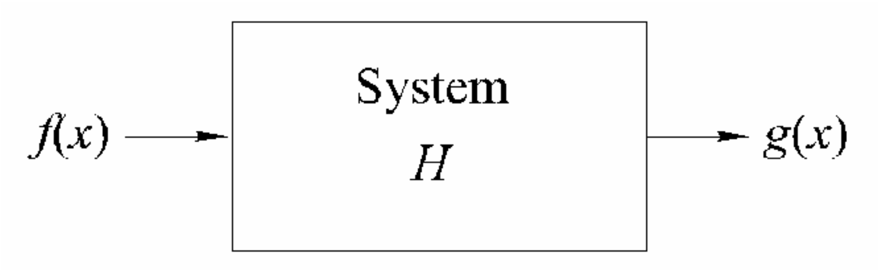
Image Enhancement

Part 1: pixel-based operations

Review: Linear Systems

- We define a system as a unit that converts an input function into an output function



$$g(x) = H[f(x)]$$

Independent variable

System operator or Transfer function

Linear Time Invariant Discrete Time Systems

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$$

$$Y_r(j\Omega) = H(j\Omega)X_c(j\Omega)$$

$$H(j\Omega) = \begin{cases} H(j\Omega) & |\Omega| < \pi/T \\ 0 & |\Omega| \geq \pi/T \end{cases}$$

IF

- The input signal is bandlimited
- The Nyquist condition for sampling is met
- The digital system is linear and time invariant

THEN

The overall continuous time system is equivalent to a LTIS whose frequency response is H.

Overview of Linear Systems

- Let $$g_i(x) = H[f_i(x)]$$

  where $f_i(x)$ is an arbitrary input in the class of all inputs $\{f(x)\}$, and $g_i(x)$ is the corresponding output.

- If $$\begin{aligned} H[a_i f_i(x) + a_j f_j(x)] &= a_i H[f_i(x)] + a_j H[f_{ji}(x)] \\ &= a_i g_i(x) + a_j g_j(x) \end{aligned}$$

  Then the system *H* is called a *linear system*.

- A linear system has the properties of *additivity* and *homogeneity*.

Linear Systems

- The system H is called *shift invariant* if

$$g_i(x) = H[f_i(x)] \text{ implies that } g_i(x + x_0) = H[f_i(x + x_0)]$$

for all $f_i(x) \in \{f(x)\}$ and for all x_0.

- This means that offsetting the independent variable of the input by x_0 causes the same offset in the independent variable of the output. Hence, the input-output relationship remains the same.

Linear Systems

- The operator *H* is said to be *causal*, and hence the system described by *H* is a *causal system*, if there is no output before there is an input. In other words,

$$f(x) = 0 \text{ for } x < x_0 \text{ implies that } g(x) = H[f(x)] = 0 \text{ for } x < x_0.$$

- A linear system *H* is said to be *stable* if its response to any bounded input is bounded. That is, if

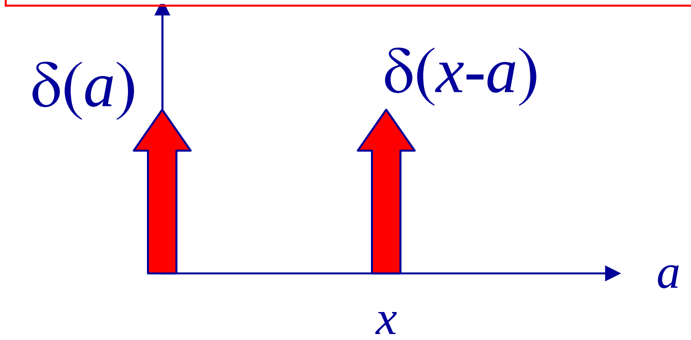
$$|f(x)| < K \text{ implies that } |g(x)| < cK$$

where *K* and *c* are constants.

Linear Systems

- A *unit impulse function*, denoted $\delta(a)$, is *defined* by the expression

$$\int_{-\infty}^{\infty} f(\alpha)\delta(x-\alpha)d\alpha = f(x).$$



- The response of a system to a unit impulse function is called the *impulse response* of the system.

$$h(x) = H[\delta(x)]$$

Linear Systems

- If *H* is a linear shift-invariant system, then we can find its response to any input signal f(x) as follows:

$$g(x) = \int_{-\infty}^{\infty} f(\alpha)h(x-\alpha)d\alpha.$$

- This expression is called the *convolution integral*. It states that the response of a linear, fixed-parameter system is completely characterized by the convolution of the input with the system impulse response.

