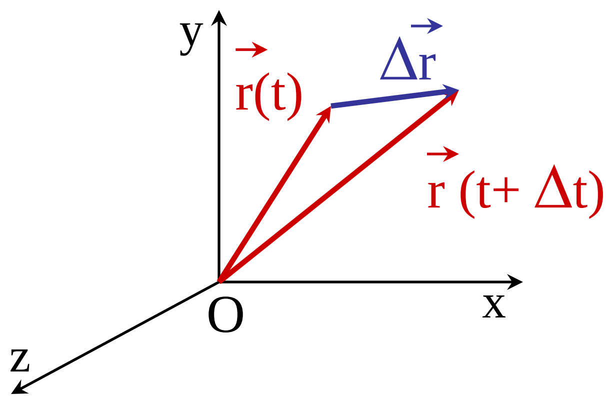
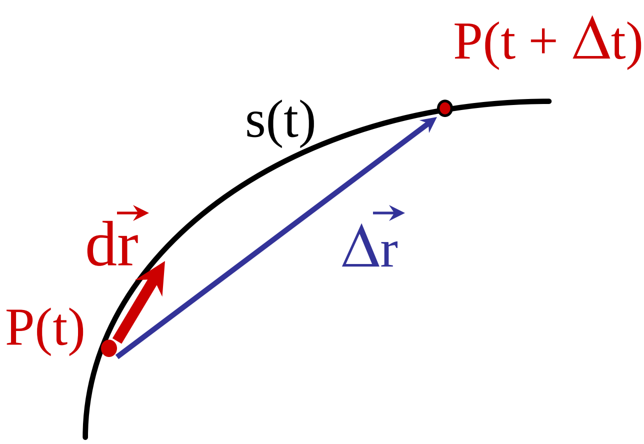
Moto curvilineo nello spazio tridimensionale

Vettore velocità istantanea:

$$\boxed{\vec{v}(t) \equiv \frac{d\vec{r}(t)}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}}$$



La velocità istantanea é un vettore tangente alla traiettoria :



$$\Delta\vec{r} \xrightarrow[\Delta t \to 0]{} d\vec{r} = ds\,\vec{u}_T$$

$\vec{u}_T$: versore tangente

$$\boxed{\vec{v}(t) \equiv \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta s}\frac{\Delta s}{\Delta t} = \frac{ds}{dt}\vec{u}_T = v(t)\vec{u}_T}$$

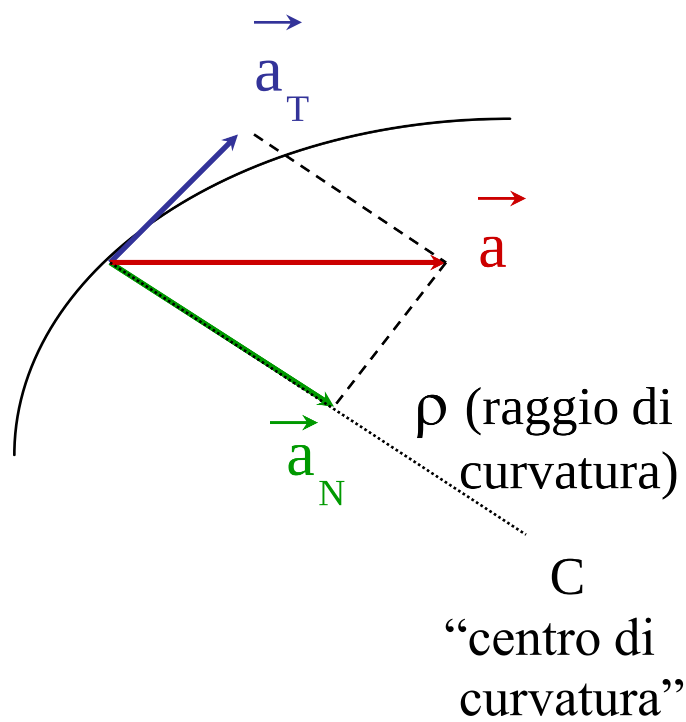
velocità scalare istantenea

Vettore accelerazione: componenti intrinseche

L'accelerazione ha una componente tangente ed una componente normale alla traiettoria :

$$\vec{a}(t) \equiv \frac{d\vec{v}(t)}{dt} = \frac{d[v(t)\vec{u}_T(t)]}{dt} =$$

$$= \frac{dv(t)}{dt}\vec{u}_T + v(t)\underbrace{\frac{d\vec{u}_T(t)}{dt}}_{\frac{v(t)}{\rho}\vec{u}_N}$$

