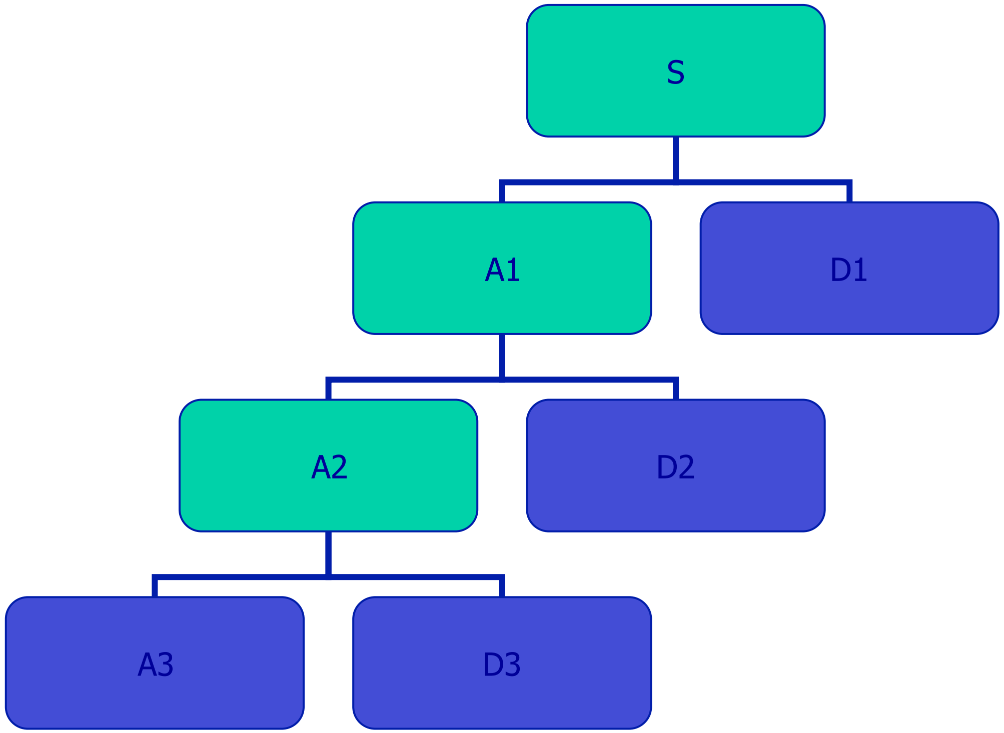
Wavelet Packets

Motivation

- Goal
  - Get minimal representation of data relative to particular cost function
- Usage
  - Data compression
  - Noise reduction

Wavelet Transform

- Wavelet transform is applied to low pass results (approximations) only:

