

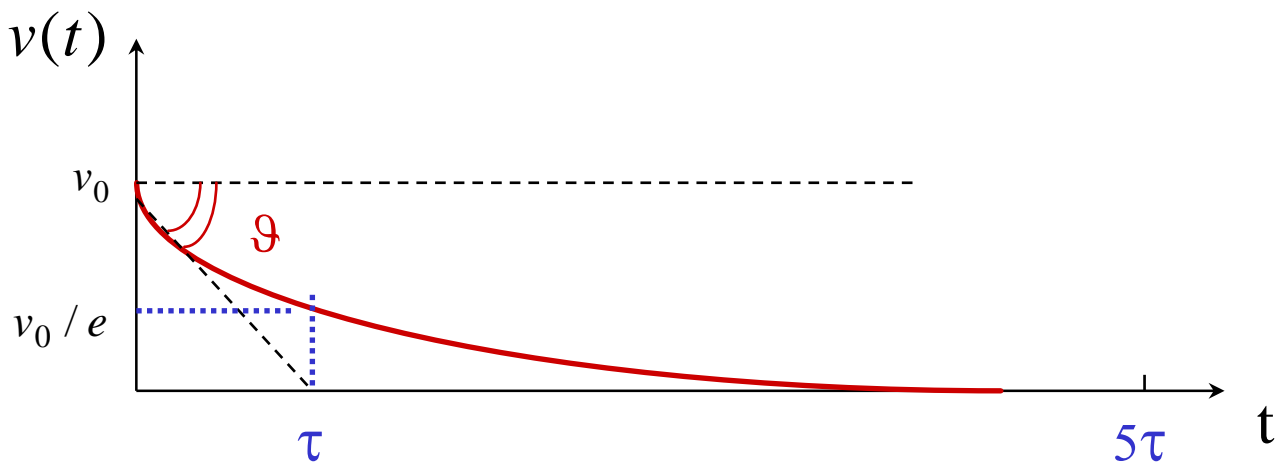
## Moto smorzato esponenzialmente:

si verifica in presenza di una decelerazione di tipo “viscoso”,  
ossia **proporzionale alla velocità** :

$$a(t) = \frac{dv(t)}{dt} = -kv(t)$$

$$\Rightarrow \frac{dv(t)}{v} = -k dt \quad \Rightarrow \quad \int_{v_0}^v \frac{dv}{v} = -k \int_0^t dt$$

$$\Rightarrow \ln\left(\frac{v}{v_0}\right) = -kt \quad \Rightarrow \quad v(t) = v_0 e^{-kt}$$



$$v(t = 1/k) = v_0 e^{-1} \quad \Rightarrow \quad \tau \equiv 1/k \quad \text{“costante di tempo” dello smorzamento}$$

$$\text{Per } t \approx 5 \tau : v(t = 5\tau) = v_0 e^{-5} \approx 0.006 v_0 \approx 0$$

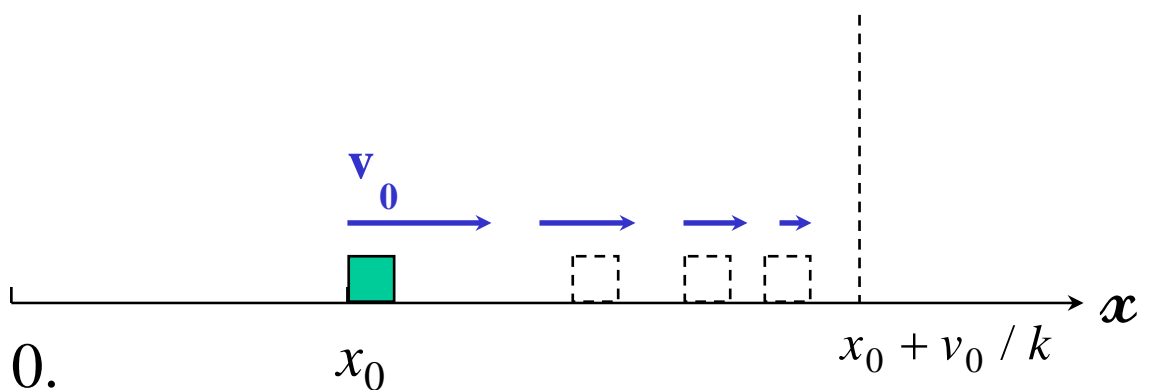
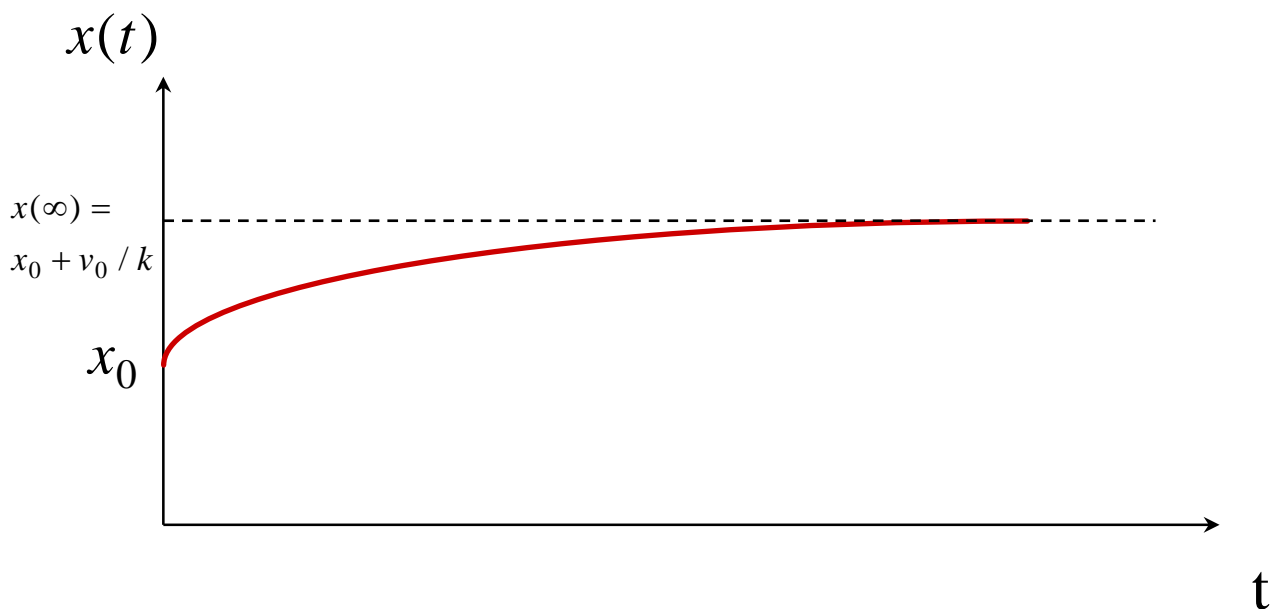
$\tau$  è l'intersezione con l'asse dei tempi della retta tangente alla curva  $v(t)$  al tempo  $t = 0$  :

