SCRITTO 19/6/2015 - TRACCIA A

EX. 1

$X \sim Bin(n,p)$, con $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$, $k=0,\dots,n$

$$E[X] = \sum_{k=0}^{n} k \cdot P(X=k) = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=0}^{n} k \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k} = \sum_{k=0}^{n} \cancel{k} \frac{n\,(n-1)!}{\cancel{k}\,(k-1)!\,(n-k)!} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} \frac{n\,(n-1)!}{(k-1)!\,(n-k)!} p^k (1-p)^{n-k} = n \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!\,[(n-1)-(k-1)]!} p^{k-1+1} (1-p)^{(n-1)-(k-1)}$$

$$= n \cdot p \cdot \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} (1-p)^{(n-1)-(k-1)}$$

Poniamo $k-1=J \Rightarrow k=J+1$

$$\Rightarrow E[X] = n \cdot p \sum_{J=0}^{n-1} \binom{n-1}{J} p^J (1-p)^{n-1-J} = n \cdot p \cdot (p+1-p)^{n-1} = n \cdot p$$

EX. 2

$$f(x,\theta)=\begin{cases}\frac{1}{2\theta\sqrt{x}}\,e^{-\frac{\sqrt{x}}{\theta}}, & x\geq 0\\ 0, & \text{altrimenti}\end{cases}$$

$$L=\prod_{i=1}^{n} f(x_i,\theta)=\prod_{i=1}^{n}\left[\frac{1}{2\theta\sqrt{x_i}}\,e^{-\frac{\sqrt{x_i}}{\theta}}\right]$$

$$\Rightarrow \ell=\ln(L)=\sum_{i=1}^{n}\ln\left(\frac{1}{2\theta\sqrt{x_i}}\,e^{-\frac{\sqrt{x_i}}{\theta}}\right)=\sum_{i=1}^{n}\left[-\ln(2)-\ln(\theta)+\right.$$

$$\left.-\frac{1}{2}\ln(x_i)-\frac{\sqrt{x_i}}{\theta}\right]$$

$$=-n\ln(2)-n\ln(\theta)-\frac{1}{2}\sum_{i=1}^{n}\ln(x_i)-\frac{1}{\theta}\sum_{i=1}^{n}\sqrt{x_i}$$

$$\Rightarrow \frac{d\ell}{d\theta}=-\frac{n}{\theta}+\frac{\sum_{i=1}^{n}\sqrt{x_i}}{\theta^2}=0$$

$$\Rightarrow \frac{-n\theta+\sum_{i=1}^{n}\sqrt{x_i}}{\theta^2}=0 \quad\Rightarrow -n\theta+\sum_{i=1}^{n}\sqrt{x_i}=0$$

$$\Rightarrow \hat{\theta}=\frac{\sum_{i=1}^{n}\sqrt{x_i}}{n}$$

Occorre verificare che $\hat{\theta}$ sia effettivamente un punto di massimo per la funzione di log-verosimiglianza:

$$\frac{d^2\ell}{d\theta^2}=+\frac{n}{\theta^2}+\frac{-2\cancel{\theta}\sum_{i=1}^{n}\sqrt{x_i}}{\theta^{\cancel{4}\,3}}=\frac{n}{\theta^2}-\frac{2\sum\sqrt{x_i}}{\theta^3}$$

$$=\frac{1}{\theta^2}\left(n-\frac{2\sum\sqrt{x_i}}{\theta}\right)<0$$

Infine: $\hat{\theta}=\dfrac{\sqrt{16}+\sqrt{25}}{2}=\dfrac{4+5}{2}=\dfrac{9}{2}$.

EX. 3

$U = 4R + 6V$

Indichiamo con

$R_i = \left\{ \begin{array}{l} \text{la pallina estratta è rossa} \\ \text{alla i-esima estrazione} \end{array} \right\}, \ i = 1, 2$

$V_i = \left\{ \begin{array}{l} \text{la pallina estratta è verde} \\ \text{alla i-esima estrazione} \end{array} \right\}, \ i = 1, 2$

$E = \left\{ \begin{array}{l} \text{una pallina è rossa, l'altra} \\ \text{è verde} \end{array} \right\}$

Allora:

$E = (R_1 \cap V_2) \cup (V_1 \cap R_2)$ (eventi incompatibili)

$\Rightarrow P(E) = P(R_1 \cap V_2) + P(V_1 \cap R_2) = P(V_2|R_1) \cdot P(R_1) + P(R_2|V_1) \cdot P(V_1)$

1° CASO: estrazione con reinserimento.

$\cancel{P(E) =}\ P(V_2|R_1) = P(V_2) = \frac{6}{10}$; $P(R_2|V_1) = P(R_2) = \frac{4}{10}$

$P(R_1) = \frac{4}{10}$, $P(V_1) = \frac{6}{10}$

$\Rightarrow P(E) = \frac{6}{10} \cdot \frac{4}{10} + \frac{4}{10} \cdot \frac{6}{10} = \frac{24}{100} \cdot 2 = \frac{\cancel{48}^{24}}{\cancel{100}_{50}} = 0,48$

2° CASO: estrazione senza reinserimento.

$P(R_1) = \frac{4}{10}$; $P(V_1) = \frac{6}{10}$; $P(R_2|V_1) = \frac{4}{9}$, $P(V_2|R_1) = \frac{6}{9}$

$\Rightarrow P(E) = \frac{\cancel{6}^{2}}{\cancel{9}_{3}} \cdot \frac{\cancel{4}^{2}}{\cancel{10}_{5}} + \frac{\cancel{4}^{2}}{\cancel{9}_{3}} \cdot \frac{\cancel{6}^{2}}{\cancel{10}_{5}} = \frac{4}{15} \cdot 2 = \frac{8}{15} = 0,53$

Ex. 4

Consideriamo i seguenti eventi.