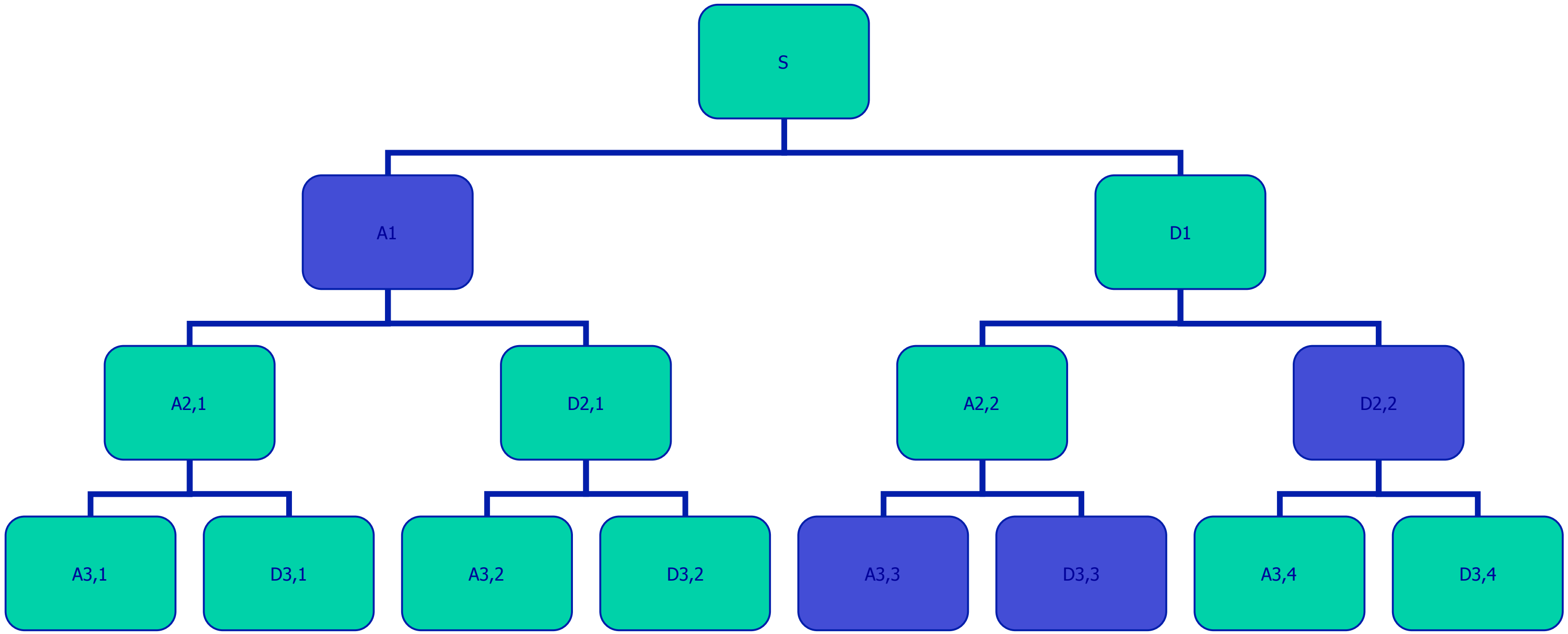


Not optimal

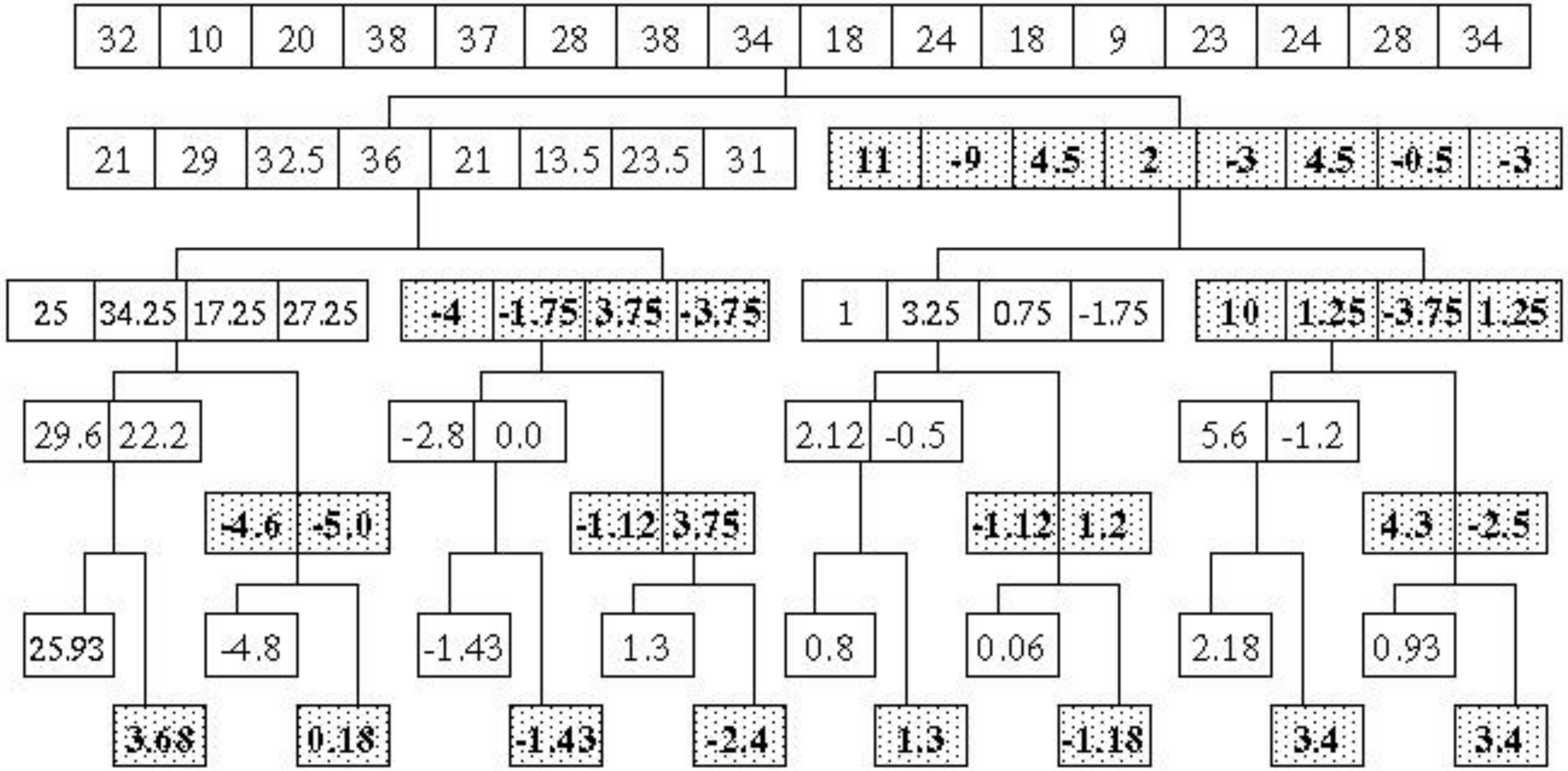
- From the point of view of compression, where we want as many small values as possible, the standard wavelet transform may not produce the best result, since it is limited to wavelet bases (the plural of basis) that increase by a power of two with each step.
- It could be that another combination of bases produce a more desirable representation.

Wavelet Packet Transform

Wavelet packet transform is applied to both low pass results (approximations) and high pass results (details)



Wavelet Packet Transform example (Haar)



Wavelet Packet Tree

Best basis

- The best basis algorithm finds a set of wavelet bases that provide the most desirable representation of the data relative to a particular cost function.
- A cost function may be chosen to fit a particular application.
- For example, in a compression algorithm the cost function might be the number of bits needed to represent the result.

Cost function

- The value of the cost function is a real number.
- Given two vectors of finite length, **a** and **b**, we denote their concatenation by [**a** **b**]. This vector simply consists of the elements in **a** followed by the elements in **b**.
- We require the following two properties:
  - The cost function is additive in the sense that $K([\mathbf{a}\ \mathbf{b}]) = K(\mathbf{a}) + K(\mathbf{b})$ for all finite length vectors **a** and **b**.
  - $K(\mathbf{0}) = 0$, where **0** denotes the zero vector