Linear Systems

- Convolution of two functions of a continuous variable is defined as

$$f(x) * h(x) = \int_{-\infty}^{\infty} f(\alpha)h(x-\alpha)d\alpha$$

- In the discrete case

$$f[n] * h[n] = \sum_{m=-\infty}^{\infty} f[m]h[n-m]$$

Linear Systems

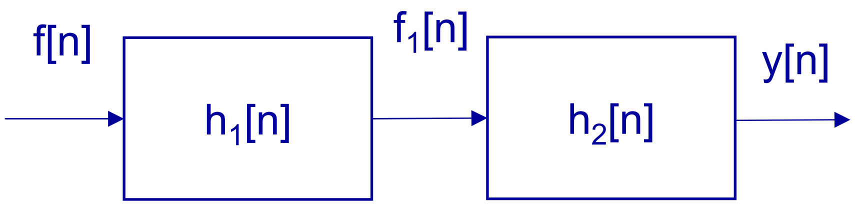
- In the 2D discrete case

$$f[n_1,n_2]*h[n_1,n_2]=\sum_{m_1=-\infty}^{\infty}\sum_{m_2=-\infty}^{\infty} f[m_1,m_2]h[n_1-m_1,n_2-m_2]$$

$h[n_1,n_2]$ is a linear filter.

Linear systems

- Cascade ("in serie")

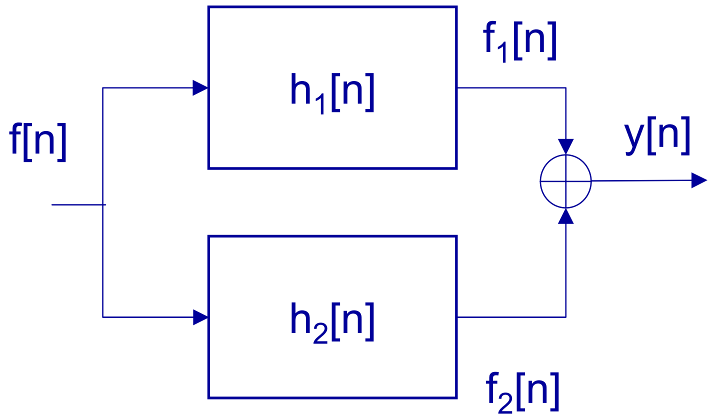


$$h[n] = h_1[n] * h_2[n]$$
$$H(\Omega) = H_1(\Omega) H_2(\Omega)$$
$$H[k] = H_1[k] H_2[k]$$

Proof

$$y[n] = f_1[n] * h_2[n] = f[n] * h_1[n] * h_2[n] =$$
$$= f[n] * (h_1[n] * h_2[n])$$

- Parallel ("in parallelo")



$$h[n] = h_1[n] + h_2[n]$$
$$H(\Omega) = H_1(\Omega) + H_2(\Omega)$$
$$H[k] = H_1[k] + H_2[k]$$

IP Algorithms

Spatial domain

- Operations are performed in the image domain
- Image ⇔ matrix of number
- Examples
  - luminance adaptation
  - chromatic adaptation
  - contrast enhancement
  - spatial filtering
  - edge detection
  - noise reduction

Transform domain

- Some operators are used to project the image in another space
- Operations are performed in the transformed domain
  - Fourier (DCT, FFT)
  - Wavelet (DWT,CWT)
- Examples
  - coding
  - denoising
  - image analysis

Most of the tasks can be implemented both in the image and in the transformed domain. The choice depends on the context and the specific application.

Spatial domain processing

Pixel-wise

- Operations involve the single pixel
- Operations:
  - histogram equalization
  - change of the colorspace
  - addition/subtraction of images
  - get negative of an image
- Applications:
  - luminance adaptation
  - contrast enhancement
  - *chromatic adaptation*

Local-wise

- The neighbourhood of the considered pixel is involved
  - Any operation involving digital filters is local-wise
- Operations:
  - correlation
  - convolution
  - filtering
  - transformation
- Applications
  - smoothing
  - sharpening
  - noise reduction
  - edge detection

Image enhancement

- There is no general unifying theory of image enhancement at present because there is no general standard of image quality that can serve as a design criterion for an image enhancement processor.
  - Consideration is given here to a variety of techniques that have proved useful for human observation improvement and image analysis.
- [Pratt, Chapter 10]

Pixel-wise operations

- Contrast enhancement
  - –Amplitude scaling
  - – Histogram straching/shrinking, sliding, equalization
- Contrast can often be improved by amplitude rescaling of each pixel