$C = \{$uno studente scelto a caso è sostenitore di Carlo$\}$

$L = \{$uno studente scelto a caso è sostenitore di Luca$\}$

$I = \{$uno studente scelto a caso è sostenitore di Ivano$\}$

$V = \{$uno studente scelto a caso ha votato$\}$

Dai dati si ricava che:

$P(I) = 0,2$

$P(C) = 0,3$; $P(L) = 0,5$; $P(L|\bar{V}) = 0,305$

$P(V|C) = 0,65$; $P(V|L) = 0,82$;

$\Rightarrow P(V|I) = ?$

Dalla formula di Bayes, si ha

$$P(L|\bar{V}) = \frac{P(\bar{V}|L) \cdot P(L)}{P(\bar{V})}$$

e $\bar{V} = (\bar{V} \cap C) \cup (\bar{V} \cap L) \cup (\bar{V} \cap I)$ (eventi incompatibili)

$$\Rightarrow P(\bar{V}) = P(\bar{V} \cap C) + P(\bar{V} \cap L) + P(\bar{V} \cap I)$$
$$= P(\bar{V}|C) \cdot P(C) + P(\bar{V}|L) \cdot P(L) + P(\bar{V}|I) \cdot P(I)$$
$$= [1 - P(V|C)] \cdot P(C) + [1 - P(V|L)] \cdot P(L) + [1 - P(V|I)] \cdot P(I)$$

$$\Rightarrow P(L|\bar{V}) = \frac{(1 - P(V|L)) \cdot P(L)}{(1 - P(V|C)) \cdot P(C) + (1 - P(V|L)) \cdot P(L) + (1 - P(V|I)) \cdot P(I)}$$

$$\Rightarrow 0,305 = \frac{(1 - 0,82) \cdot 0,5}{(1 - 0,65) \cdot 0,3 + (1 - 0,82) \cdot 0,5 + (1 - P(V|I)) \cdot 0,2}$$

Poniamo $X := 1 - P(V|I)$

$$\Rightarrow 0{,}305 = \frac{0{,}18 \cdot 0{,}5}{0{,}35 \cdot 0{,}3 + 0{,}18 \cdot 0{,}5 + X \cdot 0{,}2}$$

$$\Rightarrow \frac{1}{0{,}305} = \frac{0{,}35 \cdot 0{,}3 + 0{,}18 \cdot 0{,}5 + X \cdot 0{,}2}{0{,}18 \cdot 0{,}5}$$

$$\Rightarrow X = \left[\frac{0{,}18 \cdot 0{,}5}{0{,}305} - 0{,}35 \cdot 0{,}3 - 0{,}18 \cdot 0{,}5\right] \cdot \frac{1}{0{,}2} = 0{,}5$$

$$\Rightarrow 1 - P(V|I) = 0{,}5 \Rightarrow P(V|I) = 1 - 0{,}5 = 0{,}5$$

SCRITTO 19/6/2015 - TRACCIA B

EX. 1

$X \sim Po(\lambda)$, $P(X=k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$, $k \geq 0$

$$E(X) = \sum_{k=0}^{+\infty} k \cdot P(X=k) = \sum_{k=0}^{+\infty} k \cdot \frac{e^{-\lambda}\lambda^k}{k!} = e^{-\lambda}\sum_{k=0}^{+\infty} \frac{k\lambda^k}{k(k-1)!}$$

$$= e^{-\lambda}\sum_{k=1}^{+\infty} \frac{\lambda^k}{(k-1)!} = e^{-\lambda}\sum_{k=1}^{+\infty} \frac{\lambda^{k-1+1}}{(k-1)!} = e^{-\lambda}\cdot\lambda\cdot\sum_{k=1}^{+\infty} \frac{\lambda^{k-1}}{(k-1)!}$$

Poniamo $k-1=J \Rightarrow k=J+1$

$$\Rightarrow E[X] = e^{-\lambda}\cdot\lambda\cdot\sum_{J=0}^{+\infty}\frac{\lambda^J}{J!} = e^{-\lambda}\cdot\lambda\cdot e^{\lambda} = \lambda.$$

EX. 2

$$f(x;\theta) = \begin{cases} \dfrac{2x}{\theta} e^{-\frac{x^2}{\theta}}, & x \geq 0 \\ 0, & \text{altrimenti} \end{cases}$$

$$L = \prod_{i=1}^{n} f(x_i;\theta) = \prod_{i=1}^{n}\left[\frac{2x_i}{\theta} e^{-\frac{x_i^2}{\theta}}\right]$$

$$\Rightarrow l = \ln(L) = \sum_{i=1}^{n} \ln\left(\frac{2x_i}{\theta} e^{-\frac{x_i^2}{\theta}}\right)$$

$$= \sum_{i=1}^{n}\left[\ln(2) + \ln(x_i) - \ln(\theta) - \frac{x_i^2}{\theta}\right]$$

$$= n\ln(2) + \sum_{i=1}^{n}\ln(x_i) - n\cdot\ln(\theta) - \frac{1}{\theta}\sum_{i=1}^{n} x_i^2$$

$$\Rightarrow \frac{dl}{d\theta} = -\frac{n}{\theta} + \frac{\sum x_i^2}{\theta^2} = 0$$

$$\Rightarrow \frac{-n\theta + \sum_{i=1}^{n} x_i^2}{\theta^2} = 0 \quad \Rightarrow -n\theta + \sum_{i=1}^{n} x_i^2 = 0$$

$$\Rightarrow \hat{\theta} = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

Inoltre:

$$\hat{\theta} = \frac{(-5)^2 + (-2)^2 + 1^2 + 2^2 + 5^2}{5} = \frac{25+4+1+4+25}{5} = \frac{59}{5} = 11,8$$

$\boxed{\text{EX. 3}}$

$U = 4R + 6V$

Consideriamo i seguenti eventi

$R_i = \left\{ \begin{array}{l} \text{la pallina estratta è} \\ \text{rossa alla i-sima estrazione} \end{array} \right\} \quad i = 1, 2$

$V_i = \left\{ \begin{array}{l} \text{la pallina estratta è verde} \\ \text{alla i-sima estrazione} \end{array} \right\} \quad i = 1, 2$

$E = \left\{ \begin{array}{l} \text{almeno una pallina} \\ \text{è rossa} \end{array} \right\}$

Allora:

$E = (R_1 \cap R_2) \cup (R_1 \cap V_2) \cup (V_1 \cap R_2)$ (eventi incompatibili)

$$\Rightarrow P(E) = P(R_1 \cap R_2) + P(R_1 + V_2) + P(V_1 \cap R_2)$$

$$= P(R_2 | R_1) \cdot P(R_1) + P(V_2 | R_1) \cdot P(R_1) + P(R_2 | V_1) \cdot P(V_1)$$