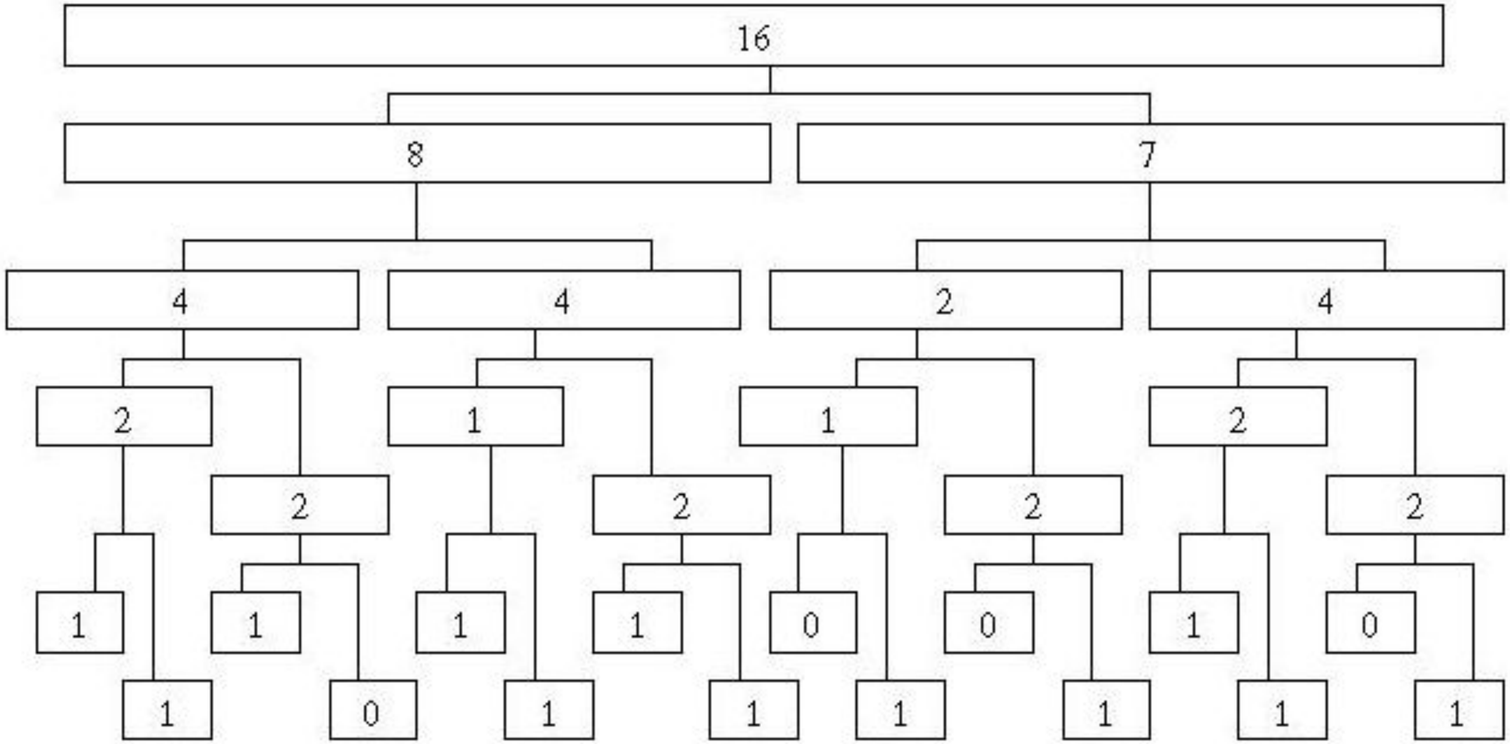
Cost functions: threshold

- The threshold cost function counts the number of *values* in a wavelet packet tree node whose absolute value is greater than a threshold value ***t***.

$$\text{cost}_{\text{threshold}} = \sum_{i=0}^{N-1} (|s[i]| > t)\ ?\ 1 : 0;$$

➔ Promoting sparsity!

Threshold function (t = 1)