Amplitude scaling

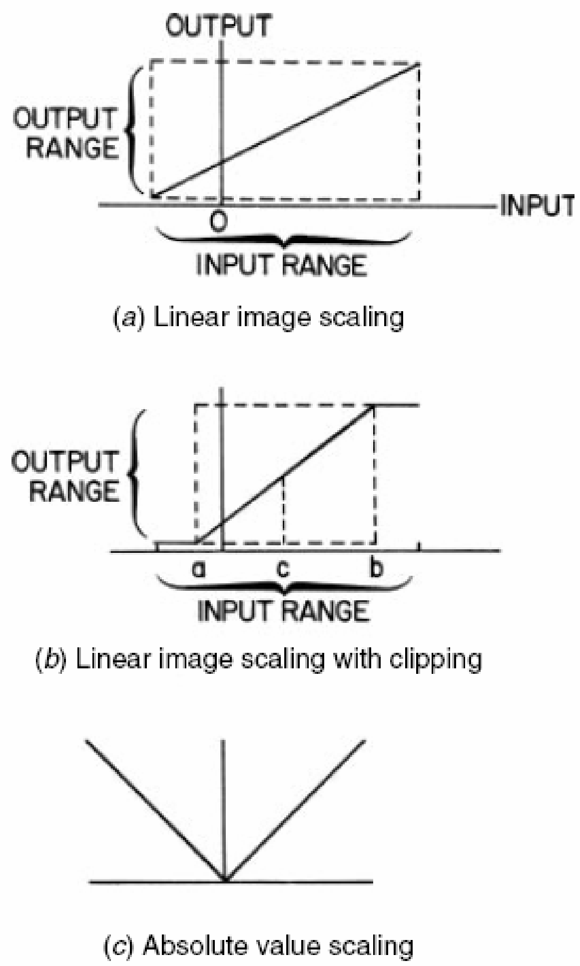


FIGURE 10.1-2. Image scaling methods.

Window-level transformation. The window value is the width of the linearslope; the level is located at the midpoint *c* of the slope line. Very common in medical imaging.

Amplitude scaling

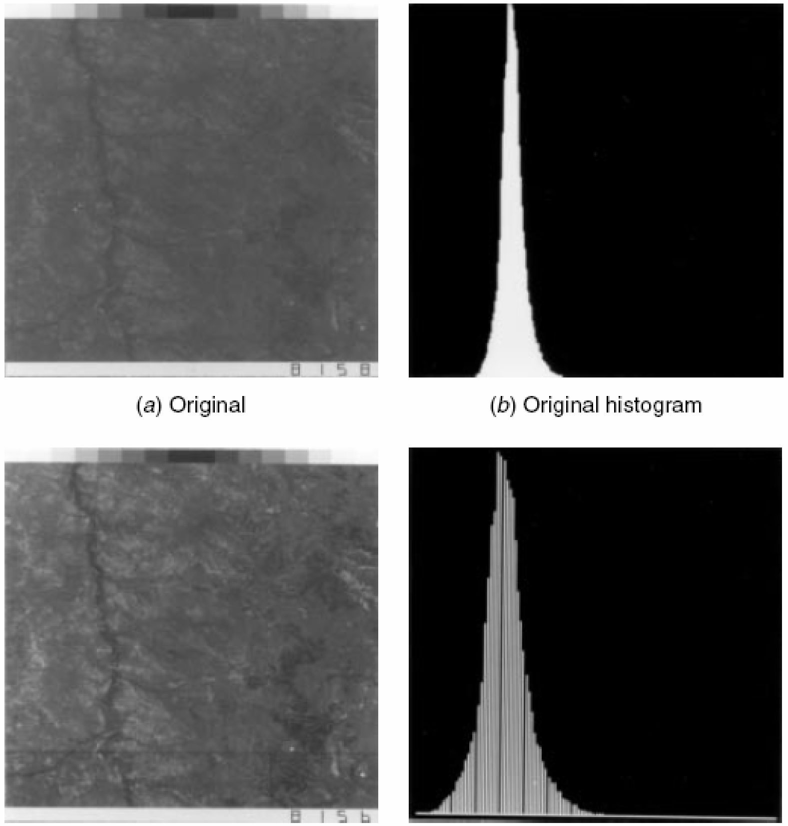
Q component of a YIQ image representation.

(*a*) Linear, full range, – 0.147 to 0.169

(*b*) Clipping, 0.000 to 0.169

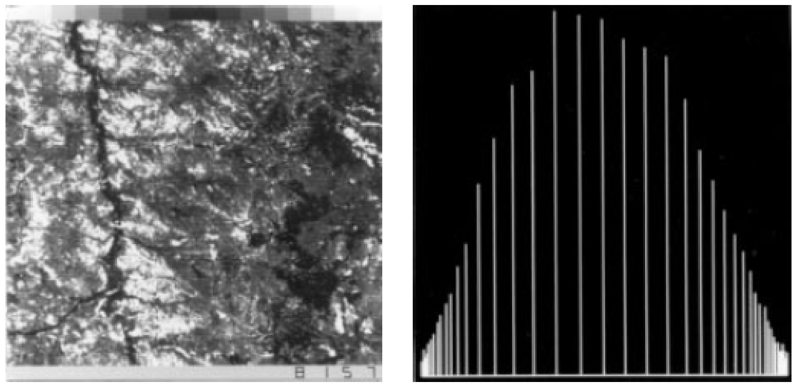
(*c*) Absolute value, 0.000 to 0.169

Window level transformation: ex.



(*a*) Original (*b*) Original histogram

(*c*) Min. clip = 0.17, max. clip = 0.64 (*d*) Enhancement histogram



(*e*) Min. clip = 0.24, max. clip = 0.35 (*f*) Enhancement histogram

FIGURE 10.1-4. Window-level contrast stretching of an earth satellite image.

Gray scale contouring is at the threshold of visibility.

Contrast enhancement via graylevel transf.