<u>1° caso</u>: estrazione con reinserimento

$P(R_2 | R_1) = P(R_2) = \frac{4}{10}$; $P(V_2 | R_1) = P(V_2) = \frac{6}{10}$,

$P(R_2 | V_1) = P(R_2) = \frac{4}{10}$; $P(R_1) = \frac{4}{10}$; $P(V_1) = \frac{6}{10}$

$$\Rightarrow P(E) = \frac{4}{10} \cdot \frac{4}{10} + \frac{6}{10} \cdot \frac{4}{10} + \frac{4}{10} \cdot \frac{6}{10} = \frac{16+24+24}{100} = \frac{64}{100} = 0,64$$

2° caso: estrazione senza reinserimento

$$P(R_1) = \frac{4}{10}, \; P(R_2|R_1) = \frac{3}{9}; \; P(V_1) = \frac{6}{10};$$

$$P(R_2|V_1) = \frac{6}{9}; \; P(V_2|R_1) = \frac{4}{9}$$

$$\Rightarrow P(E) = \frac{3}{9}\cdot\frac{4}{10} + \frac{4}{9}\cdot\frac{4}{10} + \frac{6}{9}\cdot\frac{6}{10} = \frac{12+16+36}{90}$$

$$= \frac{64}{90} = 0,71$$

$\boxed{EX.4}$

Consideriamo i seguenti eventi

$C = \{$studente sostenitore di Carlo$\}$; $I = \{$studente sostenitore di Ivano$\}$

$L = \{$studente sostenitore di Luca$\}$; $V = \{$studente ha votato$\}$

Dai dati si ricava che:

$$P(C) = 0,15; \; P(L) = 0,55; \; P(I) = 0,30$$

$$P(V|C) = 0,82; \; P(V|L) = 0,65; \; P(L|\overline{V}) = 0,63$$

$$\Rightarrow P(V|I) = ?$$

Dalla formula di Bayes:

$$P(L|\overline{V}) = \frac{P(\overline{V}|L)\cdot P(L)}{P(\overline{V})}$$

e $\overline{V} = (\overline{V}\cap C)\cup(\overline{V}\cap L)\cup(\overline{V}\cap I)$ (eventi incompatibili)

1

$$\Rightarrow P(\bar{V}) = P(\bar{V} \cap C) + P(\bar{V} \cap L) + P(\bar{V} \cap I)$$

$$= P(\bar{V} | C) \cdot P(C) + P(\bar{V} | L) \cdot P(L) + P(V | I) \cdot P(I)$$

$$= [1 - P(V|C)] \cdot P(C) + [1 - P(V|L)] \cdot P(L) + [1 - P(V|I)] \cdot P(I)$$

$$\cancel{\Rightarrow P(L|\bar{V}) =}$$

$$\Rightarrow 0{,}63 = \frac{(1 - 0{,}65) \cdot 0{,}55}{(1 - 0{,}82) \cdot 0{,}15 + (1 - 0{,}65) \cdot 0{,}55 + [1 - P(V|I)] \cdot 0{,}3}$$

$$\Rightarrow 1 - P(V|I) = \left[\frac{0{,}35 \cdot 0{,}55}{0{,}63} - 0{,}18 \cdot 0{,}15 - 0{,}35 \cdot 0{,}55 \right] \cdot \frac{1}{0{,}3}$$

$$\Rightarrow 1 - P(V|I) = 0{,}287 \Rightarrow P(V|I) = 0{,}713$$

I SCRITTO 19/6/2015 – TRACCIA e

$\boxed{EX. 1}$

$X \sim Exp(\mu)$, $f(x) = \mu e^{-\mu x}$, $x \geq 0$

$$E[X] = \int_{\mathbb{R}} x \cdot f(x)\,dx = \int_0^{+\infty} x\,\mu e^{-\mu x}\,dx = -\int_0^{+\infty} x\left(-\mu e^{-\mu x}\right)dx$$

Integriamo per parti:

$f(x) = x \Rightarrow f'(x) = 1$

$g'(x) = -\mu e^{-\mu x} \Rightarrow g(x) = e^{-\mu x}$

$$\Rightarrow E[X] = -\left[\cancel{x e^{-\mu x}}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-\mu x}\,dx\right] = \int_0^{+\infty} e^{-\mu x}\,dx$$

$$= -\frac{1}{\mu}\int_0^{+\infty} -\mu e^{-\mu x}\,dx = -\frac{1}{\mu}\left[e^{-\mu x}\right]_0^{+\infty} = \frac{1}{\mu}.$$

~~Var(X)~~

$\boxed{EX. 2}$

$$f(x,\theta) = \begin{cases} \dfrac{1}{18\theta^4} e^{-\frac{\sqrt{x}}{\theta^2}}, & x \geq 0 \\ 0, & \text{altrimenti} \end{cases}$$

$$L = \prod_{i=1}^{n} f(x_i;\theta) = \prod_{i=1}^{n}\left[\frac{1}{18\theta^4} e^{-\frac{\sqrt{x_i}}{\theta^2}}\right]$$

$$\Rightarrow \ell = \ln(L) = \sum_{i=1}^{n} \ln\left(\frac{1}{18\theta^4} e^{-\frac{\sqrt{x_i}}{\theta^2}}\right) =$$

$$= \sum_{i=1}^{n} \left[-\ln(18) - 4\ln(\theta) - \frac{\sqrt{x_i}}{\theta^2} \right]$$

$$= -n\ln(18) - 4n\ln(\theta) - \frac{1}{\theta^2} \sum_{i=1}^{n} \sqrt{x_i}$$

$$\Rightarrow \frac{d\ell}{d\theta} = -\frac{4n}{\theta} + \frac{2\cancel{\theta} \sum_{i=1}^{n} \sqrt{x_i}}{\theta^{\cancel{4}\,3}} = -\frac{4n}{\theta} + \frac{2\sum \sqrt{x_i}}{\theta^3} = 0$$

$$\Rightarrow \frac{-4n\theta^2 + 2\sum \sqrt{x_i}}{\theta^3} = 0 \Rightarrow \theta^2 = \frac{\cancel{2} \sum_{i=1}^{n} \sqrt{x_i}}{2\cancel{4}n}$$

$$\Rightarrow \hat{\theta} = \sqrt{\frac{\sum_{i=1}^{n} \sqrt{x_i}}{2n}}$$

$$\Rightarrow \hat{\theta} = \sqrt{\frac{4+5}{2 \cdot 2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$\boxed{\text{Ex. 3}}$