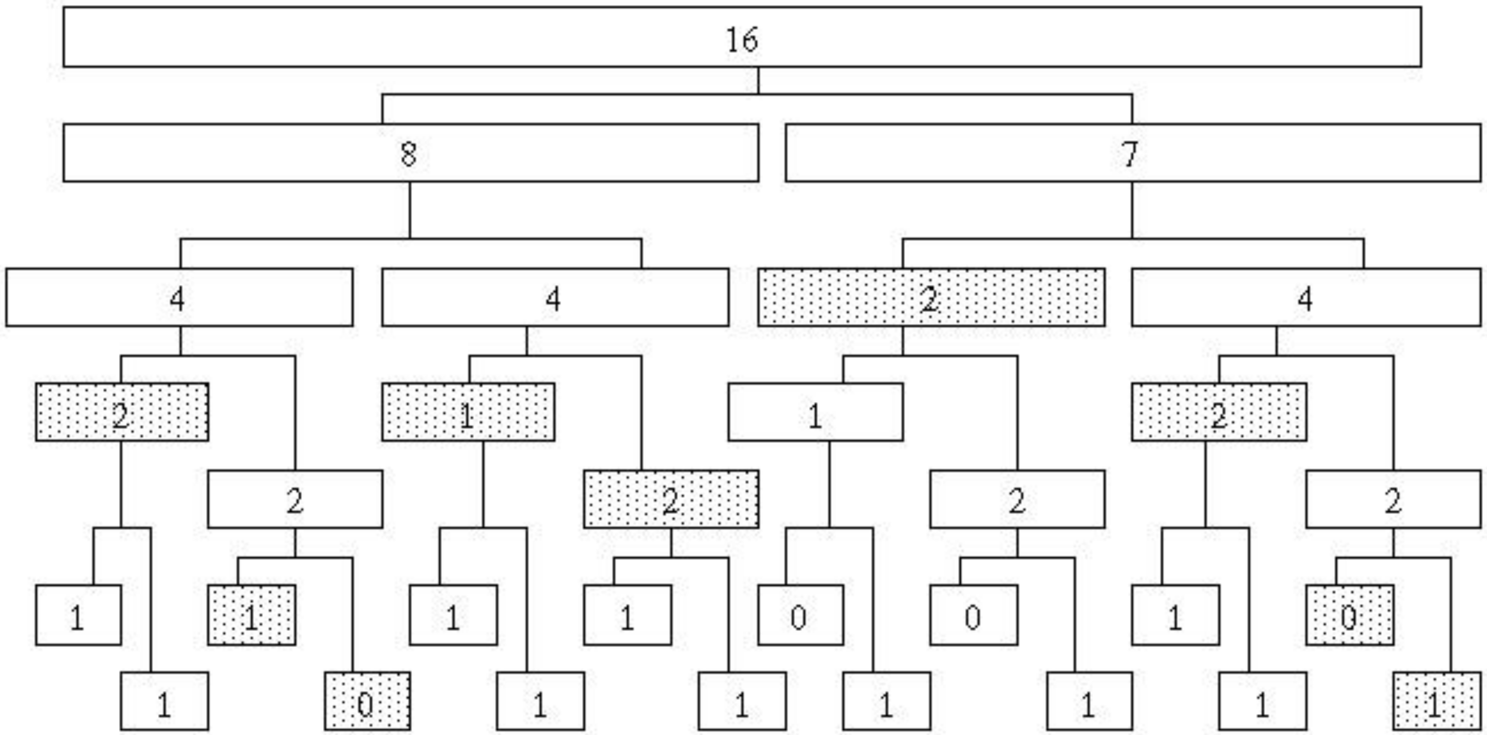


Wavelet packet tree showing cost function result

Best basis algorithm

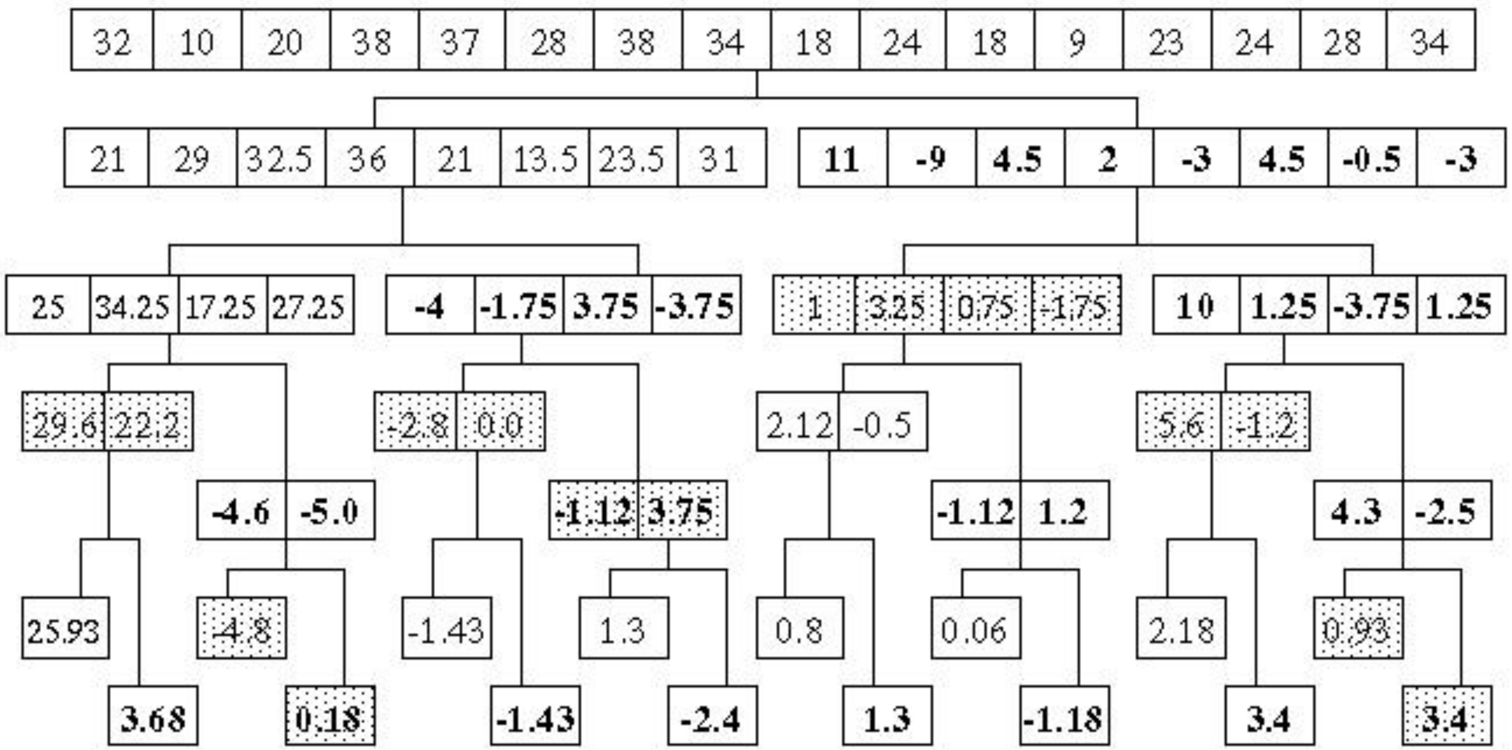
- Compute cost value for each node
- When the wavelet packet tree is constructed, all the leaves are marked with a flag. The best basis calculation is performed bottom up (that is, from the leaves of the tree toward the root):
  - A leaf (a node at the bottom of the tree with no children) returns its cost value.
  - As the calculation recurses up the tree toward the root, if there is a non-leaf node, **v1** is the cost value for that node. The value **v2** is the sum of the cost values of the children of the node.
    - If (**v1** <= **v2**) then we mark the node as part of the best basis set and remove any marks in the nodes in the sub-tree of the current node.
    - If (**v1** > **v2**) then the cost value of the node is replaced with v2.

Best basis (threshold 1)



Wavelet packet tree "best basis"

Best basis (threshold 1)



"best basis" = {29.6, 22.2, -4.8, 0.18, -2.8, 0.0, -1.23, 3.75, 1.0, 3.25, 0.75, -1.75, 5.6, -1.25, 0.93, 3.43}

Best basis contd.

- The best basis set selected by the best basis algorithm is relative to a particular cost function.
- In some cases the best basis set may be the same set yielded by the wavelet transform (in which case we could have used the simpler algorithm).
- In other cases the best basis function may not yield a result that differs from the original data set (e.g., the original data set is already a minimal representation in terms of the cost function).

Other cost functions

- Nonnormalized Shannon (0 log(0)=0)

$$\text{cost}_{shannon} = -\sum_n s[n]^2 \log(s[n]^2)$$

  – The Shannon entropy function provides a measure of the economy of representation of a signal

