EX. 1.5

$$T = X_1^2 - X_1 X_2 X_3 \;, \quad X_i \text{ v.a. i.i.d.} \;, \quad \mu = E(X_i)$$
$$\sigma^2 = Var(X_i)$$

$$E[T] = E[X_1^2 - X_1X_2X_3] \underset{\text{INDIP.}}{=} E[X_1^2] - E[X_1]\cdot E[X_2]\cdot E[$$

$$= Var(X_1) - \left(E[X_1]\right)^2 - E[X_1]\cdot E[X_2]\cdot E[X_3]$$

$$= \sigma^2 - \mu^2 - \mu^3 \neq \sigma^2$$

$\Rightarrow$ T è stimatore non corretto e la distorsione è pari a $\mu^2 + \mu^3$.

EX. 1.6

$$T = \frac{1}{2}X_1 + \frac{1}{4}X_2 + a X_3$$

$$E[T] = \frac{1}{2}E[X_1] + \frac{1}{4}E[X_2] + a E[X_3] \underset{\text{UNBIASED}}{=} \mu$$

$$\Rightarrow \frac{1}{2}\mu + \frac{1}{4}\mu + a\mu = \mu$$

$$\Rightarrow a = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\Rightarrow Var(T) = Var\left(\frac{1}{2}X_1 + \frac{1}{4}X_2 + \frac{1}{4}X_3\right) = \frac{6}{16}\sigma^2$$

Infine, $\hat{T} = \frac{1}{\cancel{2}}\cdot\overset{2}{\cancel{4}} + \frac{1}{\underset{2}{\cancel{4}}}\cdot\cancel{8} + \frac{1}{\underset{2}{\cancel{4}}}\cdot\overset{3}{\cancel{6}} = 2 + 2 =$