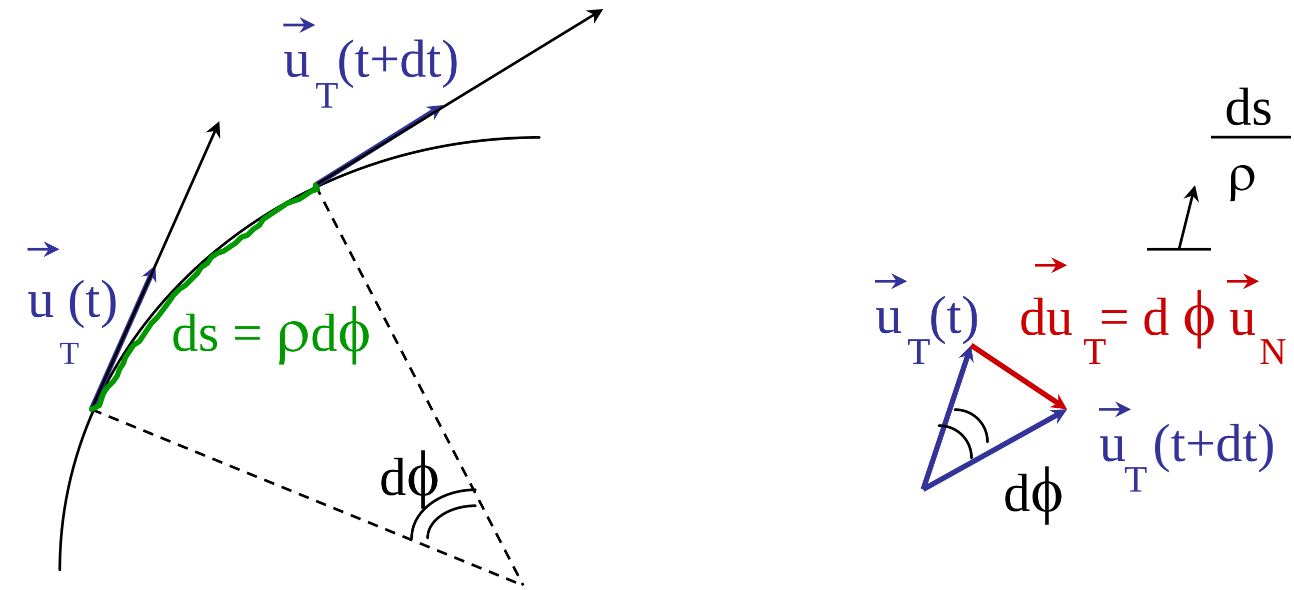


$$\boxed{\vec{a} = a_T \vec{u}_T + a_N \vec{u}_N}$$

accelerazione tangente : $a_T(t) = \dfrac{dv(t)}{dt}$

accelerazione normale : $a_N(t) = \dfrac{v^2(t)}{\rho(t)}$

Accelerazione normale:

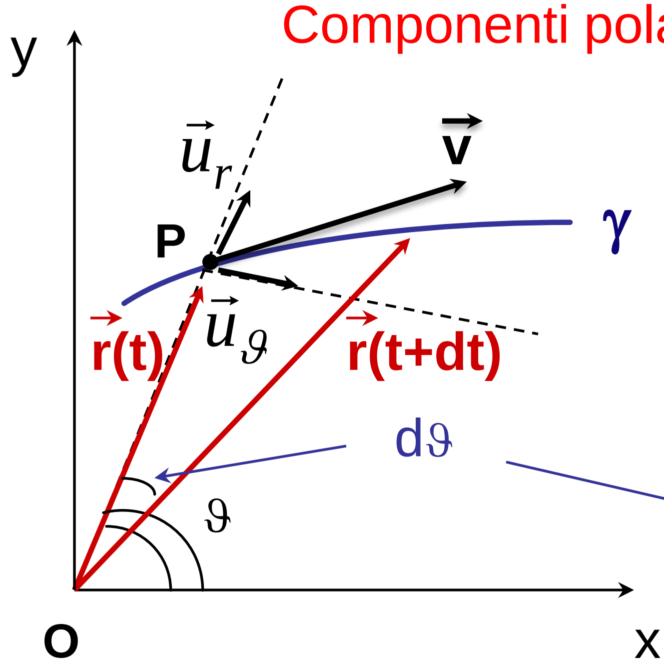


$$d(\vec{u}_T^{\,2}) = 2\vec{u}_T \bullet d\vec{u}_T = 0 \quad \Rightarrow \quad d\vec{u}_T \perp \vec{u}_T$$