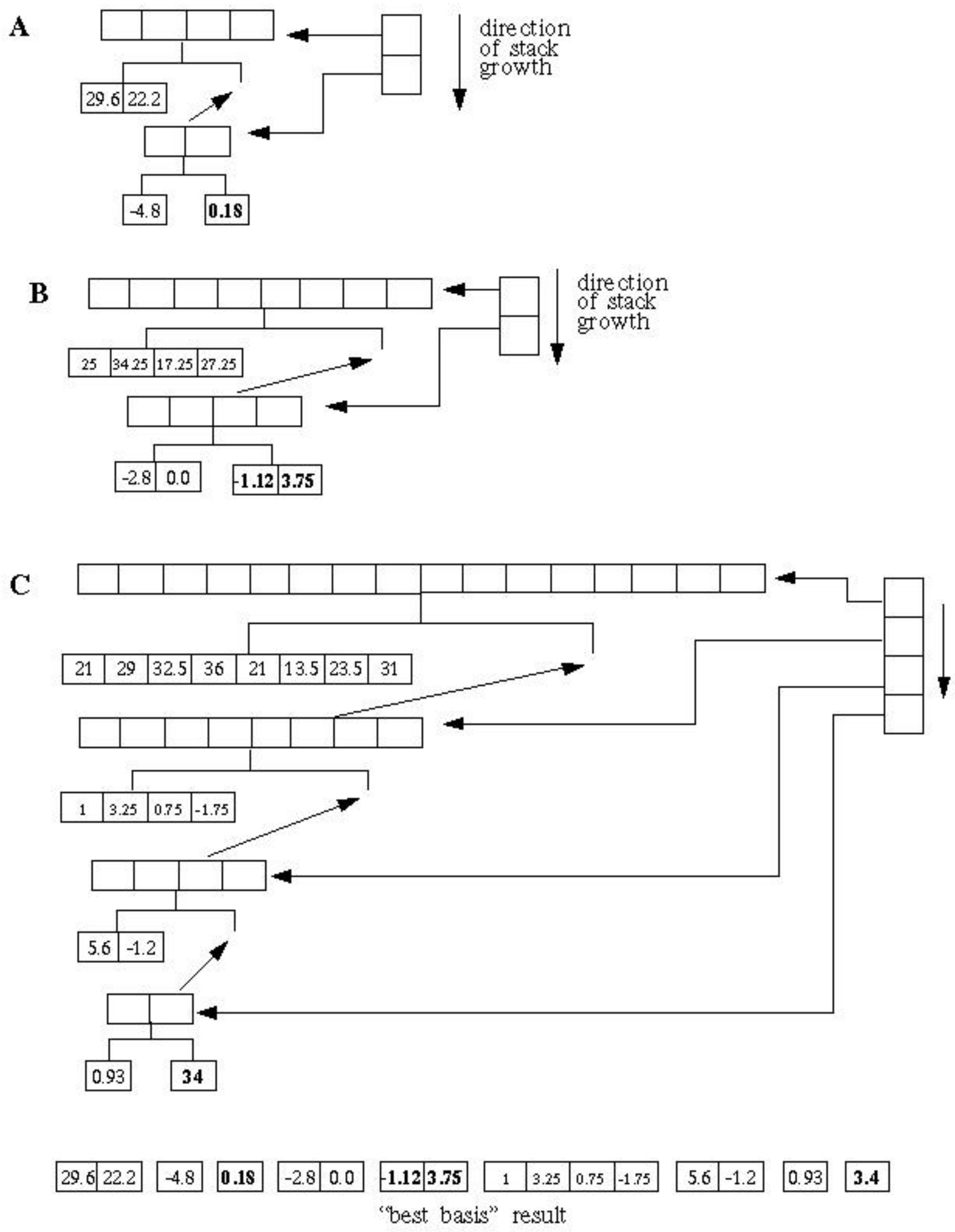
- Concentration in l_p norm $(1 \leq p)$

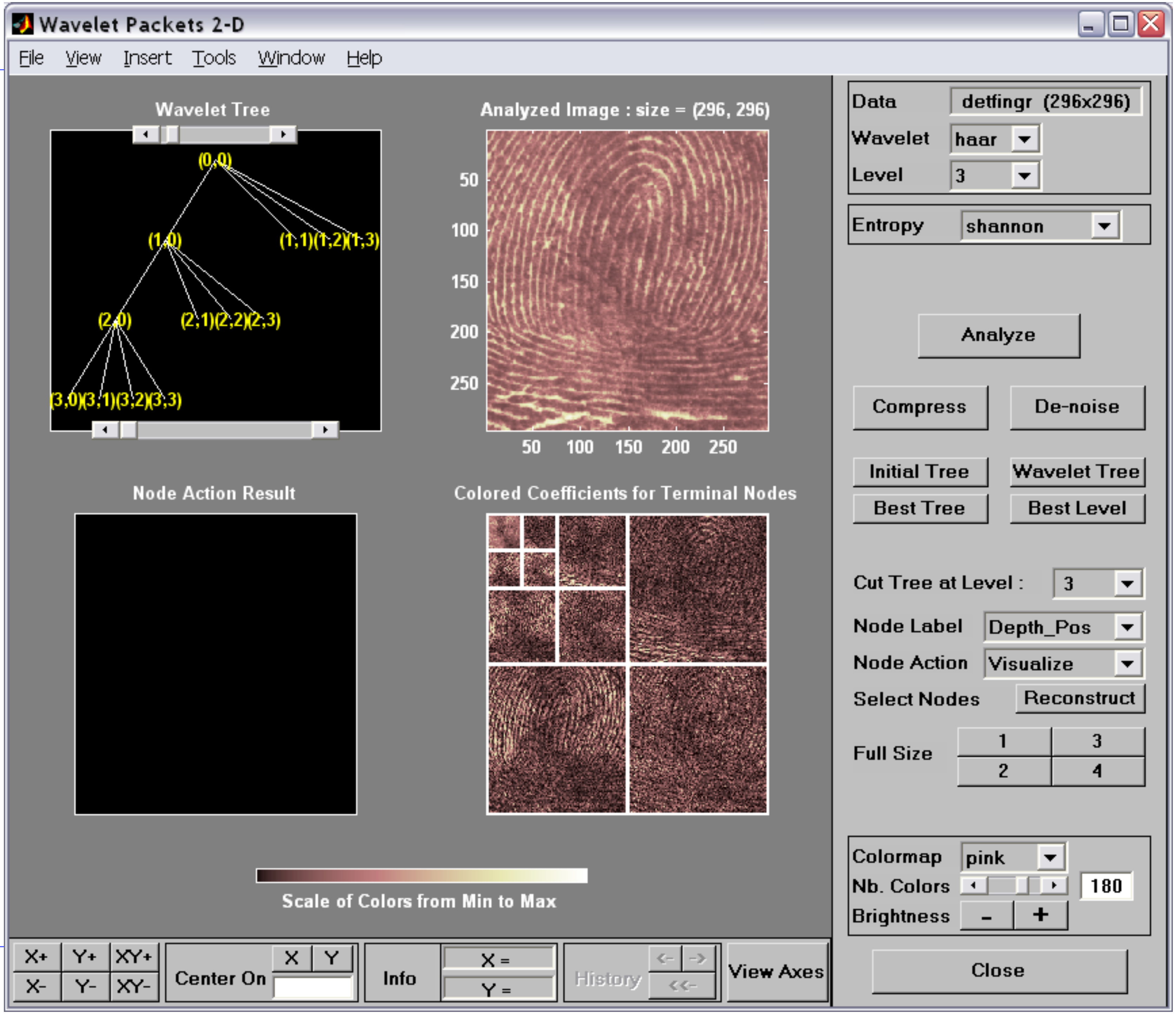
$$\text{cost}_{norm\ p} = \sum_n |s[n]|^p$$