$U = 4R + 6V$

Consideriamo i seguenti eventi

$R_i = \{$ la pallina estratta è rossa, all'i-esima estrazione $\}$ $i = 1, 2$

$V_i = \{$ la pallina estratta è verde, all'i-esima estrazione $\}$ $i = 1, 2$

$E = \{$ almeno una pallina è verde $\}$

Allora:

$E = (V_1 \cap V_2) \cup (V_1 \cap R_2) \cup (R_1 \cap V_2)$ (eventi incompatibili)

$$\Rightarrow P(E) = P(V_1 \cap V_2) + P(V_1 \cap R_2) + P(R_1 \cap V_2)$$

$$= P(V_2 | V_1) \cdot P(V_1) + P(R_2 | V_1) \cdot P(V_1) + P(V_2 | R_1) \cdot P(R_1)$$

1° caso: estrazioni con reinserimento

$$P(V_1) = \frac{6}{10}\ ;\ P(R_1) = \frac{4}{10}\ ;\ P(V_2|V_1) = P(V_2) = \frac{6}{10},$$

$$P(R_2|V_1) = P(R_2) = \frac{4}{10},\ P(V_2|R_1) = P(V_2)$$

$$\Rightarrow P(E) = \frac{6}{10}\cdot\frac{6}{10} + \frac{4}{10}\cdot\frac{6}{10} + \frac{6}{10}\cdot\frac{4}{10}$$

$$= \frac{36+24+24}{100} = \frac{84}{100} = 0{,}84$$

2° caso: estrazioni senza reinserimento

$$P(V_1) = \frac{6}{10},\ P(R_1) = \frac{4}{10},\ P(V_2|V_1) = \frac{5}{9},$$

$$P(R_2|V_1) = \frac{4}{9},\ P(V_2|R_1) = \frac{6}{9}$$

$$\Rightarrow P(E) = \frac{5}{9}\cdot\frac{6}{10} + \frac{4}{9}\cdot\frac{6}{10} + \frac{6}{9}\cdot\frac{4}{10}$$

$$= \frac{30+24+24}{90} = \frac{78}{90} = 0{,}87$$

$\boxed{\text{EX. 4}}$

Consideriamo i seguenti eventi

$C = \{$studente sostenitore di Carlo$\}$, $I = \{$studente sostenitore di Ivano$\}$

$L = \{$studente sostenitore di Luca$\}$, $V = \{$studente ha votato$\}$

Dai dati si ricava che:

$$P(C)=0,3;\ P(L)=0,5;\ P(I)=0,2$$

$$P(V|I)=0,65;\ P(V|L)=0,82;\ P(I|\overline{V})=0,305$$

$$\Rightarrow P(V|C)=?$$

$$P(I|\overline{V})=\frac{P(\overline{V}|I)\cdot P(I)}{P(\overline{V})}$$

$$e\quad \overline{V}=(\overline{V}\cap C)\cup(\overline{V}\cap L)\cup(\overline{V}\cap I)\qquad \text{(eventi incompatibili)}$$

$$\Rightarrow P(\overline{V})=P(\overline{V}\cap C)+P(\overline{V}\cap L)+P(\overline{V}\cap I)$$

$$=P(\overline{V}|C)\cdot P(C)+P(\overline{V}|L)\cdot P(L)+P(\overline{V}|I)\cdot P(I)$$

$$=[1-P(V|C)]\cdot P(C)+[1-P(V|L)]\cdot P(L)+[1-P(V|I)]\cdot P(I)$$

$$\Rightarrow 0,305=\frac{(1-0,65)\cdot 0,2}{[1-P(V|C)]\cdot 0,3+(1-0,82)\cdot 0,5+(1-0,65)\cdot 0,2}$$

$$\Rightarrow 1-P(V|C)=\left[\frac{0,35\cdot 0,2}{0,305}-0,18\cdot 0,5-0,35\cdot 0,2\right]\cdot\frac{1}{0,3}$$

$$\Rightarrow 1-P(V|C)=0,232\Rightarrow P(V|C)=0,768$$

SCRITTO DEL 19/6/2015 – TRACCIA D

EX. 1

$$Var(X) = E(X^2) - (E(X))^2 \text{ . Infatti:}$$

$$Var(X) = E((X - E(X))^2] = E[X^2 + (E(X))^2 - 2X \cdot E(X)]$$

$$= E[X^2] + (E(X))^2 - 2E(X) \cdot E(X)$$

↓ Linearità della media

$$= E(X^2) + (E(X))^2 - 2(E(X))^2$$

$$= E(X^2) - (E(X))^2 .$$

EX. 2

$$f(x;\theta) = \begin{cases} \theta^2 (x-1) e^{-\theta(x-1)}, & x > 1 \\ 0, & \text{altrimenti} \end{cases}$$

$$L = \prod_{i=1}^{n} f(x_i, \theta) = \prod_{i=1}^{n} \left[\theta^2 (x_i - 1) e^{-\theta(x_i - 1)}\right]$$

$$\Rightarrow \ell = \ln(L) = \sum_{i=1}^{n} \left[2\ln(\theta) + \ln(x_i - 1) - \theta(x_i - 1)\right]$$

$$= 2n\ln(\theta) + \sum_{i=1}^{n} \ln(x_i - 1) - \theta \sum_{i=1}^{n} \ln(x_i - 1)$$

$$\Rightarrow \frac{d\ell}{d\theta} = \frac{2n}{\theta} - \sum_{i=1}^{n} \ln(x_i - 1) = 0$$

$$\Rightarrow \frac{2n}{\theta} = \sum_{i=1}^{n} \ln(x_i - 1) \Rightarrow \theta = \frac{2n}{\sum_{i=1}^{n} \ln(x_i - 1)}$$

$$\Rightarrow \hat{\theta} = \frac{2 \cdot 3}{\ln(1) + \ln(e) + \ln(e^2)} = \frac{6}{1+2} = 2$$