$$\tan \theta \equiv \left. \frac{dv}{dt} \right|_{t=0} = -kv_0 e^{-kt} \Big|_{t=0} = -kv_0 = -\frac{v_0}{\tau}$$

## Spazio percorso in un moto smorzato esponenzialmente :

$$x(t) = x_0 + \int_0^t v(t) dt = x_0 + \int_0^t v_0 e^{-kt} dt = x_0 - \frac{v_0}{k} e^{-kt} \Big|_0^t$$

$$x(t) = x_0 + \frac{v_0}{k} (1 - e^{-kt})$$



**Moto accelerato** in presenza di un **attrito viscoso**:

$$\frac{dv(t)}{dt} = a - kv(t)$$

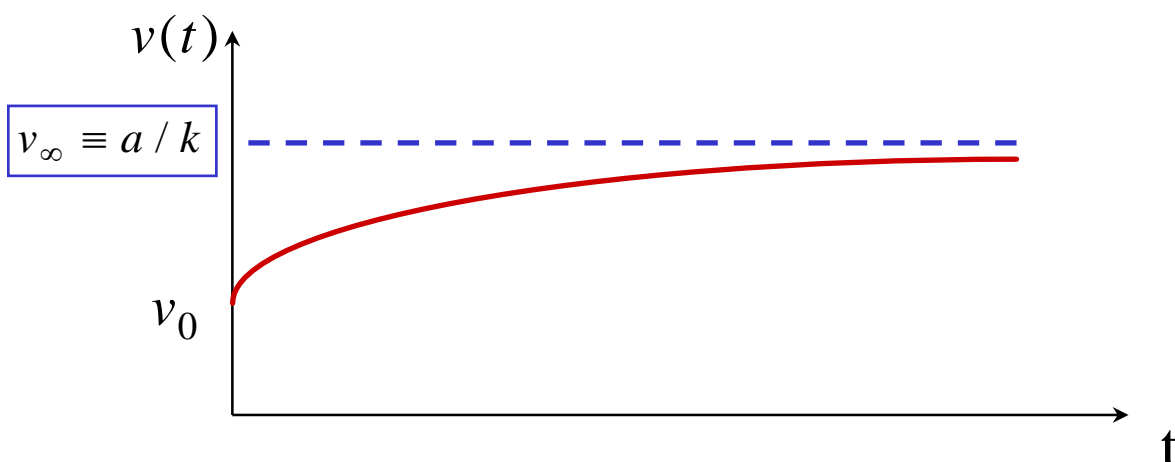
↑  
termine costante (es: g)

$$\Rightarrow \frac{dv(t)}{a - kv(t)} = dt \quad \text{Posto: } w(t) \equiv a - kv(t) \\ \rightarrow dw \equiv -kdv$$

$$\Rightarrow \frac{1}{k} \frac{dw(t)}{w} = -dt \Rightarrow \ln\left(\frac{w}{w_0}\right) = -kt \Rightarrow w(t) = w_0 e^{-kt}$$

$$\Rightarrow a - kv(t) = (a - kv_0)e^{-kt} \Rightarrow v(t) = \frac{a}{k} - \left(\frac{a}{k} - v_0\right)e^{-kt}$$

$$\Rightarrow v(t) = \frac{a}{k} + \left(v_0 - \frac{a}{k}\right)e^{-kt}$$



“velocità limite” :  $v_\infty \equiv \lim_{t \rightarrow \infty} v(t) = a/k$  (indipendente da  $v_0$ )