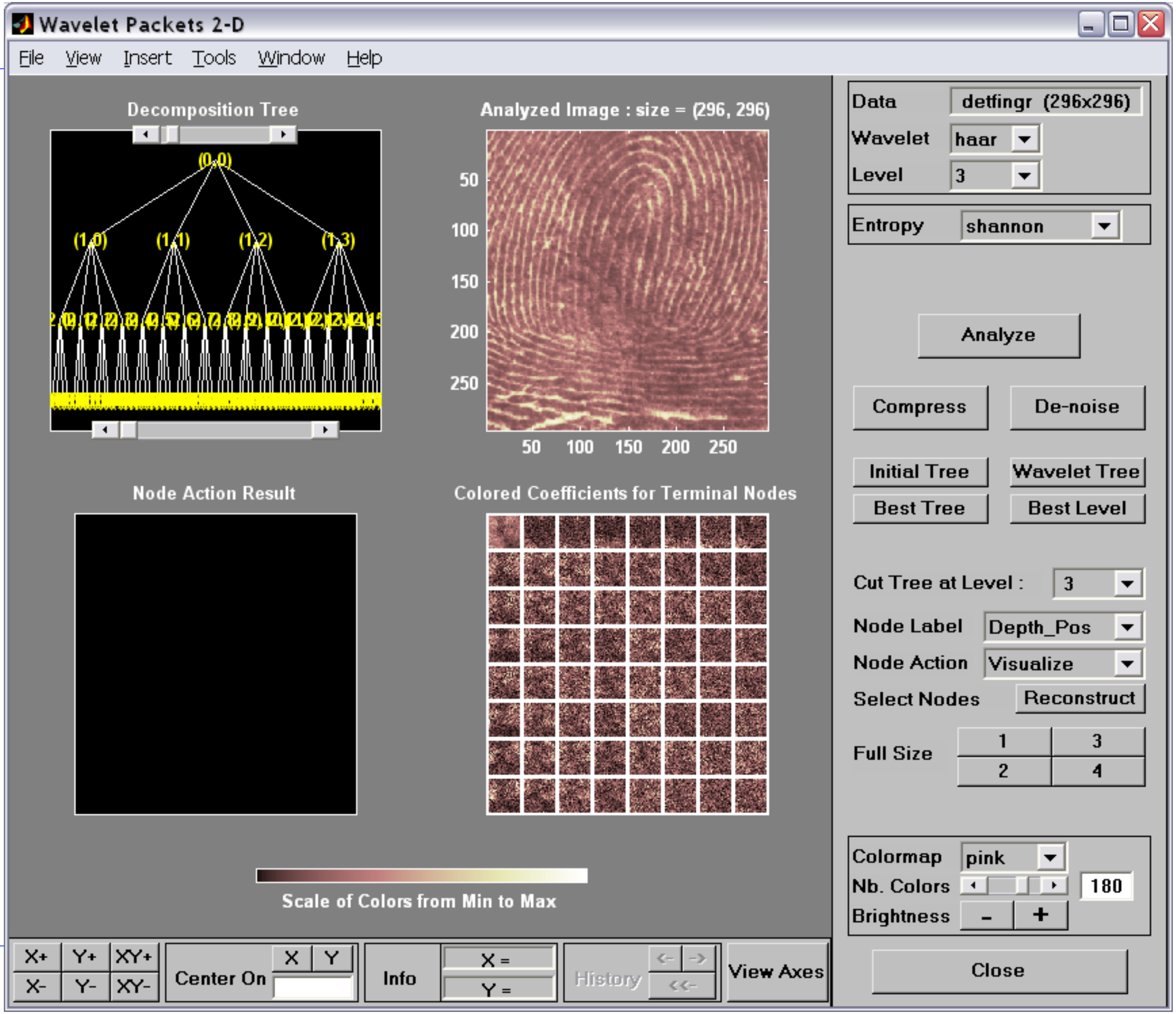
- Logarithm of “energy” (log(0)=0)

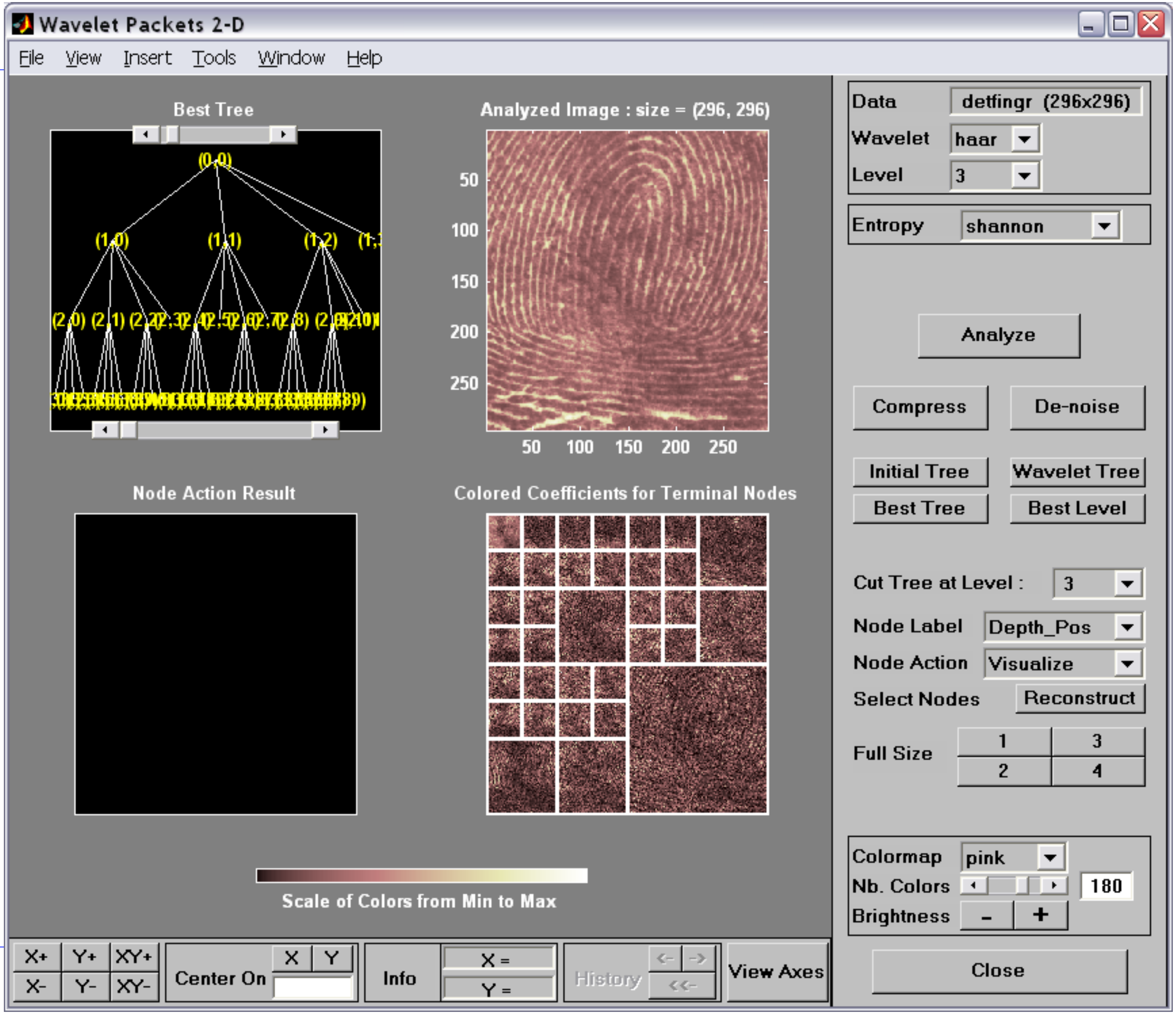
$$\text{cost}_{\log} = \sum_n \log(s[n]^2)$$

