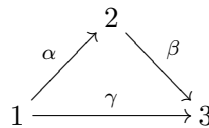


(submit your solutions during the lecture on December 18, 2014)

Exercise 1. Let K be a field.

- (a) Given the quiver $Q : 1 \longrightarrow 2 \longrightarrow 3$ (no relations), compute ¹ the representations S_2, I_2, I_1 and their transposes $\text{Tr}S_2, \text{Tr}I_2, \text{Tr}I_1$.
- (b) Given the quiver $Q : 1 \longrightarrow 2 \longleftarrow 3$ (no relations), compute τI_i for $i = 1, 2, 3$. (10 points)

Exercise 2. Let K be a field and Q the quiver (no relations).



- (a) Determine all indecomposable projective representations and their radicals.
- (b) Determine all indecomposable injective representations and their socles.
- (c) Determine the minimal projective resolutions of the simple representations.
- (d) Compute the representation $\nu(S_1)$.
- (e) Compute the representation $\tau(S_1)$. (10 points)

Exercise 3. Let Q be the quiver $1 \begin{matrix} \xrightarrow{\alpha} \\ \xleftarrow{\beta} \end{matrix} 2$. Compute minimal projective resolutions of the simple representations S_1 and S_2 of the bound quiver (Q, I) , where

- (a) $I = \langle \beta\alpha \rangle$,
- (b) $I = \langle \alpha\beta, \beta\alpha \rangle$

(hint: in case (b) show that the minimal projective resolutions are

$$\begin{aligned} \cdots \rightarrow P_2 \rightarrow P_1 \rightarrow P_2 \rightarrow P_1 \rightarrow P_2 \rightarrow P_1 \rightarrow S_1 \rightarrow 0 \\ \cdots \rightarrow P_1 \rightarrow P_2 \rightarrow P_1 \rightarrow P_2 \rightarrow P_1 \rightarrow P_2 \rightarrow S_2 \rightarrow 0 \end{aligned}$$

thus KQ/I has infinite global dimension). (10 points)

¹All computations should be done in terms of representations (given by vector spaces and linear maps).