$\boxed{\text{EX. 3}}$

$\Omega = 4R + 6V$

Consideriamo i seguenti eventi

$R_i = \{$la pallina è rossa all'i-sima estrazione$\}$; $i = 1, 2$

$V_i = \{$la pallina è verde all'i-sima estrazione$\}$; $i = 1, 2$

$E_1 = \{$entrambe le palline sono rosse$\}$, $E_2 = \{$entrambe le palline sono verdi$\}$

<u>1° caso:</u> estrazione con reinserimento

$$E_1 = R_1 \cap R_2 \Rightarrow P(E_1) = P(R_1 \cap R_2) = P(R_2 | R_1) \cdot P(R_1)$$

$$= P(R_2) \cdot P(R_1) = \frac{4}{10} \cdot \frac{4}{10} = \frac{16}{100} = 0,16$$

<u>2° caso:</u> estrazione senza reinserimento

$$E_2 = V_1 \cap V_2 \Rightarrow P(E_2) = P(V_1 \cap V_2) = P(V_2 | V_1) \cdot P(V_1)$$

$$= \frac{5}{9} \cdot \frac{6}{10} = \frac{30}{90} = 0,33$$

EX. 4

Consideriamo i seguenti eventi:

$C = \left\{ \begin{matrix} \text{sostenitori di} \\ \text{Carlo} \end{matrix} \right\}$; $L = \left\{ \begin{matrix} \text{sostenitori di} \\ \text{Luca} \end{matrix} \right\}$; $I = \left\{ \begin{matrix} \text{sostenitori di} \\ \text{Ivano} \end{matrix} \right\}$

$V = \left\{ \begin{matrix} \text{rinuncia ha} \\ \text{votato} \end{matrix} \right\}$

Dai dati si ricava che:

$P(C) = 0{,}15$; $P(L) = 0{,}55$, $P(I) = 0{,}3$

$P(V|I) = 0{,}65$; $P(V|L) = 0{,}82$, $P(I|\overline{V}) = 0{,}45$

$\Rightarrow P(V|C) = ?$

Dalla formula di Bayes, si ha:

$$P(I|\overline{V}) = \frac{P(\overline{V}|I)\,P(I)}{P(\overline{V})}$$

e $\overline{V} = (\overline{V} \cap C) \cup (\overline{V} \cap L) \cup (\overline{V} \cap I)$

$$\Rightarrow P(\overline{V}) = P(\overline{V}|C) \cdot P(C) + P(\overline{V}|L) \cdot P(L) + P(\overline{V}|I) \cdot P(I)$$

$$= [1 - P(V|C)] \cdot P(C) + [1 - P(V|L)] \cdot P(L) + [1 - P(V|I)] \cdot P(I)$$

$$\Rightarrow 0{,}45 = \frac{(1 - 0{,}65) \cdot 0{,}3}{[1 - P(V|C)] \cdot 0{,}15 + (1 - 0{,}82) \cdot 0{,}55 + (1 - 0{,}65) \cdot 0{,}3}$$

$$\Rightarrow 1 - P(V|C) = \left[\frac{0{,}35 \cdot 0{,}3}{0{,}45} - 0{,}18 \cdot 0{,}55 - 0{,}35 \cdot 0{,}3 \right] \cdot \frac{1}{0{,}15}$$

$$\Rightarrow 1 - P(V|C) = 0{,}196 \Rightarrow P(V|C) = 0{,}804$$

I SCRITTO DEL 19/6/2015 - TRACCIA E

EX. 1

Lo stimatore della media è $\overline{X}$ (media campionaria) $\sim N\left(\mu, \frac{\sigma^2}{n}\right)$

La quantità pivotale è $Z := \frac{\overline{X} - E[\overline{X}]}{\sqrt{Var(\overline{X})}} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

Allora, l'intervallo di confidenza al livello α è t.c.

$$P\left(-z_{1-\frac{\alpha}{2}} \leq Z \leq z_{1-\frac{\alpha}{2}}\right) = 1-\alpha$$

$$\Rightarrow P\left[-z_{1-\frac{\alpha}{2}} \leq \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{1-\frac{\alpha}{2}}\right] = 1-\alpha$$

$$\Rightarrow P\left[-\frac{\sigma}{\sqrt{n}} \cdot z_{1-\frac{\alpha}{2}} \leq \overline{X} - \mu \leq \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}}\right] = 1-\alpha$$

$$\Rightarrow P\left[-\overline{X} - z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq -\mu \leq -\overline{X} + z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right] = 1-\alpha$$

$$\Rightarrow I.C. = \left[\overline{X} + z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} - z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right].$$

EX. 2

$$E[T_1] = \frac{1}{6} E[X_1] + \frac{3}{4} E[X_2] + a[X_3] = m$$

$$\Rightarrow \frac{1}{6} m + \frac{3}{4} m + a \cdot m = m$$

$$\Rightarrow a = 1 - \frac{1}{6} - \frac{3}{4} = \frac{12-2-9}{12} = \frac{1}{12}$$

$$E[T_2] = \frac{2}{3}E(X_1) + \frac{5}{7}E(X_2) + b\cdot E(X_3) = m$$

$$\Rightarrow \frac{2}{3}m + \frac{5}{7}m + b\cdot m = m$$

$$\Rightarrow b = 1 - \frac{2}{3} - \frac{5}{7} = \frac{21-14-15}{21} = -\frac{8}{21}$$

$$E[T_3] = \frac{1}{2}E(X_1) + \frac{1}{4}E(X_2) + c\cdot E(X_3) = m$$

$$\Rightarrow \frac{1}{2}m + \frac{1}{4}m + c\cdot m = m$$

$$\Rightarrow c = 1 - \frac{1}{2} - \frac{1}{4} = \frac{4-2-1}{4} = \frac{1}{4}$$