- Point transformations that modify the contrast of an image within a display's dynamic range
- Often nonlinear point transformations

$$G[j,k] = (F[j,k])^p$$

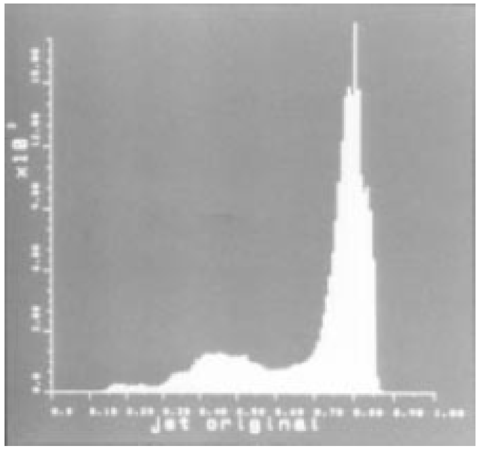
$$0 \le F[j,k] \le 1$$

$$p: \text{ power law varaible}$$

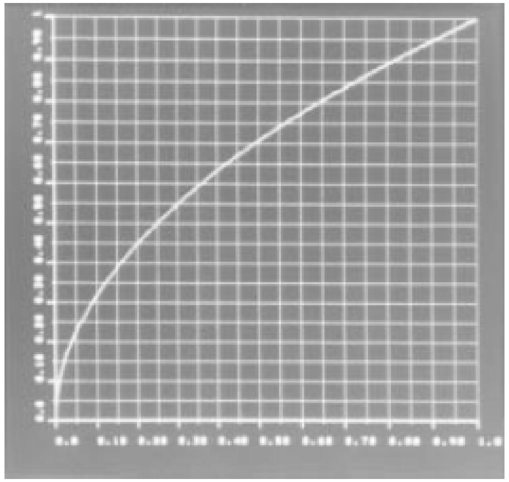
example

original

(*a*) Original

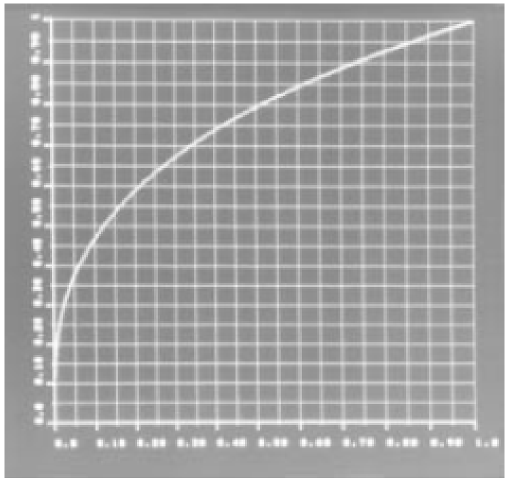


(*b*) Original histogram



(*a*) Square root function

(*b*) Square root output



(*c*) Cube root function

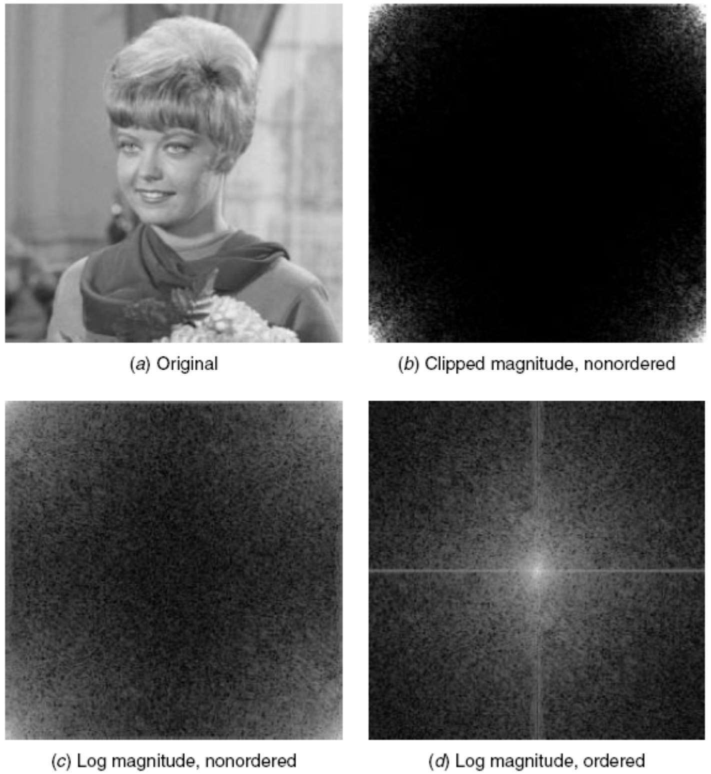
(*d*) Cube root output

log amplitude scaling

- The logarithm function is useful for scaling image arrays with a very wide dynamic range.

$$G(j,k) = \frac{\log_e\{1.0 + aF(j,k)\}}{\log_e\{2.0\}}$$

a>0


(a) Original
(b) Clipped magnitude, nonordered
(c) Log magnitude, nonordered
(d) Log magnitude, ordered