$$\Rightarrow \quad \boxed{\frac{d\vec{u}_T}{dt} = \frac{d\phi}{dt}\vec{u}_N = \frac{1}{\rho}\frac{ds}{dt}\vec{u}_N = \frac{v(t)}{\rho}\vec{u}_N}$$

Moto piano in coordinate polari Orθ,t

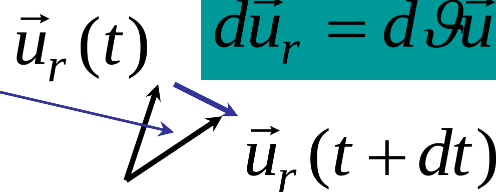
Componenti polari della velocità:



$$\vec{u}_r^{\,2} \equiv 1 = \text{ costante}$$

$$\Rightarrow \quad d\vec{u}_r^{\,2} = 2\vec{u}_r \cdot d\vec{u}_r = 0$$

$$\Rightarrow \quad \vec{u}_r \perp d\vec{u}_r$$



$$\boxed{d\vec{u}_r = d\vartheta\,\vec{u}_\vartheta}$$

$$\vec{r}(t) = r(t)\vec{u}_r(t)$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dr(t)}{dt}\vec{u}_r + r(t)\overbrace{\frac{d\vec{u}_r(t)}{dt}}^{=\frac{d\vartheta}{dt}\vec{u}_\vartheta}$$

$$\Rightarrow \quad \boxed{\vec{v}(t) = \underbrace{\frac{dr(t)}{dt}\vec{u}_r}_{\vec{v}_r} + \underbrace{r(t)\frac{d\vartheta(t)}{dt}\vec{u}_\vartheta}_{\vec{v}_\vartheta}}$$

"velocità radiale" "velocità trasversa"

$$\vec{v}(t) = \underbrace{\left(\frac{dr(t)}{dt}, r(t)\frac{d\vartheta(t)}{dt}\right)}_{\text{componenti polari}} = \underbrace{\left(\frac{dx(t)}{dt}, \frac{dy(t)}{dt}\right)}_{\text{componenti cartesiane}}$$

Componenti polari dell'accelerazione

$$\vec a \equiv \frac{d\vec v(t)}{dt} = \frac{d}{dt}\left(\frac{dr}{dt}\vec u_r + r\frac{d\vartheta}{dt}\vec u_\vartheta\right) =$$

$$= \frac{d^2r}{dt^2}\vec u_r + \underbrace{\frac{dr}{dt}\underbrace{\frac{d\vec u_r}{dt}}_{=\frac{d\vartheta}{dt}\vec u_\vartheta} + \frac{dr}{dt}\frac{d\vartheta}{dt}\vec u_\vartheta}_{=2\frac{dr}{dt}\frac{d\vartheta}{dt}\vec u_\vartheta} + r\frac{d^2\vartheta}{dt^2}\vec u_\vartheta + r\frac{d\vartheta}{dt}\underbrace{\frac{d\vec u_\vartheta}{dt}}_{=-\frac{d\vartheta}{dt}\vec u_r}$$

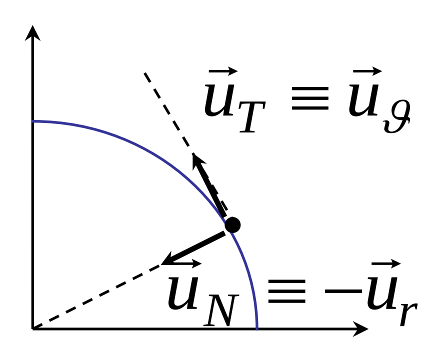
$$\Rightarrow \boxed{\vec a = \underbrace{\left(\frac{d^2r}{dt^2} - r\left(\frac{d\vartheta}{dt}\right)^2\right)\vec u_r}_{\vec a_r} + \underbrace{\left(2\frac{dr}{dt}\frac{d\vartheta}{dt} + r\frac{d^2\vartheta}{dt^2}\right)\vec u_\vartheta}_{\vec a_\vartheta}}$$