Calcoliamo le varianze:

$$Var(T_1) = \frac{1}{36}Var(X_1) + \frac{9}{16}Var(X_2) + \frac{1}{144}Var(X_3)$$

$$= \frac{4s^2 + 81s^2 + s^2}{144} = \frac{86}{144}s^2$$

$$Var(T_2) = \frac{4}{9}Var(X_1) + \frac{25}{49}Var(X_2) + \frac{64}{441}Var(X_3)$$

$$= \frac{196s^2 + 225s^2 + 64s^2}{441} = \frac{485}{441}s^2$$

$$Var(T_3) = \frac{1}{4}Var(X_1) + \frac{1}{16}Var(X_2) + \frac{1}{16}Var(X_3)$$

$$= \frac{4+1+1}{16}s^2 = \frac{6}{16}s^2$$

$\Rightarrow T_3$ è lo stimatore più efficiente

$$\Rightarrow \hat{T}_3 = \frac{1}{\cancel{2}}\cdot\cancel{4}^{2} + \frac{1}{2\cancel{4}}\cdot\cancel{8} + \frac{1}{\cancel{4}2}\cdot\cancel{6}^{3} = 4.$$

$\boxed{\text{Ex. 3}}$

$$1 = \int_{\mathbb{R}} f(x)\,dx = \int_0^2 (x - Cx^3)\,dx = \frac{x^2}{2}\Big|_0^2 - C\frac{x^4}{4}\Big|_0^2$$

$$\Rightarrow \frac{\cancel{4}^{\,2}}{\cancel{2}} - C\cdot\frac{\cancel{16}^{\,4}}{\cancel{4}} = 1 \Rightarrow 2 - 4C = 1 \Rightarrow 4C = 1 \Rightarrow C = \frac{1}{4}$$

$$\mathrm{Var}(X) = E(X^2) - (E(X))^2$$

$$E(X) = \int_0^2 x\left(x - \frac{1}{4}x^3\right)dx = \int_0^2 \left(x^2 - \frac{1}{4}x^4\right)dx$$

$$= \left[\frac{x^3}{3} - \frac{1}{4}\frac{x^5}{5}\right]_0^2 = \frac{8}{3} - \frac{1}{4}\cdot\frac{\cancel{32}^{\,8}}{5} = \frac{8}{3} - \frac{8}{5}$$

$$= \frac{40 - 24}{15} = +\frac{16}{15}.$$

$$E(X^2) = \int_0^2 x^2\left(x - \frac{1}{4}x^3\right)dx = \int_0^2 \left(x^3 - \frac{1}{4}x^5\right)dx$$

$$= \left[\frac{x^4}{4} - \frac{1}{4}\frac{x^6}{6}\right]_0^2 = \frac{16}{4} - \frac{1}{4}\cdot\frac{\cancel{64}^{\,16}}{6} = \frac{48 - 32}{12} = \frac{\cancel{16}^{\,4}}{\cancel{12}_{\,3}}$$

$$\Rightarrow \mathrm{Var}(X) = \frac{4}{3} - \left(\frac{16}{15}\right)^2 = \frac{4}{3} - \frac{256}{225} = \frac{300 - 256}{225} = \frac{44}{225}$$

EX. 4

Consideriamo i seguenti eventi:

$A_0 = \{\text{emesso } \alpha\}$; $A_1 = \{\text{ricevuto } \alpha\}$;

$B_0 = \{\text{emesso } \beta\}$; $B_1 = \{\text{ricevuto } \beta\}$.

Dai dati si ricava che:

$P(A_0) = 0{,}4$; $P(B_0) = 0{,}6$

(i) $P(B_1 | A_0) = P(A_1 | B_0) = 0{,}25$

$\Rightarrow A_1 = (A_0 \cap A_1) \cup (B_0 \cap A_1)$ (eventi incompatibili)

$\Rightarrow P(A_1) = P(A_0 \cap A_1) + P(B_0 \cap A_1) = P(A_1 | A_0) \cdot P(A_0) +$
$+ P(A_1 | B_0) \cdot P(B_0)$

$= (1 - 0{,}25) \cdot 0{,}4 + 0{,}25 \cdot 0{,}6 = 0{,}45$.

(ii) $P(B_1 | B_0) = 0{,}9$; $P(A_1 | A_0) = 0{,}8$

$E = (A_0 \cap B_1) \cup (B_0 \cap A_1)$ (eventi incompatibili)

$\Rightarrow P(E) = P(B_1 | A_0) \cdot P(A_0) + P(A_1 | B_0) \cdot P(B_0)$

$= [1 - P(A_1 | A_0)] \cdot P(A_0) + [1 - P(B_1 | B_0)] \cdot P(B_0)$

$= 0{,}2 \cdot 0{,}4 + 0{,}1 \cdot 0{,}6 = 0{,}14$

SCRITTO DEL 19/6/2015 - TRACCIA F

$\boxed{\text{EX. 1}}$

Vedi lezione 6

$\boxed{\text{EX. 2}}$

$$E[T_1] = \frac{4}{9}\mu + \frac{5}{6}\mu + a\cdot\mu = \mu$$

$$\Rightarrow a = 1 - \frac{4}{9} - \frac{5}{6} = \frac{18-8-15}{18} = \frac{-5}{18}$$

$$E[T_2] = \frac{5}{8}\mu + \frac{1}{4}\mu + b\cdot\mu = \mu$$

$$\Rightarrow b = 1 - \frac{5}{8} - \frac{1}{4} = \frac{8-5-2}{8} = \frac{1}{8}$$

$$E[T_3] = \frac{2}{3}\mu + \frac{1}{4}\mu + c\cdot\mu = \mu$$

$$\Rightarrow c = 1 - \frac{2}{3} - \frac{1}{4} = \frac{12-8-3}{12} = \frac{1}{12}$$

Calcoliamo le varianze:

$$Var(T_1) = \frac{16}{81}\sigma^2 + \frac{25}{36}\sigma^2 + \frac{25}{324}\sigma^2 = \frac{64+225+25}{324}\sigma^2 = \frac{314}{324}\sigma^2$$

$$Var(T_2) = \frac{25}{64}\sigma^2 + \frac{1}{16}\sigma^2 + \frac{1}{64}\sigma^2 = \frac{25+4+1}{64}\sigma^2 = \frac{30}{64}\sigma^2$$

$$Var(T_3) = \frac{4}{9}\sigma^2 + \frac{1}{16}\sigma^2 + \frac{1}{144}\sigma^2 = \frac{64+9+1}{144}\sigma^2 = \frac{74}{144}\sigma^2$$