Reverse and Inverse functions

- Reverse function

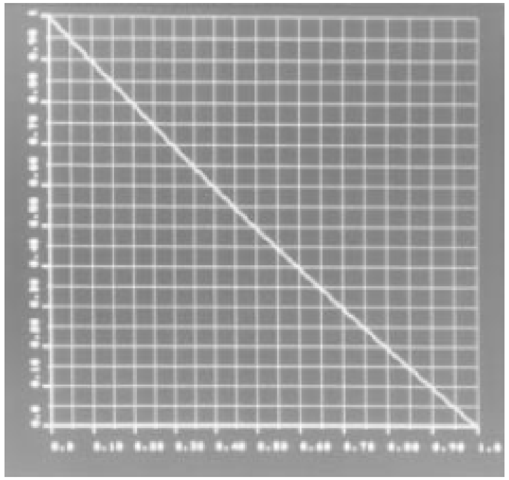
$$G[i,k] = (1 - F[i,k])$$
$$0 \leq F[i,k] \leq 1$$

- Inverse function

$$G(j,k) = \begin{cases} 1.0 & \text{for } 0.0 \leq F(j,k) < 0.1 \\ \dfrac{0.1}{F(j,k)} & \text{for } 0.1 \leq F(j,k) \leq 1.0 \end{cases}$$

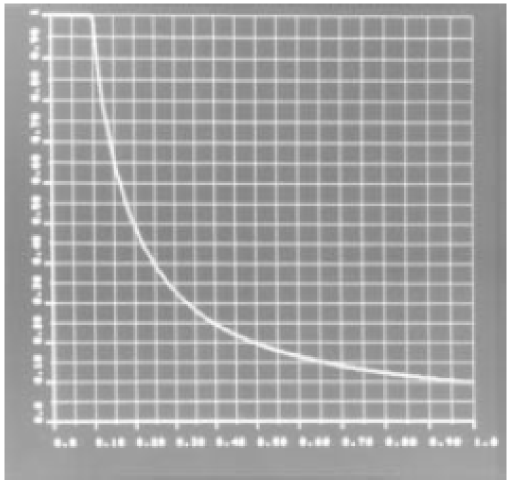
clipped below 0.1 to maintain the range (max value=1)

example



(*a*) Reverse function

(*b*) Reverse function output

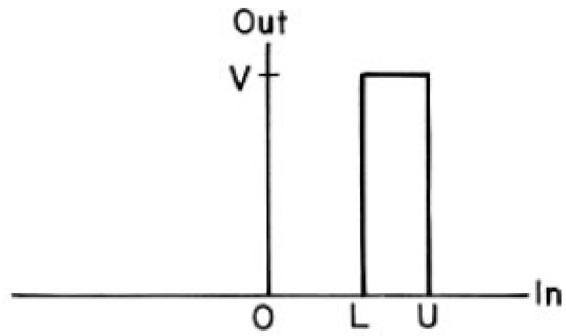


(*c*) Inverse function

(*d*) Inverse function output

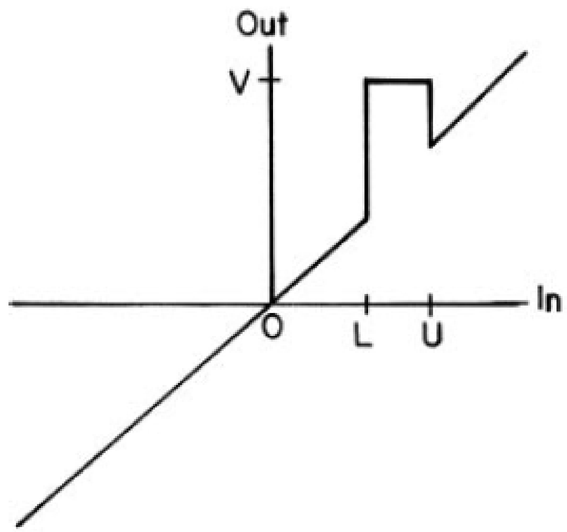
Level slicing

- Pixels within the amplitude passband are rendered maximum white in the output, and pixels outside the passband are rendered black.



(a) Zero background scaling transformation

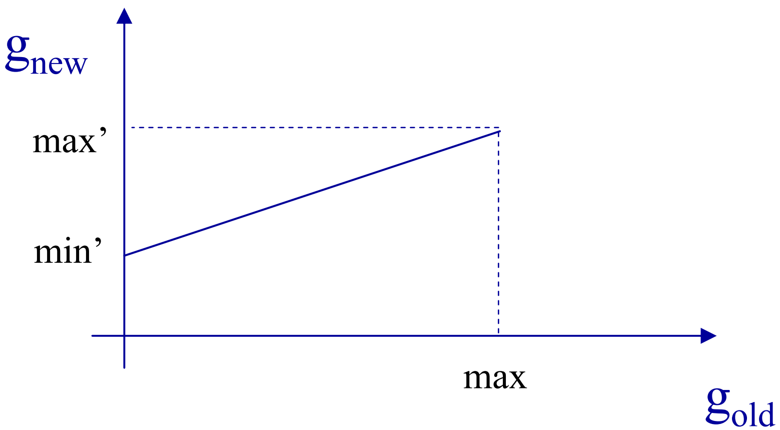
- Pixels outside the amplitude passband are displayed in their original state

