(**submit your solutions** during the lecture on December 18, 2014)

Exercise 1. Let K be a field.

(a) Given the quiver $Q : 1 \longrightarrow 2 \longrightarrow 3$ (no relations), compute [1] the representations S_2, I_2, I_1 and their transposes $\mathrm{Tr}S_2$, $\mathrm{Tr}I_2$, $\mathrm{Tr}I_1$.

(b) Given the quiver $Q : 1 \longrightarrow 2 \longleftarrow 3$ (no relations), compute τI_i for $i = 1, 2, 3$. (10 points)

Exercise 2. Let K be a field and Q the quiver

$$\begin{tikzcd} & 2 \arrow[dr, "\beta"] & \\ 1 \arrow[ur, "\alpha"] \arrow[rr, "\gamma"] & & 3 \end{tikzcd}$$

(no relations).

(a) Determine all indecomposable projective representations and their radicals.

(b) Determine all indecomposable injective representations and their socles.

(c) Determine the minimal projective resolutions of the simple representations.

(d) Compute the representation $\nu(S_1)$.

(e) Compute the representation $\tau(S_1)$. (10 points)

Exercise 3. Let Q be the quiver $\begin{tikzcd} 1 \arrow[r, bend left, "\alpha"] & 2 \arrow[l, bend left, "\beta"] \end{tikzcd}$. Compute minimal projective resolutions of the simple representations S_1 and S_2 of the bound quiver (Q, I), where

(a) $I = \langle \beta\alpha \rangle$,

(b) $I = \langle \alpha\beta,\, \beta\alpha \rangle$
(hint: in case (b) show that the minimal projective resolutions are

$$\cdots \to P_2 \to P_1 \to P_2 \to P_1 \to P_2 \to P_1 \to S_1 \to 0$$
$$\cdots \to P_1 \to P_2 \to P_1 \to P_2 \to P_1 \to P_2 \to S_2 \to 0$$

thus KQ/I has infinite global dimension). (10 points)

[1] All computations should be done in terms of representations (given by vector spaces and linear maps).