$\Rightarrow$ lo stimatore più efficiente è T_2 $\Rightarrow \hat{T}_2 = \frac{5}{2\cancel{8}}\cdot\cancel{4} + \frac{1}{2\cancel{4}}\cdot 8 + \frac{1}{2\cancel{12}}\cdot\cancel{6} = \frac{7}{2}$

Ex. 3

$$1 = \int_{\mathbb{R}} f(x)\,dx = \int_1^e C \cdot \log(x)\,dx = C \int_1^e \log(x)\,dx$$

Integrazione per parti:

$$f(x) = \log(x) \Rightarrow f'(x) = \frac{1}{x}$$

$$g'(x) = 1 \Rightarrow g(x) = x$$

$$\Rightarrow 1 = C \cdot \left[x \cdot \log(x) \Big|_1^e - \int_1^e x \cdot \frac{1}{x}\,dx \right]$$

$$= C \cdot \left[e - e + 1 \right] = C \Rightarrow C = 1.$$

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \int_1^x \log(t)\,dt$$

$$= \left[t \cdot \log(t) \Big|_1^x - \int_1^x dt \right] = x \cdot \log(x) - x - 1.$$

Ex. 4

Consideriamo gli eventi:

$A_0 = \{\text{emesso } \alpha\}$; $B_0 = \{\text{emesso } \beta\}$; $A_1 = \{\text{ricevuto } \alpha\}$; $B_1 = \{\text{ricevuto } \beta\}$

Dai dati si ricava che:

$P(A_0) = 0{,}4$; $P(B_0) = 0{,}6$; $P(A_1 | B_0) = P(B_1 | A_0) = 0{,}25$.

i) $\Rightarrow P(B_1) = P\left((B_0 \cap B_1) \cup (A_0 \cap B_1)\right) = P(B_0 \cap B_1) + P(A_0 \cap B_1)$

$$= P(B_1 | B_0) \cdot P(B_0) + P(B_1 | A_0) \cdot P(A_0)$$

$$= 0{,}75 \cdot 0{,}6 + 0{,}25 \cdot 0{,}4 = 0{,}55.$$

ii) $P(B_1 | B_0) = 0{,}7$; $P(A_1 | A_0) = 0{,}3$

$$\Rightarrow P(E) = P\left((A_0 \cap B_1) \cup (B_0 \cap A_1)\right) = P(B_1 | A_0) \cdot P(A_0) + P(A_1 | B_0) \cdot P(B_0)$$

$$= 0{,}7 \cdot 0{,}4 + 0{,}3 \cdot 0{,}6 = 0{,}46$$

I SCRITTO DEL 19/6/2015 - TRACCIA G

$\boxed{\text{EX. 1}}$ Vedi lezione 5, lezione 6.

$\boxed{\text{EX. 2}}$

$$E(T_1) = \frac{1}{8}\mu + \frac{3}{4}\mu + a\cdot\mu = \mu$$

$$\Rightarrow a = 1 - \frac{1}{8} - \frac{3}{4} = \frac{8-1-6}{8} = \frac{1}{8}$$

$$E(T_2) = \frac{2}{7}\mu + \frac{1}{5}\mu + b\cdot\mu = \mu$$

$$\Rightarrow b = 1 - \frac{2}{7} - \frac{1}{5} = \frac{35-10-7}{35} = \frac{18}{35}$$

$$E(T_3) = \frac{3}{8}\mu + \frac{4}{7}\mu + c\cdot\mu = \mu$$

$$\Rightarrow c = 1 - \frac{3}{8} - \frac{4}{7} = \frac{56-21-32}{56} = \frac{3}{56}$$

Calcoliamo le varianze:

$$Var(T_1) = \frac{1}{64}S^2 + \frac{9}{16}S^2 + \frac{1}{64}S^2 = \frac{1+36+1}{64}S^2 = \frac{38}{64}S^2$$

$$Var(T_2) = \frac{4}{49}S^2 + \frac{1}{25}S^2 + \frac{324}{1225}S^2 = \frac{100+49+324}{1225}S^2 = \frac{473}{1225}S^2$$

$$Var(T_3) = \frac{9}{64}S^2 + \frac{28}{49}S^2 + \frac{9}{3136}S^2 = \frac{441+1792+9}{3136}S^2 = \frac{2242}{3136}S^2$$

$\Rightarrow$ lo stimatore più efficiente è T_2

$$\Rightarrow \hat{T}_2 = \frac{2}{7}\cdot 4 + \frac{1}{5}\cdot 2 + \frac{18}{35}\cdot 6 = \frac{40+14+108}{35} = \frac{162}{35}$$

$\boxed{\text{Ex. 3}}$

$$1 = \int_{\mathbb{R}} f(x)\,dx = c\int_0^2 (x-1)^2\,dx = c\int_0^2 (x^2+1-2x)\,dx$$

$$= c\left[\frac{x^3}{3} + x - \frac{\cancel{2}x^2}{\cancel{2}}\right]_0^2 = c\left[\frac{8}{3}+2-4\right]$$

$$\Rightarrow c\cdot\frac{2}{3} = 1 \Rightarrow c = \frac{3}{2}$$

$$E(X) = \frac{3}{2}\int_0^2 x(x-1)^2\,dx = \frac{3}{2}\int_0^2 x(x^2+1-2x)\,dx$$

$$= \frac{3}{2}\int_0^2 (x^3+x-2x^2)\,dx = \frac{3}{2}\left[\frac{x^4}{4}+\frac{x^2}{2}-\frac{2x^3}{3}\right]_0^2$$

$$= \frac{3}{2}\cdot\left[\frac{\cancel{16}^{4}}{\cancel{4}} + \frac{\cancel{4}^{2}}{\cancel{2}} - \frac{2\cdot 8}{3}\right] = \frac{\cancel{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}} = 1$$

$$E(X^2) = \frac{3}{2}\int_0^2 x^2(x-1)^2\,dx = \frac{3}{2}\int_0^2 x^2(x^2+1-2x)\,dx$$

$$= \frac{3}{2}\int_0^2 (x^4+x^2-2x^3)\,dx = \frac{3}{2}\left[\frac{x^5}{5}+\frac{x^3}{3}-\frac{\cancel{2}x^4}{\cancel{4}2}\right]_0^2$$

$$= \frac{3}{2}\left[\frac{32}{5}+\frac{8}{3}-\frac{\cancel{16}^{8}}{\cancel{2}}\right] = \frac{3}{2}\cdot\frac{96+40-120}{\cancel{15}\,5} = \frac{16}{5}$$

~~$P(2 \leq X \leq 4) = 1 - P(X \leq 2, X \geq 4) = 1 - P(|X-3| \geq 1)$~~

~~d'altro canto, $P(|X-3| \geq 1) \leq \frac{Var(X)}{1^2}$~~

~~dove $Var(X) = \frac{16}{5} - 9 =$~~

$P(2 \leq X \leq 4) = 0$, poiché $f(x) = 0,\ \forall x > 2$.