"accelerazione radiale" "accelerazione trasversa"

In un <u>moto circolare</u> (r = R, costante):

$$a_r = -R\left(\frac{d\vartheta}{dt}\right)^2 = -R\omega^2 \equiv -a_N$$

$$a_\vartheta = R\frac{d^2\vartheta}{dt^2} = R\frac{d\omega}{dt} = R\alpha \equiv a_T$$



Moto circolare uniforme:

coordinata curvilinea
$s(t)=R\vartheta(t)$

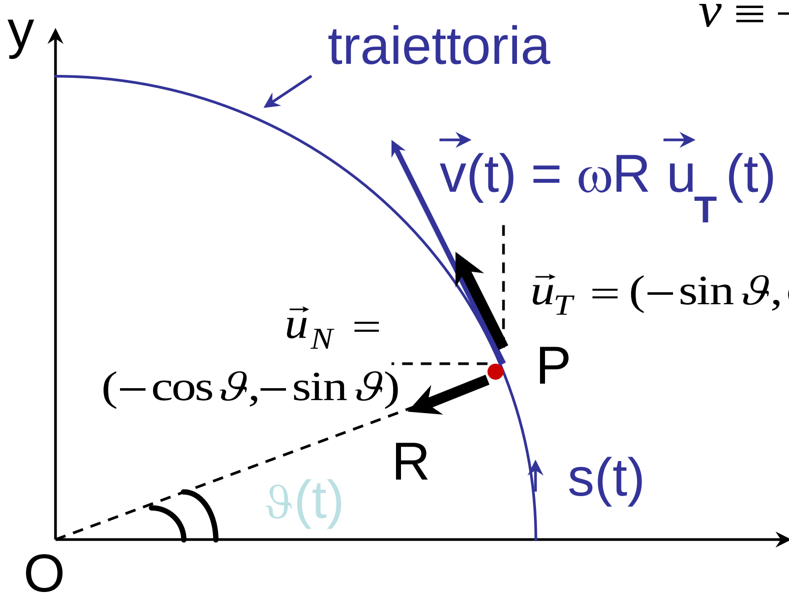
velocità con modulo costante:

$$v \equiv \frac{ds(t)}{dt} = R\frac{d\vartheta(t)}{dt} = \omega R$$

"velocità angolare"

$$\boxed{\omega \equiv \frac{d\vartheta(t)}{dt}}$$

$$\boxed{\vartheta(t) = \vartheta_0 + \omega t}$$



$$x(t) = R\cos\vartheta(t) \qquad v_x(t) = \frac{dx(t)}{dt} = -R\sin\vartheta(t)\frac{d\vartheta}{dt} \equiv -R\omega\sin\vartheta(t)$$

$$\Rightarrow$$

$$y(t) = R\sin\vartheta(t) \qquad v_y(t) = \frac{dy(t)}{dt} = R\cos\vartheta(t)\frac{d\vartheta}{dt} \equiv R\omega\cos\vartheta(t)$$

$$\Rightarrow \qquad \vec{v}(t) = (v_x(t), v_y(t)) = R\omega\underbrace{(-\sin\vartheta(t), \cos\vartheta(t))}_{\vec{u}_T}$$

$$\Rightarrow \qquad \boxed{\vec{v}(t) = R\omega\vec{u}_T(t) = v\vec{u}_T(t)}$$

$$a_x(t) = \frac{dv_x(t)}{dt} = -R\omega\cos\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\cos\vartheta(t)$$

$$a_y(t) = \frac{dv_y(t)}{dt} = -R\omega\sin\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\sin\vartheta(t)$$

$$\Rightarrow \qquad \vec{a}(t) = (a_x(t), a_y(t)) = R\omega^2\underbrace{(-\cos\vartheta(t), -\sin\vartheta(t))}_{\vec{u}_N}$$

$$\Rightarrow \qquad \boxed{\vec{a}(t) = R\omega^2\vec{u}_N(t) = \frac{v^2}{R}\vec{u}_N(t)}$$