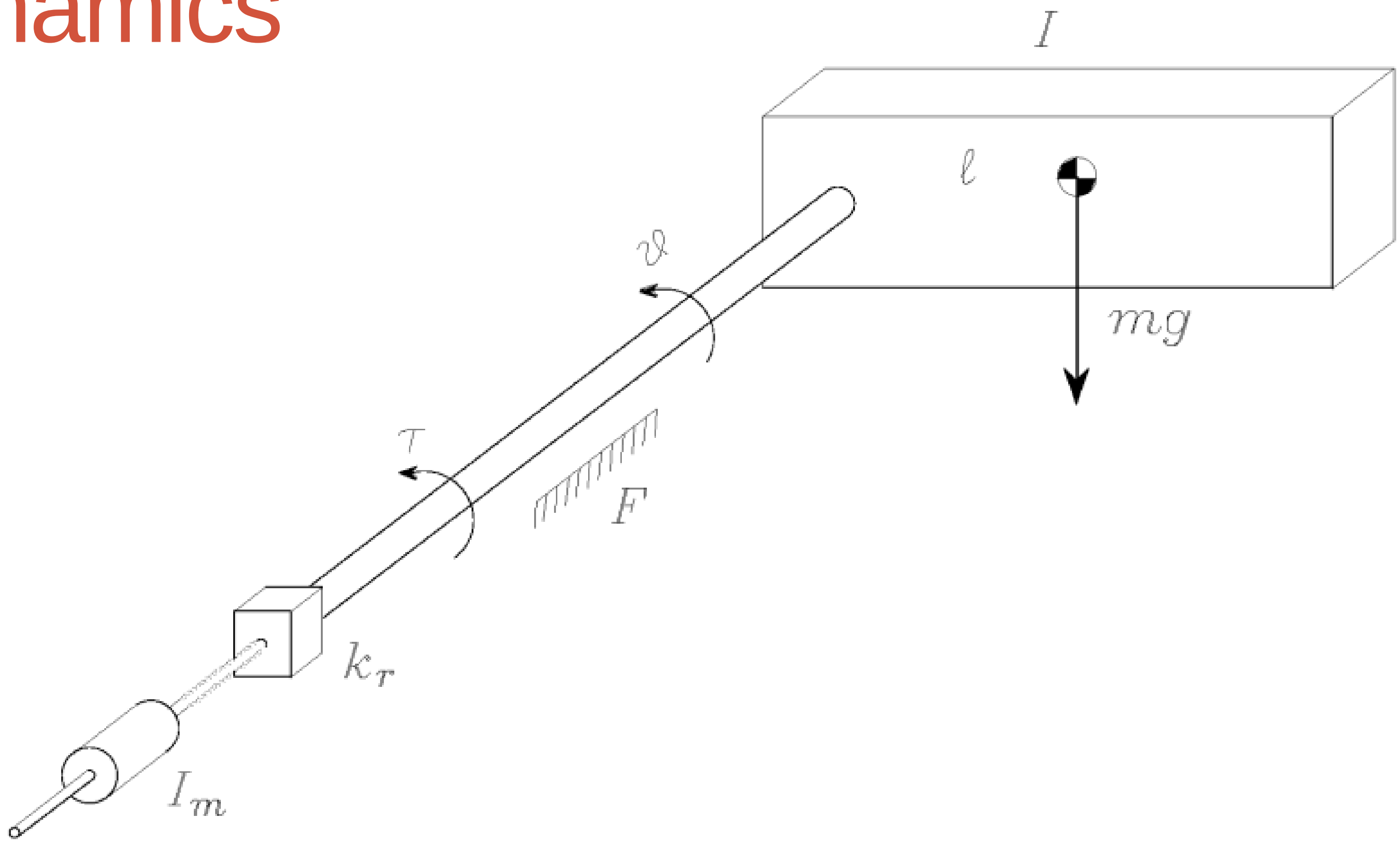
SCHEMAS AND NOTES ON DYNAMICS

Andrea Calanca

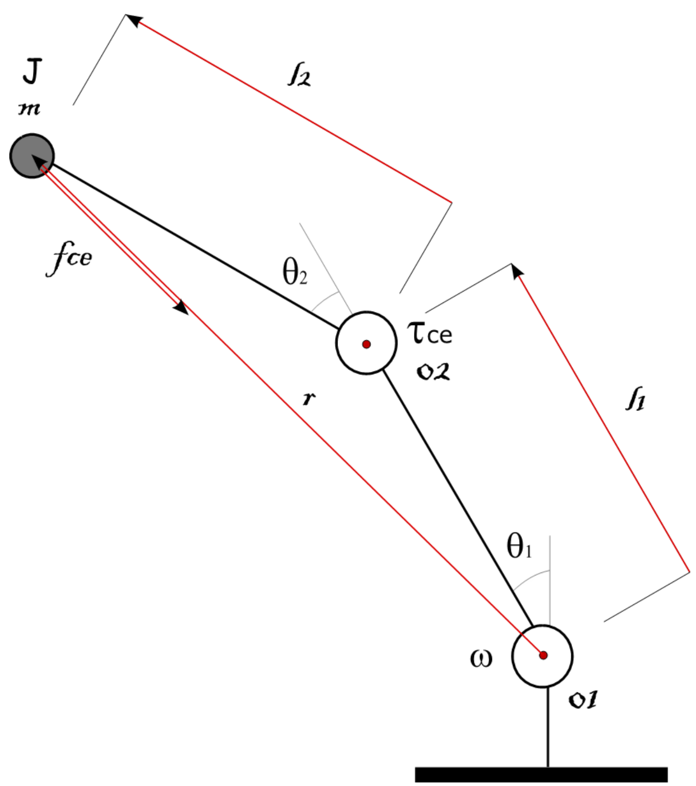
Dynamics

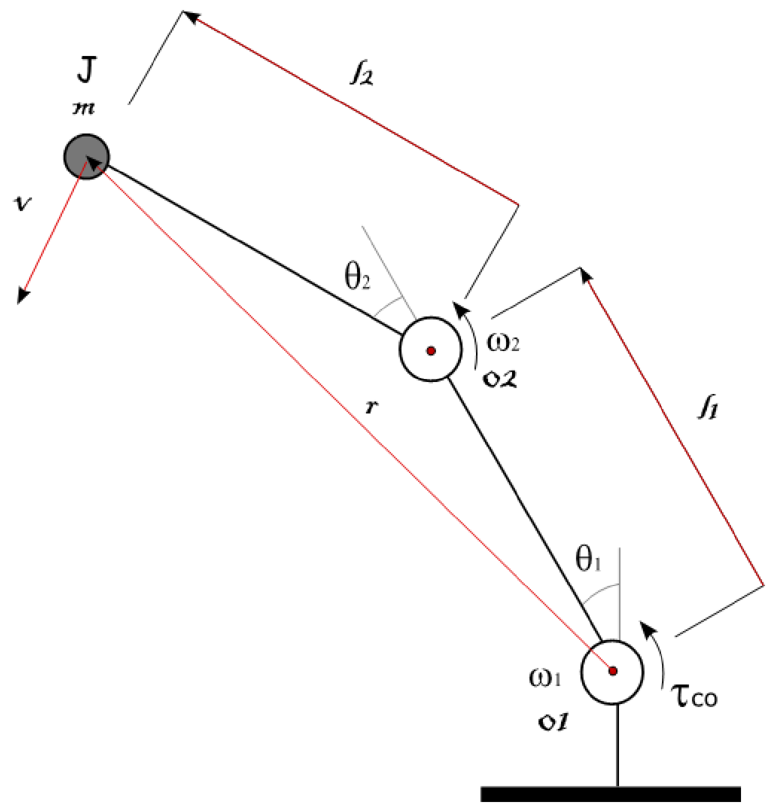


$$(I + I_m k_r^2)\ddot{\vartheta} + F\dot{\vartheta} + mg\ell \sin\vartheta = \tau$$

Dynamics

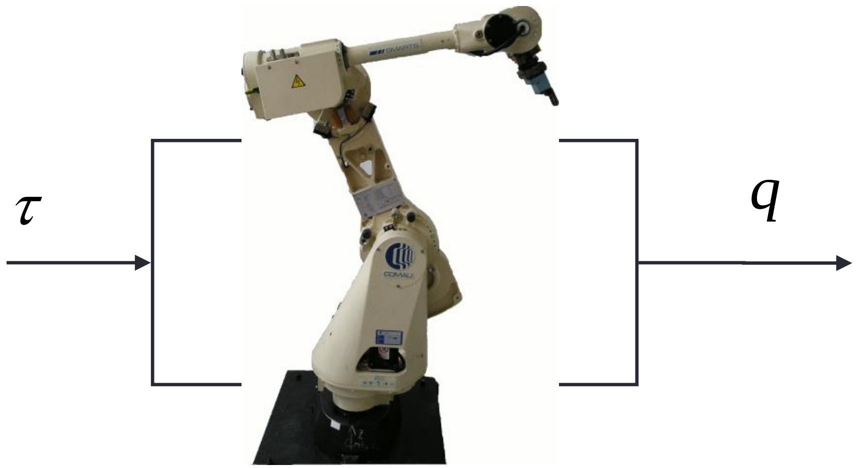
- Why we do not see any centripetal force?
- Why we do not see any Coriolis force?





Dynamics

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$