EX. 4

Consideriamo i seguenti eventi:

$A_0 = \{\text{emesso } \alpha\}$; $B_0 = \{\text{emesso } \beta\}$; $A_1 = \{\text{ricevuto } \alpha\}$; $B_1 = \{\text{ricevuto } \beta\}$

$P(A_0) = 0{,}45$, $P(B_0) = 0{,}55$.

(i) $P(A_1|B_0) = P(B_1|A_0) = 0{,}3$

$$\Rightarrow P(B_1) = P(B_1 \cap A_0) + P(B_1 \cap B_0)$$
$$= P(B_1|A_0)\cdot P(A_0) + P(B_1|B_0)\cdot P(B_0)$$
$$= 0{,}3 \cdot 0{,}45 + 0{,}7 \cdot 0{,}55 = 0{,}52$$

(ii) $P(A_1|A_0) = 0{,}8$; $P(B_1|B_0) = 0{,}9$

$$\Rightarrow P(E) = P(A_1 \cap B_0) + P(B_1 \cap A_0) = P(A_1|B_0)\cdot P(B_0) +$$
$$+ P(B_1|A_0)\cdot P(A_0)$$
$$= 0{,}1 \cdot 0{,}55 + 0{,}2 \cdot 0{,}45 = 0{,}145$$

SCRITTO DEL 19/6/2015 - TRACCIA 4

$\boxed{EX. 1}$

$X \sim N(\mu, \sigma^2)$, $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$$E(X) = \int_{\mathbb{R}} x \cdot f(x)\, dx = \int_{\mathbb{R}} x \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Poniamo $\frac{x-\mu}{\sigma} = y \Rightarrow x = \sigma y + \mu \Rightarrow dx = \sigma\, dy$

$$\Rightarrow E(X) = \int_{\mathbb{R}} (\sigma y + \mu) \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-\frac{y^2}{2}} \cancel{\sigma}\, dy = \underbrace{\sigma \int_{\mathbb{R}} y \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy}_{=0} +$$

$$+ \mu \underbrace{\int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy}_{=1} = \mu.$$

$\boxed{EX. 2}$

$E(T_1) = \frac{1}{6} m + a\, m + \frac{3}{4} m = m \Rightarrow a = 1 - \frac{1}{6} - \frac{3}{4} = \frac{1}{12}$

$E(T_2) = \frac{2}{3} m + b \cdot m + \frac{5}{7} m = m \Rightarrow b = 1 - \frac{2}{3} - \frac{5}{7} = -\frac{8}{21}$

$E(T_3) = \frac{1}{2} m + c \cdot m + \frac{1}{4} m = m \Rightarrow c = \frac{1}{4}$

Calcoliamo le varianze:

$Var(T_1) = \frac{86}{144} s^2$, $Var(T_2) = \frac{485}{441} s^2$, $Var(T_3) = \frac{6}{16} s^2$

$\Rightarrow$ lo stimatore più efficiente è T_3

$$\Rightarrow \hat{T}_3 = \frac{1}{2} \cdot 4 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 6 = 4$$

EX 3

$$1 = \int_{\mathbb{R}} x \cdot f(x) = \int_1^2 \frac{C}{x^2}\,dx = C\int_1^2 x^{-2}\,dx$$

$$= C \cdot \left[\frac{x^{-1}}{-1}\right]_1^2 = -\left[\frac{C}{x}\right]_1^2 = -C\left[\frac{1}{2} - 1\right] = C \cdot \frac{1}{2}$$

$$\Rightarrow C = 2.$$

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt = \int_1^x \frac{2}{t^2}\,dt$$

$$= 2\int_1^x t^{-2}\,dt = 2\left[-t^{-1}\right]_1^x = 2\left[1 - \frac{1}{x}\right]$$

$$\cancel{\Rightarrow P(X \le \pi/4) = 2\left(1 - \frac{1}{x}\right)\Big|_{x=\pi/4} = 2\left(1 - \frac{4}{\pi}\right)}$$

$$\Rightarrow P\left(X \le \frac{\pi}{4}\right) = 0, \text{ poiché } \frac{\pi}{4} < 1.$$

EX. 4

Consideriamo i seguenti eventi: $A_1 = \{\text{ricevuto } \alpha\}$; $B_1 = \{\text{ricevuto } \beta\}$

$A_0 = \{\text{emesso } \alpha\}$, $B_0 = \{\text{emesso } \beta\}$;

$P(A_0) = 0{,}45$; $P(B_0) = 0{,}55$.

(i) $P(A_1 | B_0) = P(B_1 | A_0) = 0{,}25$

$P(A_1) = P(A_1 \cap B_0) + P(A_1 \cap A_0) = P(A_1|B_0) \cdot P(B_0) + P(A_1|A_0) \cdot P(A_0)$

$= 0{,}25 \cdot 0{,}55 + 0{,}75 \cdot 0{,}45 = 0{,}475$

(ii) $P(A_1|A_0) = 0{,}3$; $P(B_1|B_0) = 0{,}7$

$\Rightarrow P(E) = P(A_1 \cap B_0) + P(B_1 \cap A_0) = P(A_1|B_0) \cdot P(B_0) + P(B_1|A_0) \cdot P(A_0)$

$= 0{,}3 \cdot 0{,}55 + 0{,}7 \cdot 0{,}45 = 0{,}48$