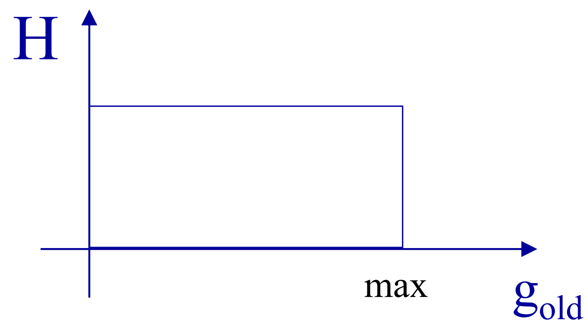


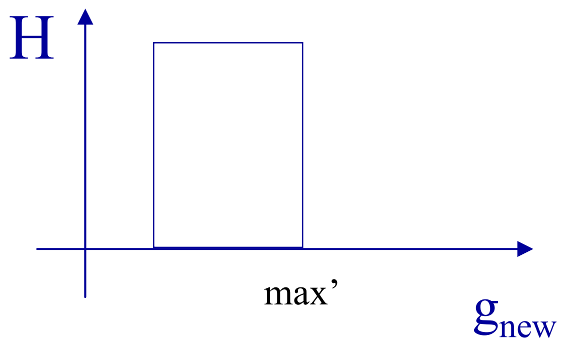
(b) Image background scaling transformation