$$\boldsymbol{q} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$\boldsymbol{\tau} = \boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{F}_d sign(\dot{\boldsymbol{q}}) + \boldsymbol{F}_v\dot{\boldsymbol{q}}$$

$$\boldsymbol{\tau}_{tot} = \boldsymbol{B}(q)\ddot{q}$$

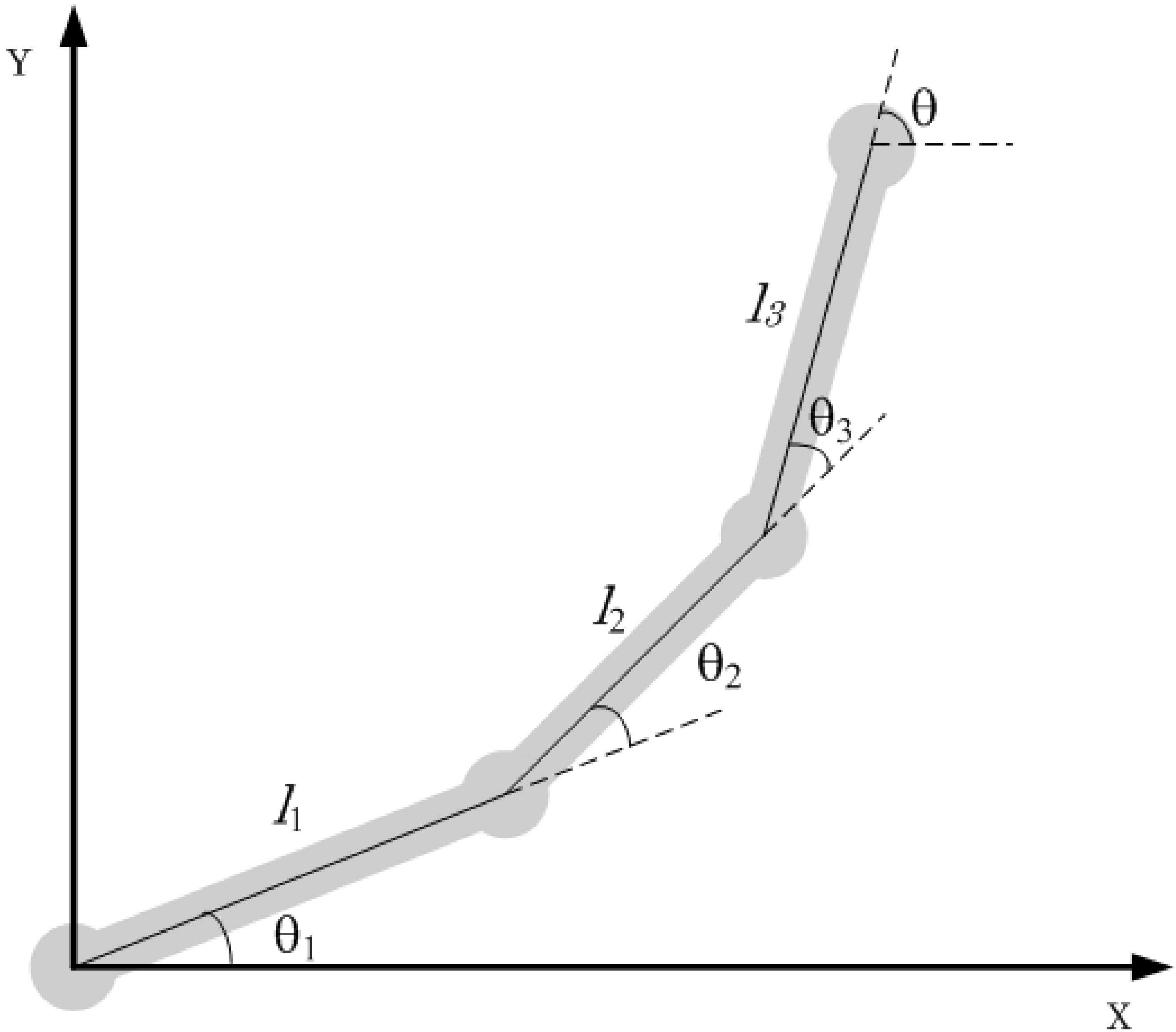
$$\boldsymbol{\tau}_{tot} = \boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{F}_d sign(\dot{\boldsymbol{q}}) - \boldsymbol{F}_v\dot{\boldsymbol{q}}$$