Inverse wavelet packet transform

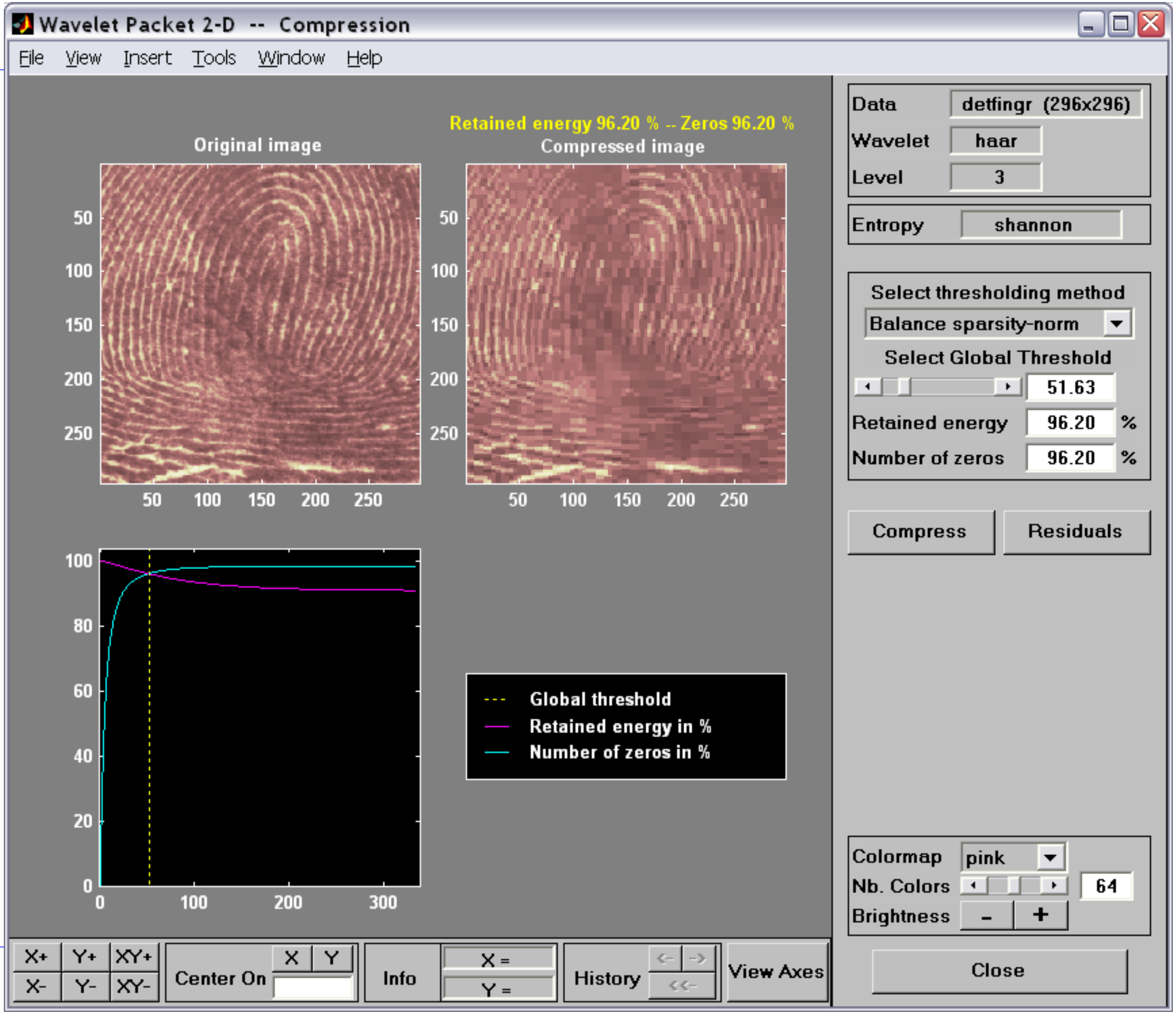


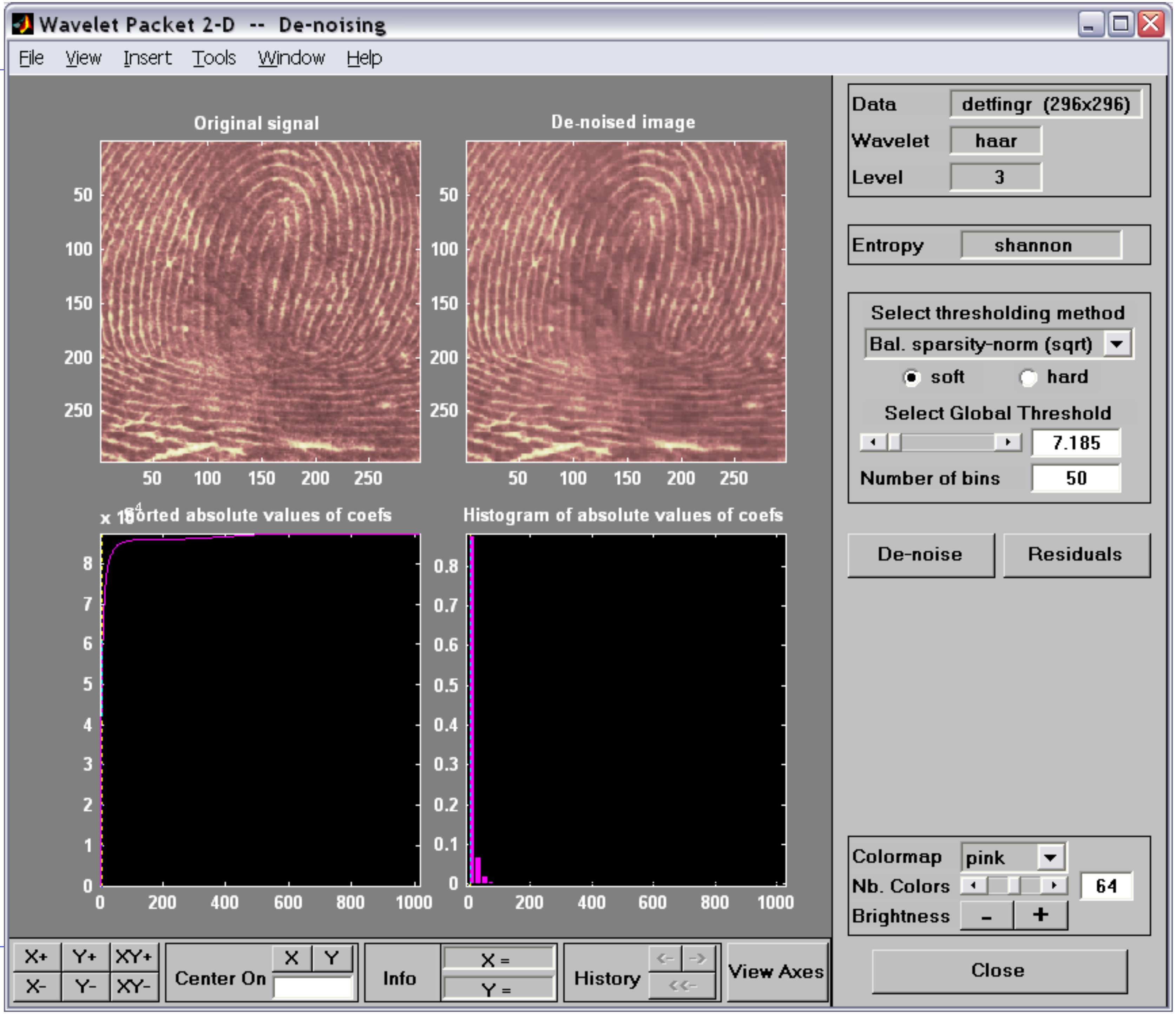

Wavelet Packets 2-D
File View Insert Tools Window Help
Wavelet Tree
(0,0)
(1,0) (1,1) (1,2) (1,3)
(2,0) (2,1) (2,2) (2,3)
(3,0) (3,1) (3,2) (3,3)
Analyzed Image : size = (296, 296)
50 100 150 200 250
Node Action Result
Colored Coefficients for Terminal Nodes
Scale of Colors from Min to Max
Data detfingr (296x296)
Wavelet haar
Level 3
Entropy shannon
Analyze
Compress
De-noise
Initial Tree
Wavelet Tree
Best Tree
Best Level
Cut Tree at Level : 3
Node Label Depth_Pos
Node Action Visualize
Select Nodes Reconstruct
Full Size 1 3 2 4
Colormap pink
Nb. Colors 180
Brightness - +
Close
X+ Y+ XY+
X- Y- XY-
Center On X Y
Info X = Y =
History
View Axes


Wavelet Packets 2-D
File
View
Insert
Tools
Window
Help
Decomposition Tree
(0,0)
(1,0)
(1,1)
(1,2)
(1,3)
Analyzed Image : size = (296, 296)
50
100
150
200
250
50
100
150
200
250
Node Action Result
Colored Coefficients for Terminal Nodes
Scale of Colors from Min to Max
Data
detfingr (296x296)
Wavelet
haar
Level
3
Entropy
shannon
Analyze
Compress
De-noise
Initial Tree
Wavelet Tree
Best Tree
Best Level
Cut Tree at Level :
3
Node Label
Depth_Pos
Node Action
Visualize
Select Nodes
Reconstruct
Full Size
1
3
2
4
Colormap
pink
Nb. Colors
180
Brightness
-
+
Close
X+
Y+
XY+
X-
Y-
XY-
Center On
X
Y
Info
X =
Y =
History
View Axes



Wavelet Packets 2-D
File View Insert Tools Window Help
Best Tree
