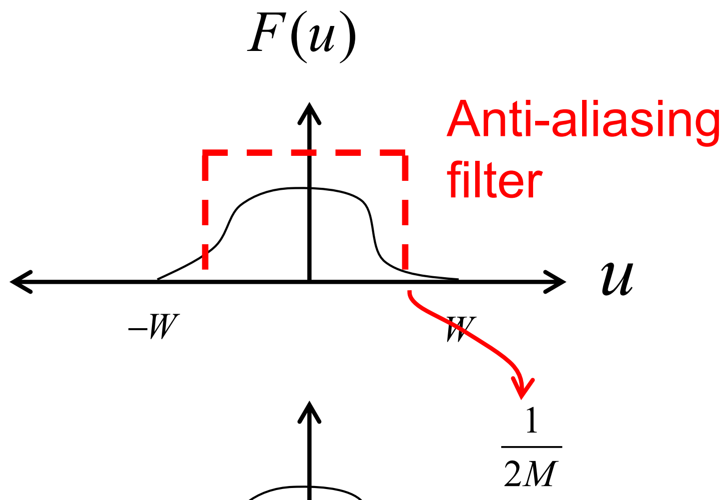


Aliased

Sampling

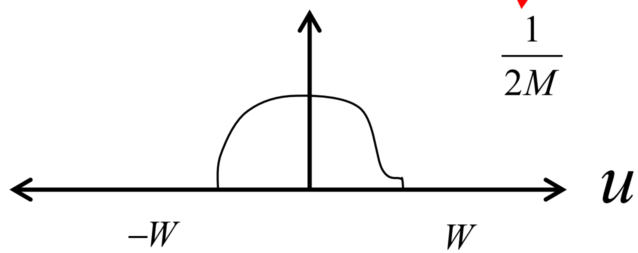


$\Longleftrightarrow$



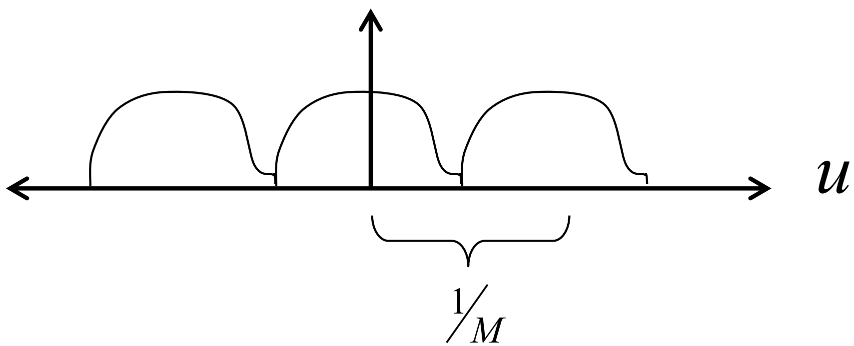
$$f(x) * h(x)$$

$\Longleftrightarrow$



$$\left[f(x) * h(x)\right] comb_M(x)$$

$\Longleftrightarrow$

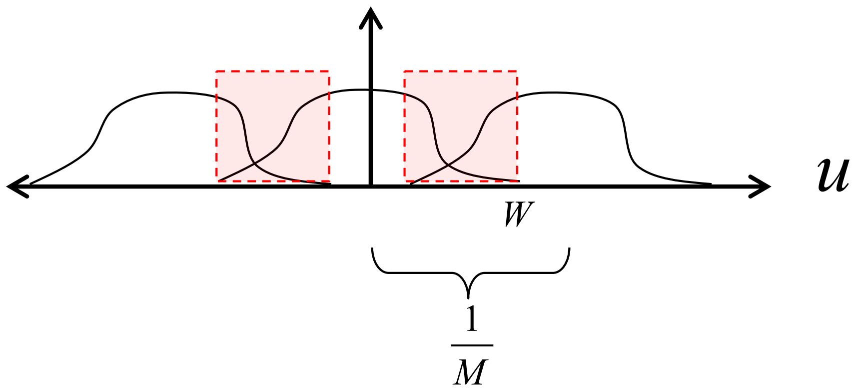


Sampling

- Without anti-aliasing filter:

$$f(x)comb_M(x)$$

$\Longleftrightarrow$



- With anti-aliasing filter:

$$[f(x)*h(x)]comb_M(x)$$

$\Longleftrightarrow$