Dynamics

Three-link planar manipulator

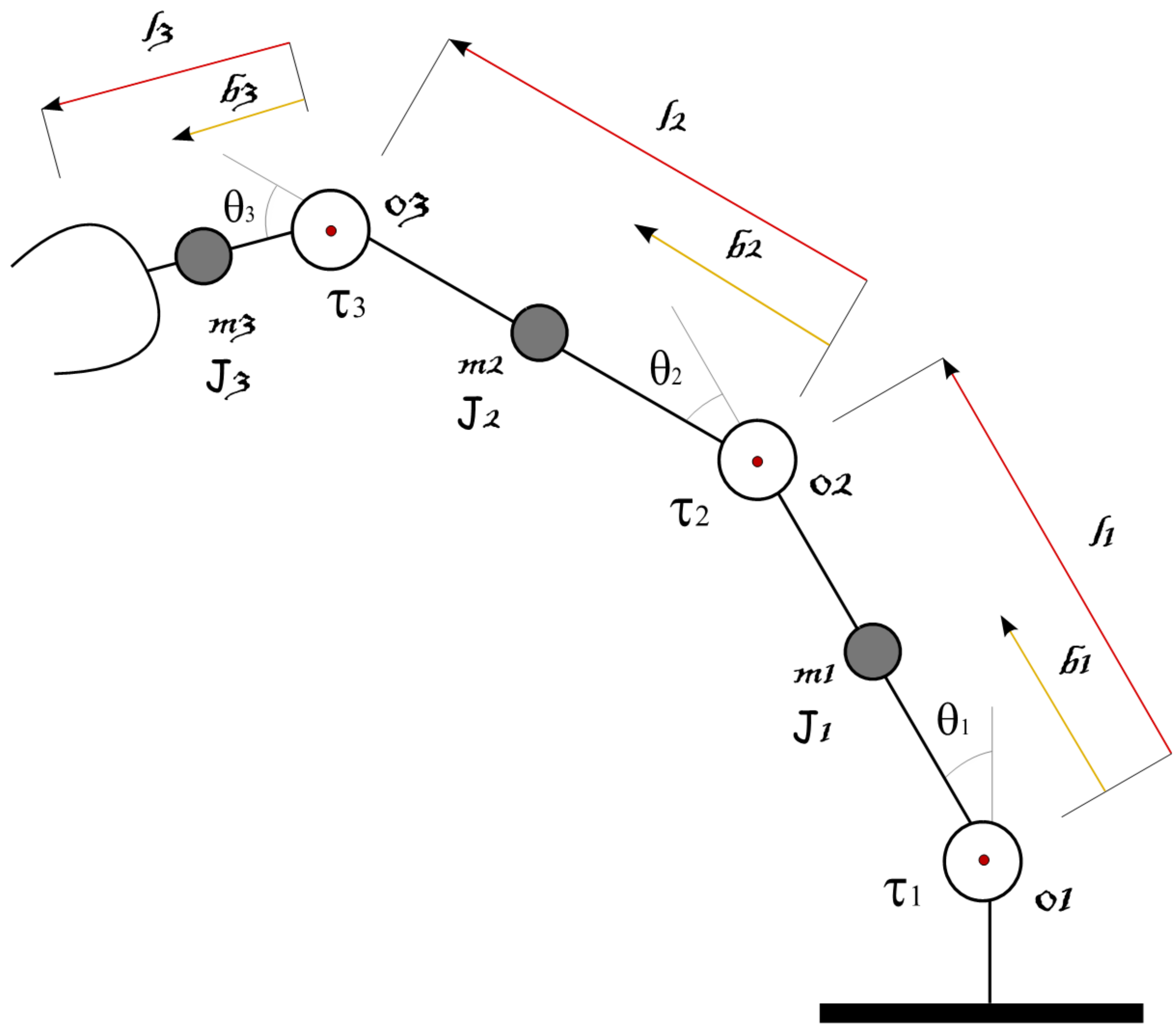
$$\boldsymbol{q} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$



Dynamics

… with lumped masses



Dynamics

$$\boldsymbol{\tau} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \ddot{\boldsymbol{q}} + \begin{bmatrix} 0 & C_{12} & C_{13} \\ C_{21} & 0 & C_{23} \\ C_{31} & C_{32} & 0 \end{bmatrix} \dot{\boldsymbol{q}} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} +$$

$$+ \begin{bmatrix} f_{d1} & 0 & 0 \\ 0 & f_{d2} & 0 \\ 0 & 0 & f_{d3} \end{bmatrix} sign(\dot{\boldsymbol{q}}) + \begin{bmatrix} f_{v1} & 0 & 0 \\ 0 & f_{v2} & 0 \\ 0 & 0 & f_{v3} \end{bmatrix} \dot{\boldsymbol{q}}$$