Histogram changes

- Graylevel transformations induce histogram changes



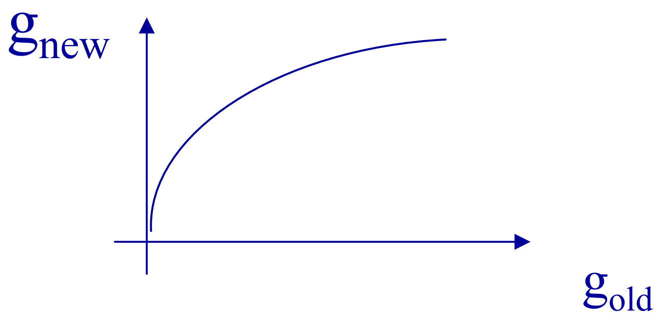


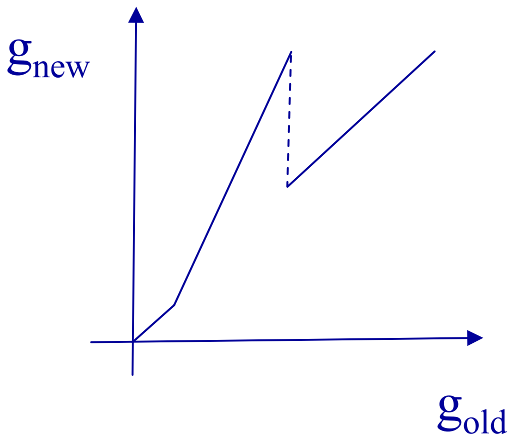


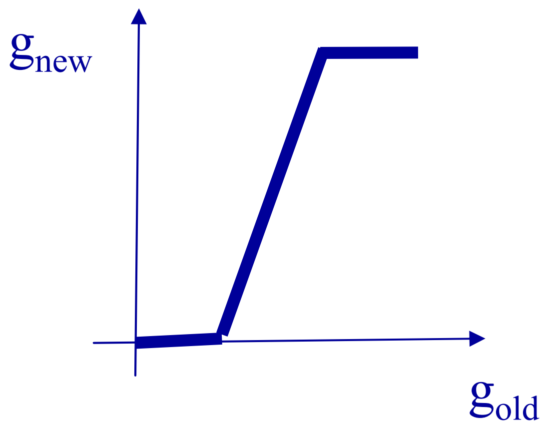
Other non-linear transformations

- Used to emphasize mid-range levels

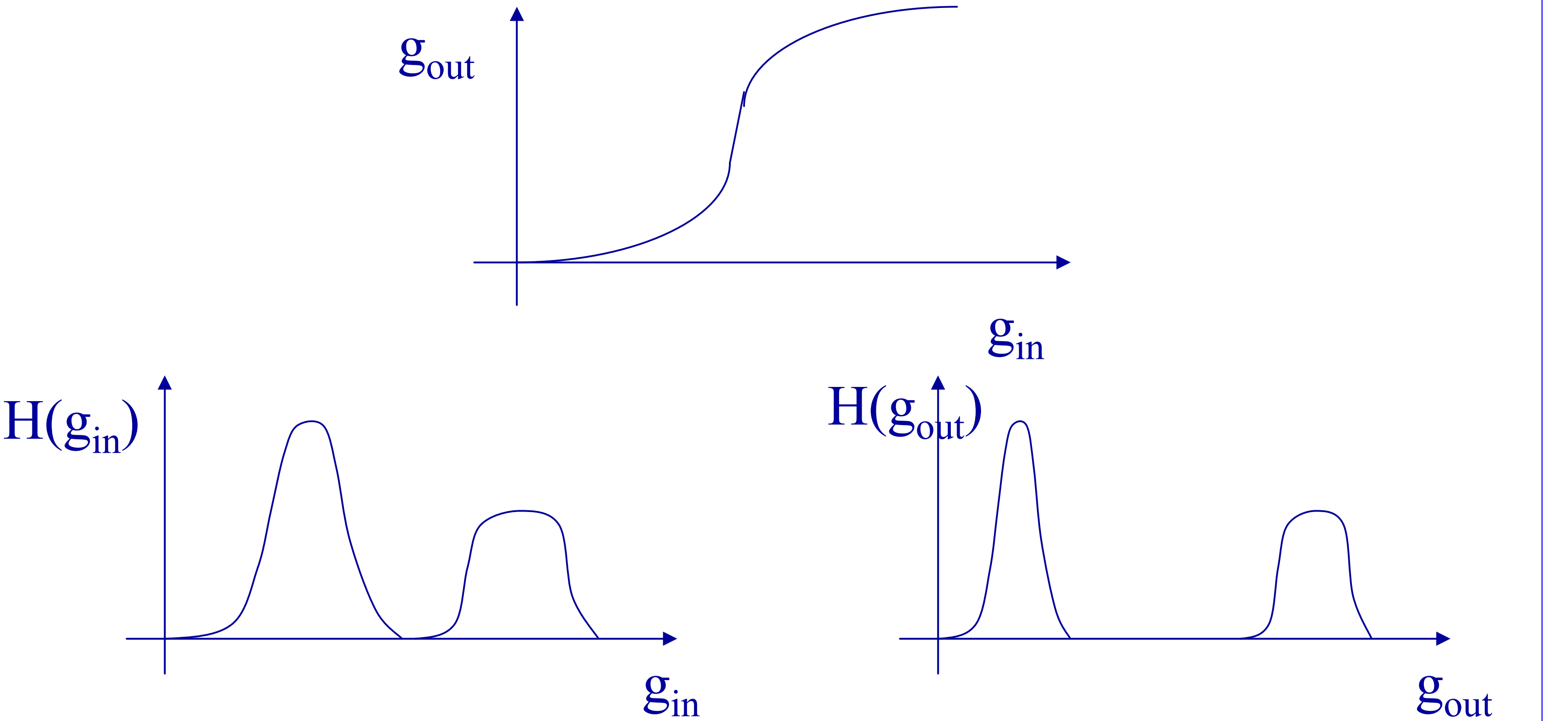
  $g_{new} = g_{old} + g_{old}\ C\ (g_{old,max} - g_{old})$





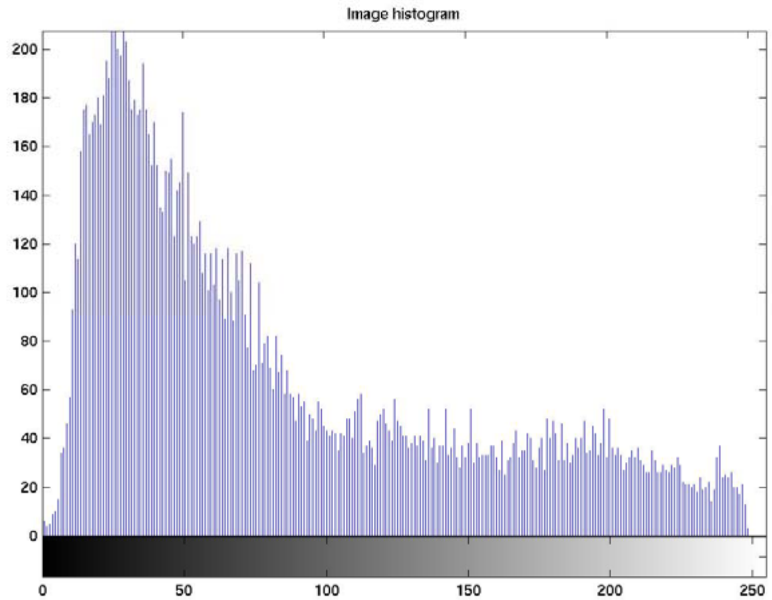


Sigmoid transformation (*soft thresholding*)



Pixel-wise: Histogram equalization

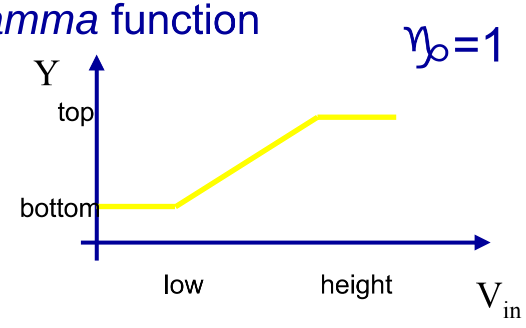
- Pixel features: luminance, color,
- Histogram equalization: shapes the intensity histogram to approximate a specified distribution
  - It is often used for enhancing contrast by shaping the image histogram to a uniform distribution over a given number of grey levels. The grey values are redistributed over the dynamic range to have a constant number of samples in each interval (i.e. histogram bin).
  - Can also be applied to colormaps of color images.