Analyzed Image : size = (296, 296)
Node Action Result
Colored Coefficients for Terminal Nodes
Scale of Colors from Min to Max
Data detfingr (296x296)
Wavelet haar
Level 3
Entropy shannon
Analyze
Compress
De-noise
Initial Tree
Wavelet Tree
Best Tree
Best Level
Cut Tree at Level : 3
Node Label Depth_Pos
Node Action Visualize
Select Nodes Reconstruct
Full Size 1 2 3 4
Colormap pink
Nb. Colors 180
Brightness - +
Close
X+ Y+ XY+ X- Y- XY-
Center On X Y
Info X = Y =
History
View Axes


Wavelet Packet 2-D -- Compression
File View Insert Tools Window Help
Retained energy 96.20 % -- Zeros 96.20 %
Original image
Compressed image
Global threshold
Retained energy in %
Number of zeros in %
Data detfingr (296x296)
Wavelet haar
Level 3
Entropy shannon
Select thresholding method
Balance sparsity-norm
Select Global Threshold
51.63
Retained energy 96.20 %
Number of zeros 96.20 %
Compress
Residuals
Colormap pink
Nb. Colors 64
Brightness - +
Close
X+ Y+ XY+
X- Y- XY-
Center On X Y
Info
X =
Y =
History
View Axes


Wavelet Packet 2-D -- De-noising
File View Insert Tools Window Help
Original signal
De-noised image
Sorted absolute values of coefs
Histogram of absolute values of coefs
Data
detfingr (296x296)
Wavelet
haar
Level
3
Entropy
shannon
Select thresholding method
Bal. sparsity-norm (sqrt)
soft
hard
Select Global Threshold
7.185
Number of bins
50
De-noise
Residuals
Colormap
pink
Nb. Colors
64
Brightness
Close
X+ Y+ XY+
X- Y- XY-
Center On
Info
X =
Y =
History
View Axes