Dynamics

$$
\begin{aligned}
B_{11} &= m_1 b_1^2 + J_1 + m_2 l_1^2 + m_2 b_2^2 + m_2 l_1 b_2 \cos\theta_2 + J_2 + m_3 l_1^2 + m_3 l_2^2 + m_3 b_3^2 + \\
&\quad 2 m_3 l_1 b_3 \cos(\theta_2 + \theta_3) + 2 m_3 l_1 l_2 \cos\theta_2 + 2 m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{21} &= m_2 b_2^2 + m_2 l_1 b_2 \cos\theta_2 + J_2 + m_3 l_2^2 + m_3 b_3^2 + m_3 l_1 b_3 \cos(\theta_2 + \theta_3) + \\
&\quad m_3 l_1 l_2 \cos\theta_2 + 2 m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{31} &= m_3 b_3^2 + m_3 l_1 b_3 \cos(\theta_2 + \theta_3) + m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{12} &= m_2 b_2^2 + m_2 l_1 b_2 \cos\theta_2 + J_2 + m_3 l_2^2 + m_3 b_3^2 + m_3 l_1 b_3 \cos(\theta_2 + \theta_3) + \\
&\quad m_3 l_1 l_2 \cos\theta_2 + 2 m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{22} &= m_2 b_2^2 + J_2 + m_3 l_2^2 + m_3 b_3^2 + 2 m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{32} &= m_3 b_3^2 + m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{13} &= m_3 b_3^2 + m_3 l_1 b_3 \cos(\theta_2 + \theta_3) + m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{23} &= m_3 b_3^2 + m_3 l_2 b_3 \cos\theta_3 + J_3 \\
B_{33} &= m_3 b_3^2 + J_3
\end{aligned}
$$

Dynamics

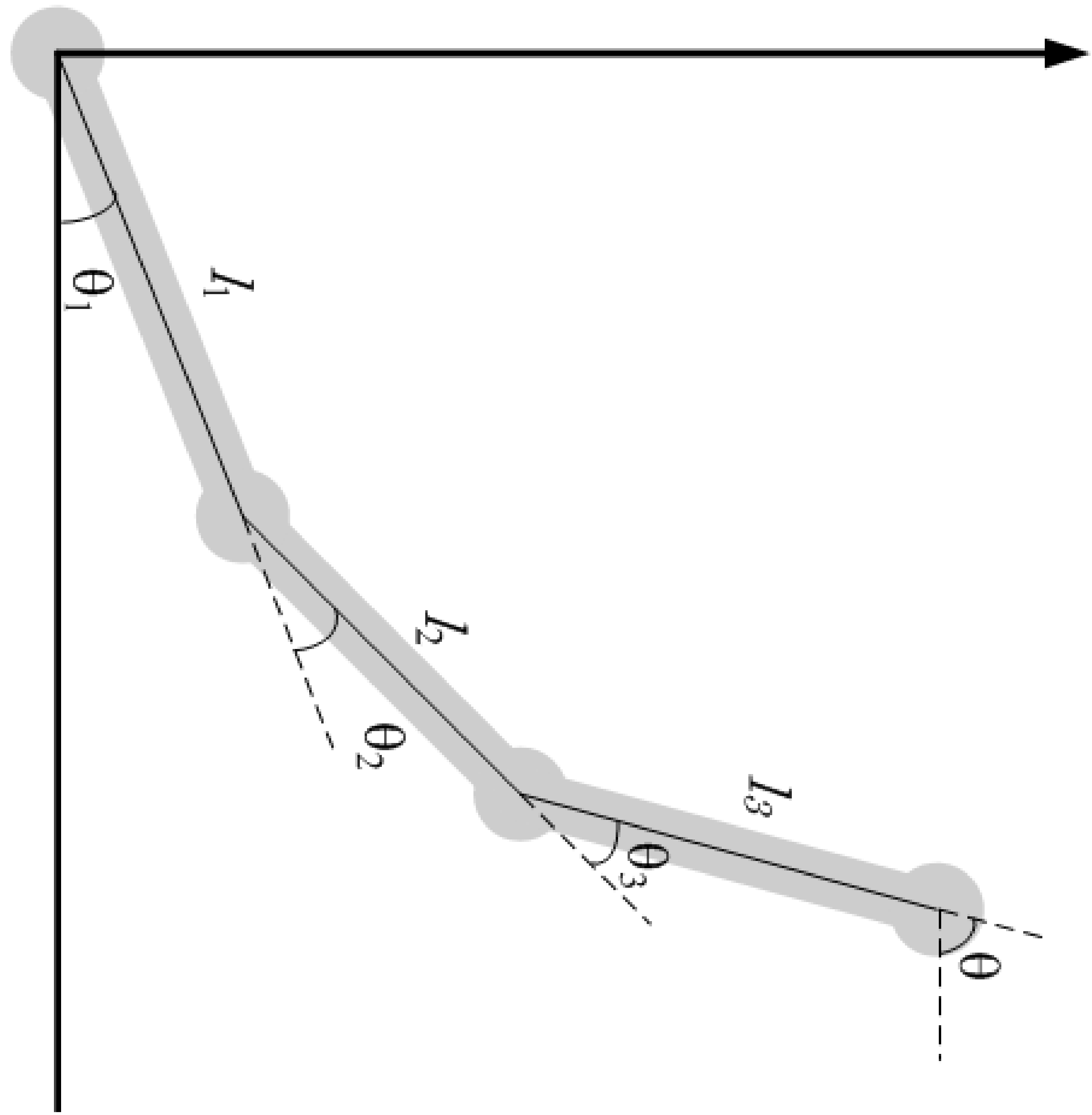
$$
\begin{aligned}
C_{21} &= m_2 l_1 b_2 \dot{\theta}_1 \sin\theta_2 + m_3 l_1 l_2 \dot{\theta}_1 \sin\theta_2 + m_3 l_1 b_3 \dot{\theta}_1 \sin(\theta_2+\theta_3) - 2 m_3 l_2 b_3 \dot{\theta}_3 \sin(\theta_3) \\
C_{31} &= m_3 l_1 b_3 \dot{\theta}_1 \sin(\theta_2+\theta_3) + m_3 l_2 b_3 \dot{\theta}_1 \sin\theta_3 \\
C_{12} &= -m_2 l_1 b_2 \dot{\theta}_2 \sin\theta_2 - m_3 l_1 l_2 \dot{\theta}_2 \sin\theta_2 - m_3 l_1 b_3 \dot{\theta}_2 \sin(\theta_2+\theta_3) - 2 m_2 l_1 b_2 \dot{\theta}_1 \sin\theta_2 \\
&\quad -2 m_3 l_1 l_2 \dot{\theta}_1 \sin\theta_2 - 2 m_3 l_1 b_3 \dot{\theta}_1 \sin(\theta_2+\theta_3) - 2 m_3 l_2 b_3 \dot{\theta}_3 \sin(\theta_3) \\
C_{32} &= m_3 l_2 b_3 \dot{\theta}_2 \sin\theta_3 + 2 m_3 l_2 b_3 \dot{\theta}_1 \sin(\theta_3) \\
C_{13} &= -m_3 l_1 b_3 \dot{\theta}_3 \sin(\theta_2+\theta_3) - m_3 l_2 b_3 \dot{\theta}_3 \sin\theta_3 - 2 m_3 l_1 b_3 \dot{\theta}_1 \sin(\theta_2+\theta_3) \\
&\quad -2 m_3 l_2 b_3 \dot{\theta}_1 \sin(\theta_3) \\
C_{23} &= -m_3 l_2 b_3 \dot{\theta}_3 \sin\theta_3 - 2 m_3 l_2 b_3 \sin(\theta_3) \dot{\theta}_2 \\
g_1 &= m_1 g b_1 \sin\theta_1 + m_2 g b_2 \sin(\theta_1+\theta_2) + m_2 g l_1 \sin\theta_1 + m_3 g b_3 \sin(\theta_1+\theta_2+\theta_3) \\
&\quad + m_3 g l_2 \sin(\theta_1+\theta_2) + m_3 g l_1 \sin\theta_1 \\
g_2 &= m_2 g b_2 \sin(\theta_1+\theta_2) + m_3 g b_3 \sin(\theta_1+\theta_2+\theta_3) + m_3 g l_2 \sin(\theta_1+\theta_2) \\
g_3 &= m_3 g b_3 \sin(\theta_1+\theta_2+\theta_3)
\end{aligned}
$$

Dynamics

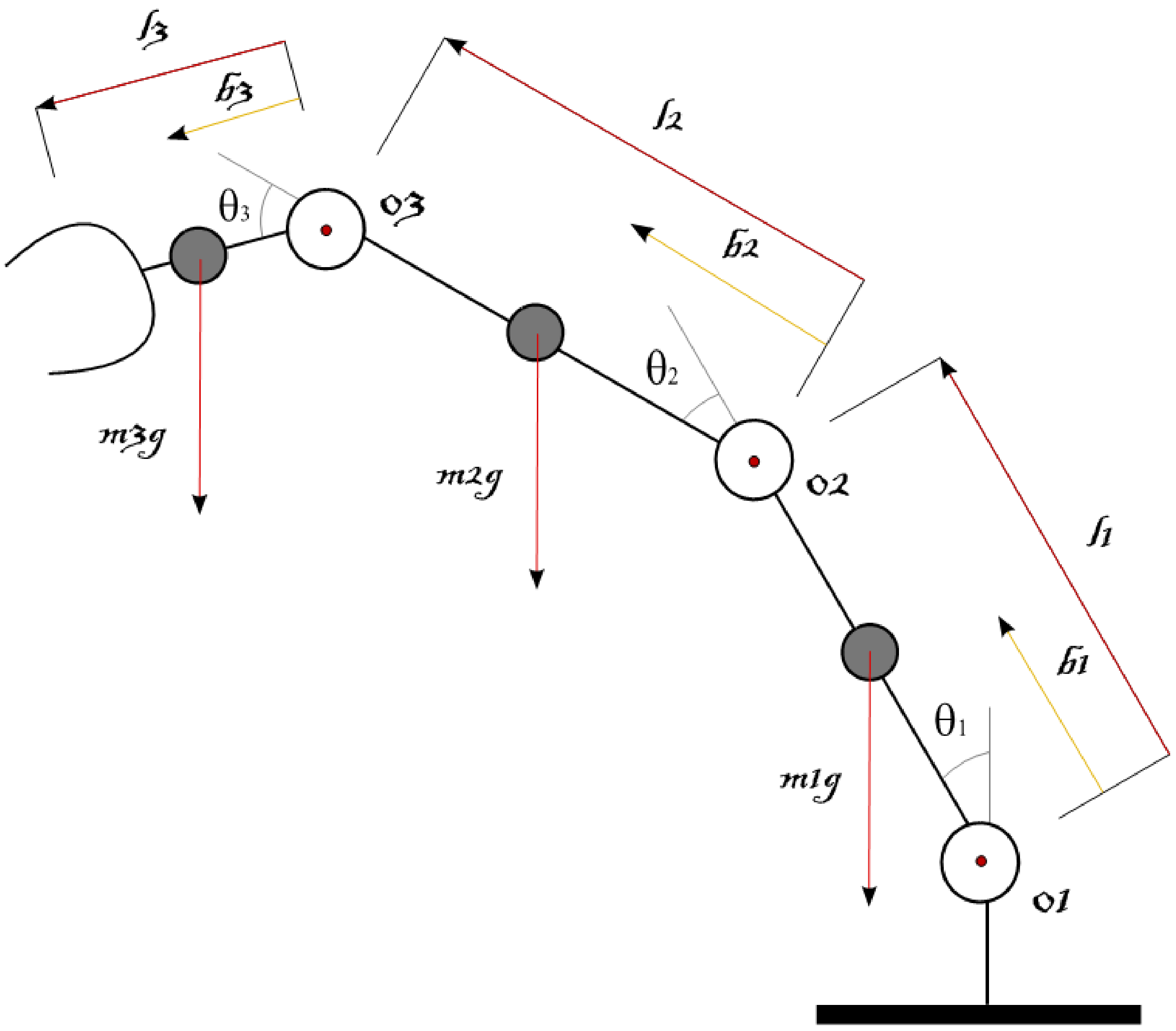
Three-link planar manipulator

$$\boldsymbol{q} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$



Gravity Forces

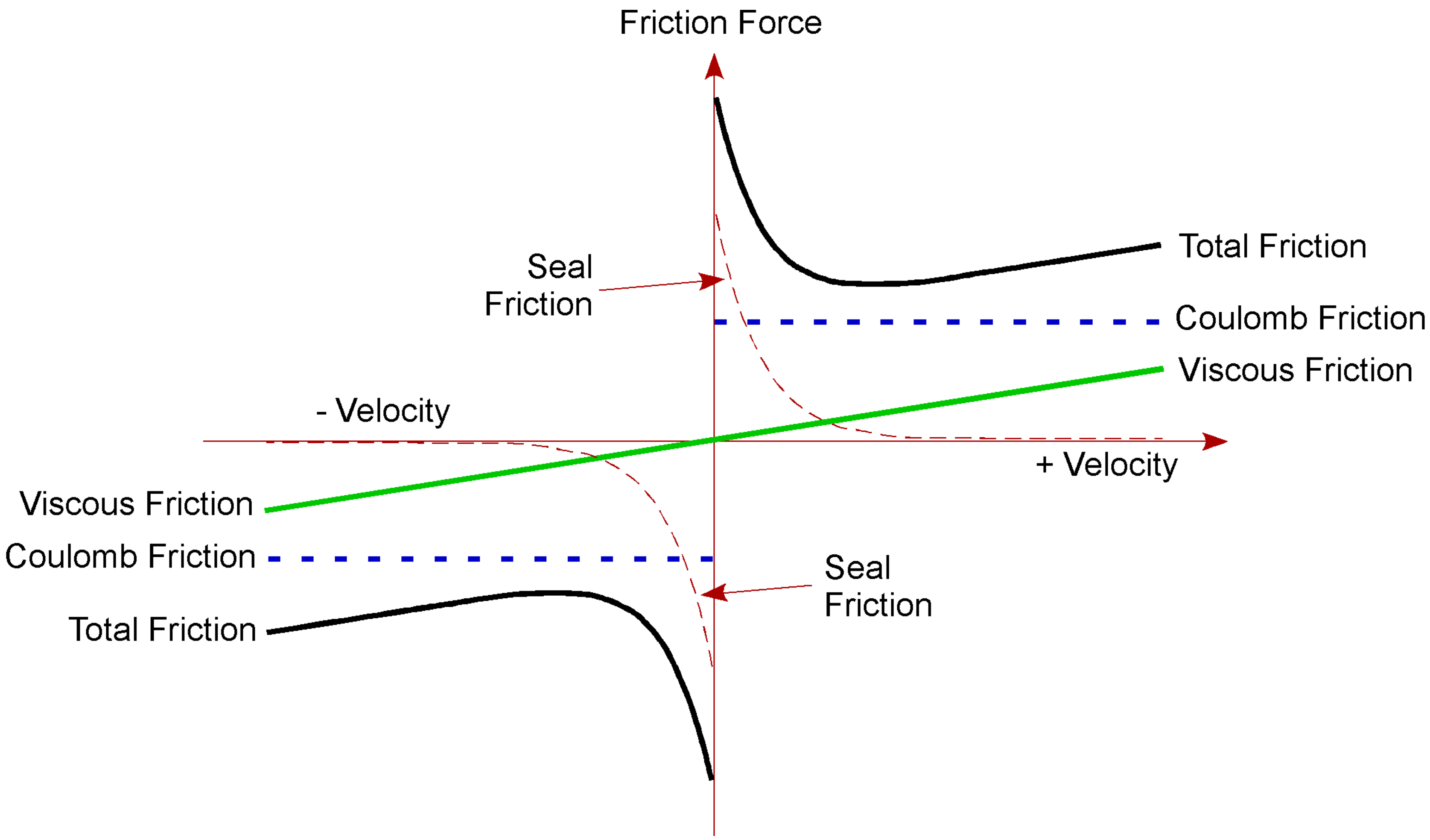


Dynamics

$$\boldsymbol{\tau} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \ddot{\boldsymbol{q}} + \begin{bmatrix} 0 & C_{12} & C_{13} \\ C_{21} & 0 & C_{23} \\ C_{31} & C_{32} & 0 \end{bmatrix} \dot{\boldsymbol{q}} + \boxed{\begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}} +$$

$$+ \begin{bmatrix} f_{d1} & 0 & 0 \\ 0 & f_{d2} & 0 \\ 0 & 0 & f_{d3} \end{bmatrix} sign(\dot{\boldsymbol{q}}) + \begin{bmatrix} f_{v1} & 0 & 0 \\ 0 & f_{v2} & 0 \\ 0 & 0 & f_{v3} \end{bmatrix} \dot{\boldsymbol{q}}$$

Friction

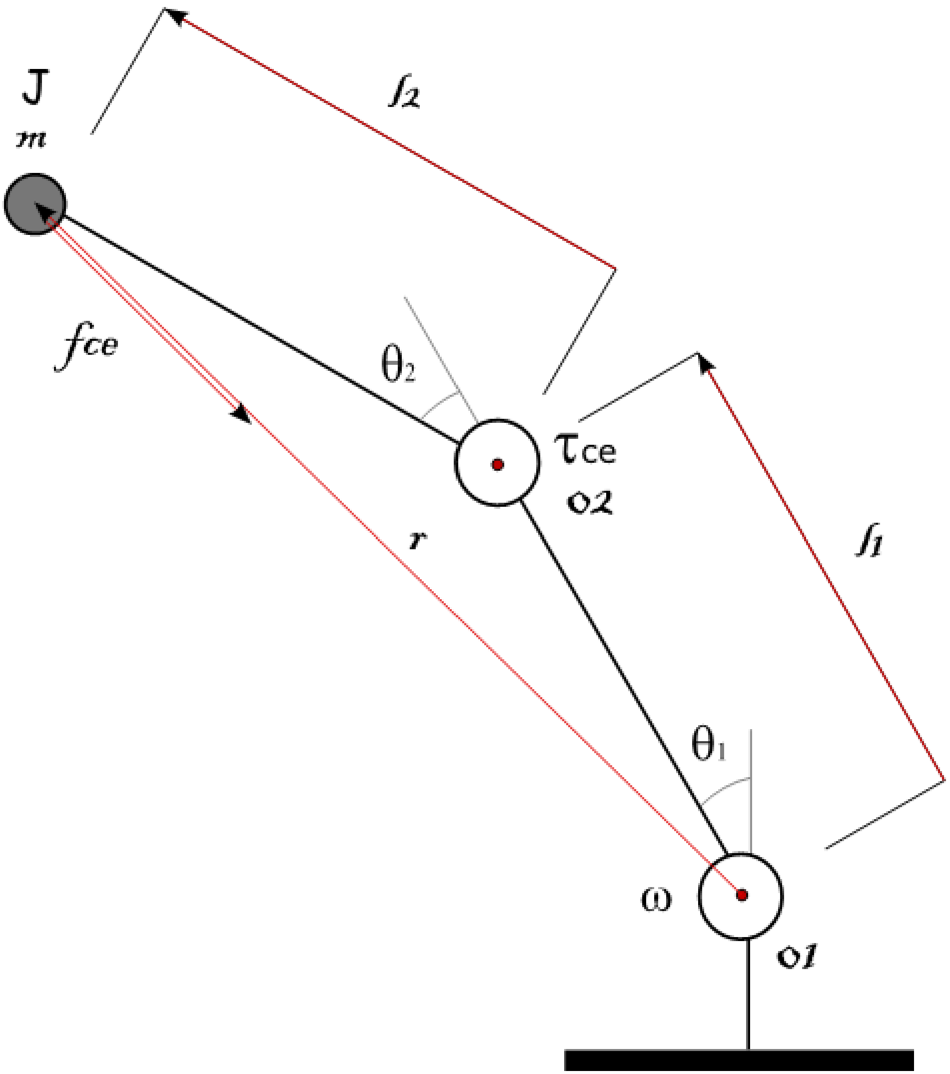

Friction Force
Seal
Friction
Total Friction
Coulomb Friction
Viscous Friction
- Velocity
+ Velocity
Viscous Friction
Coulomb Friction
Seal
Friction
Total Friction

Dynamics

$$\boldsymbol{\tau} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \ddot{\boldsymbol{q}} + \begin{bmatrix} 0 & C_{12} & C_{13} \\ C_{21} & 0 & C_{23} \\ C_{31} & C_{32} & 0 \end{bmatrix} \dot{\boldsymbol{q}} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} +$$

$$+ \begin{bmatrix} f_{d1} & 0 & 0 \\ 0 & f_{d2} & 0 \\ 0 & 0 & f_{d3} \end{bmatrix} sign(\dot{\boldsymbol{q}}) + \begin{bmatrix} f_{v1} & 0 & 0 \\ 0 & f_{v2} & 0 \\ 0 & 0 & f_{v3} \end{bmatrix} \dot{\boldsymbol{q}}$$

Centripetal Forces

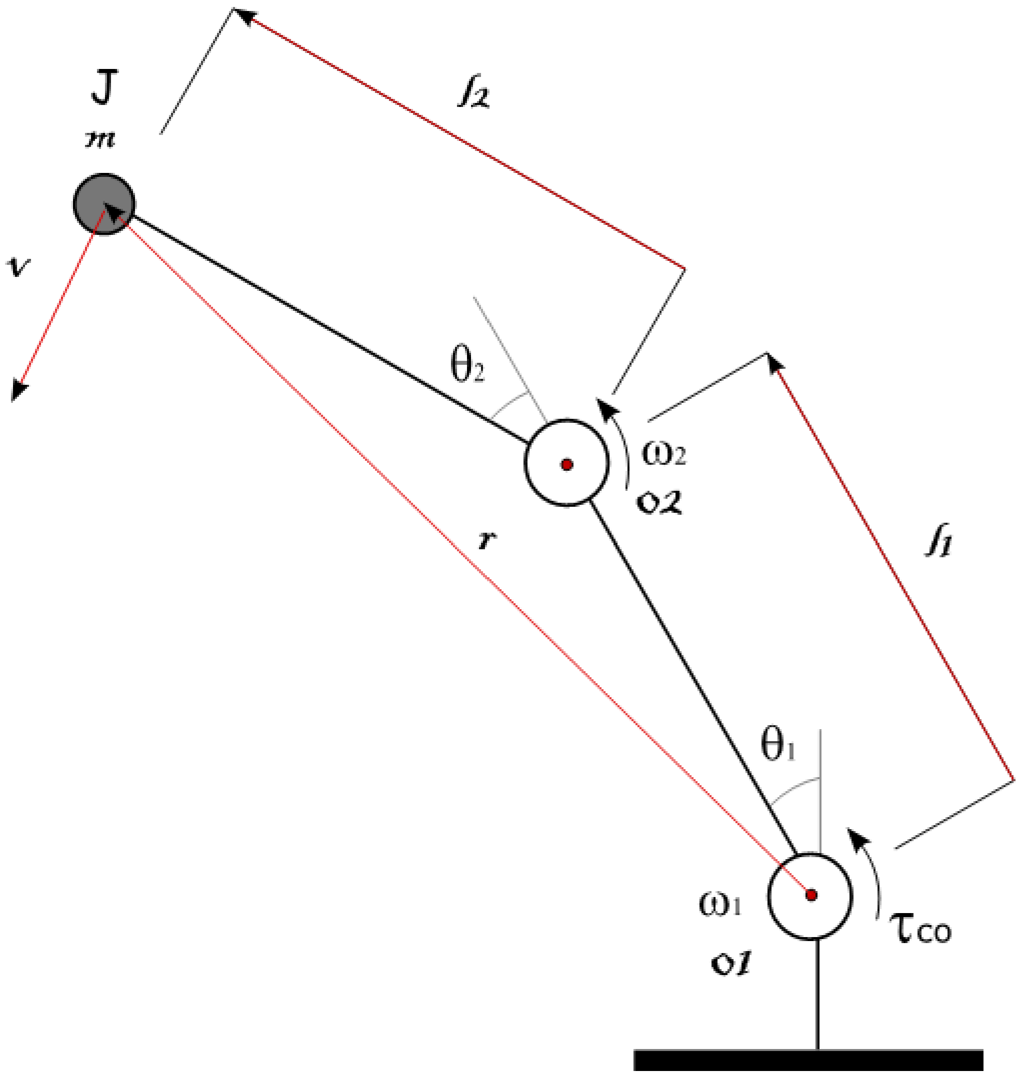


Dynamics

$$\boldsymbol{\tau} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \ddot{\boldsymbol{q}} + \boxed{\begin{bmatrix} 0 & C_{12} & C_{13} \\ C_{21} & 0 & C_{23} \\ C_{31} & C_{32} & 0 \end{bmatrix} \dot{\boldsymbol{q}}} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} +$$

$$+ \begin{bmatrix} f_{d1} & 0 & 0 \\ 0 & f_{d2} & 0 \\ 0 & 0 & f_{d3} \end{bmatrix} sign(\dot{\boldsymbol{q}}) + \begin{bmatrix} f_{v1} & 0 & 0 \\ 0 & f_{v2} & 0 \\ 0 & 0 & f_{v3} \end{bmatrix} \dot{\boldsymbol{q}}$$

Coriolis Forces

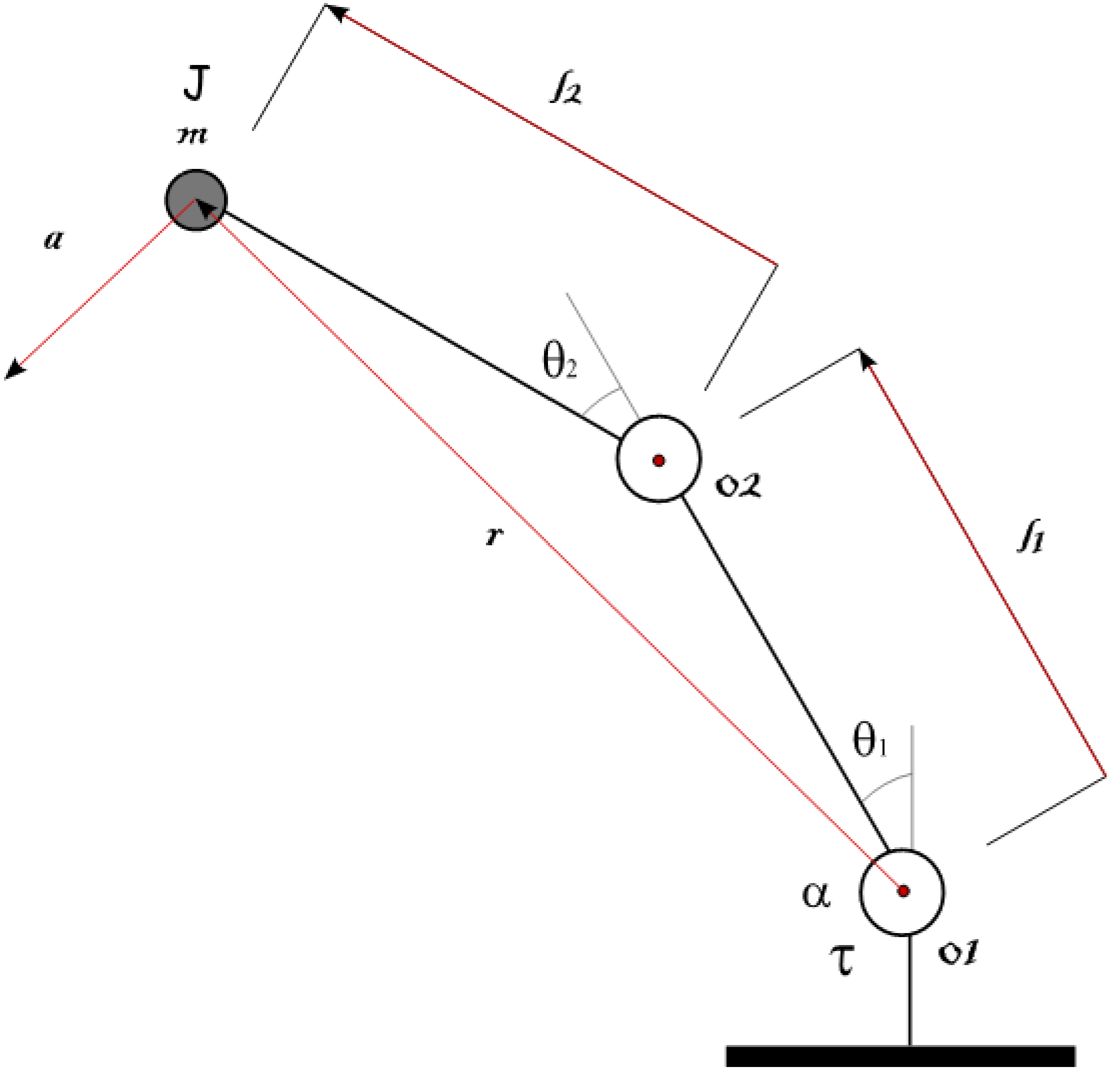


Dynamics

$$\boldsymbol{\tau} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \ddot{\boldsymbol{q}} + \boxed{\begin{bmatrix} 0 & C_{12} & C_{13} \\ C_{21} & 0 & C_{23} \\ C_{31} & C_{32} & 0 \end{bmatrix}} \dot{\boldsymbol{q}} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} +$$

$$+ \begin{bmatrix} f_{d1} & 0 & 0 \\ 0 & f_{d2} & 0 \\ 0 & 0 & f_{d3} \end{bmatrix} sign(\dot{\boldsymbol{q}}) + \begin{bmatrix} f_{v1} & 0 & 0 \\ 0 & f_{v2} & 0 \\ 0 & 0 & f_{v3} \end{bmatrix} \dot{\boldsymbol{q}}$$

"Inertial" Forces



Dynamics

$$\boldsymbol{\tau} = \boxed{\begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \ddot{\boldsymbol{q}}} + \begin{bmatrix} 0 & C_{12} & C_{13} \\ C_{21} & 0 & C_{23} \\ C_{31} & C_{32} & 0 \end{bmatrix} \dot{\boldsymbol{q}} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} +$$

$$+ \begin{bmatrix} f_{d1} & 0 & 0 \\ 0 & f_{d2} & 0 \\ 0 & 0 & f_{d3} \end{bmatrix} sign(\dot{\boldsymbol{q}}) + \begin{bmatrix} f_{v1} & 0 & 0 \\ 0 & f_{v2} & 0 \\ 0 & 0 & f_{v3} \end{bmatrix} \dot{\boldsymbol{q}}$$