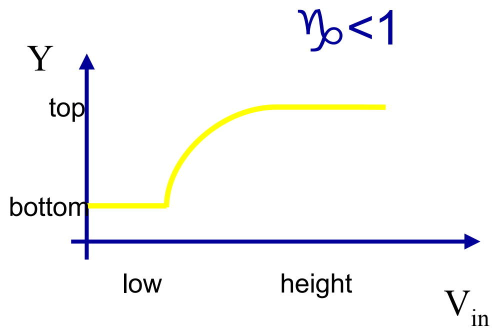


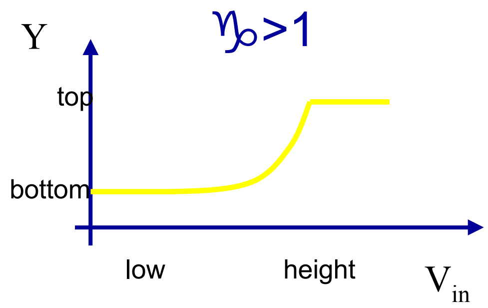
Histogram equalization

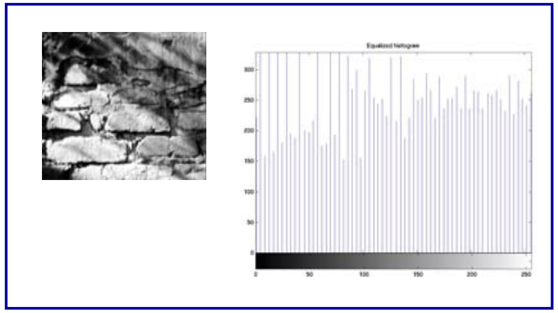
Can be used to compensate the distortions in the gray level distribution due to the non-linearity of a system component

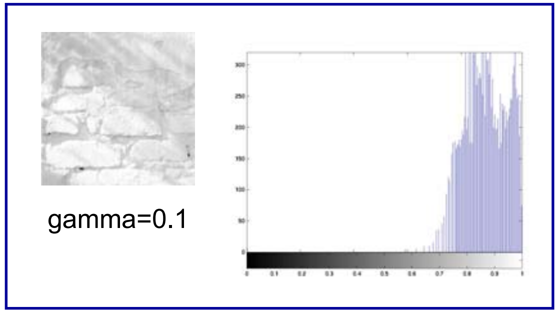
Gamma function



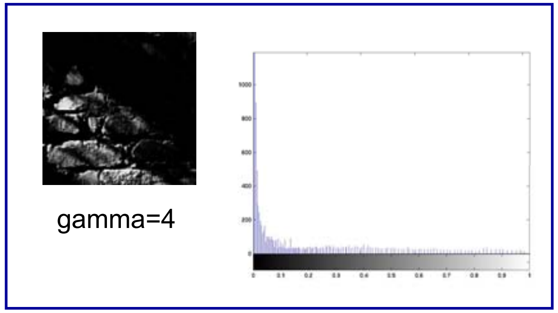








gamma=0.1

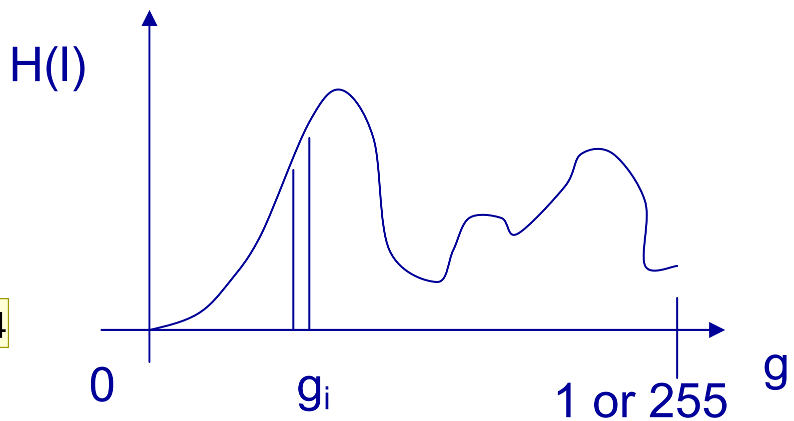


gamma=4

Histogram

- Function H=H(g) indicating the number of pixels having gray-value equal to g
  - Non-normalized images: $0 \le g \le 255 \rightarrow$ bin-size≥ 1, can be integer
  - Normalized images: $0 \le g \le 1 \rightarrow$ bin-size<1

VM4



$$A = \int_0^{\max} H(g)dg$$

area under the curve=number of pixels

$$A = \sum_{i=1}^{N_g} H[g_i]$$

In the continuous case

$$H(g) = -\frac{dA(g)}{dg} = \lim_{\Delta g \to 0} \frac{A(g) - A(g+\Delta g)}{\Delta g}$$

VM4 fare check!
swan; 21/05/2003

Histogram transformation

$$g_{out} = f(g_{in}) \Rightarrow g_{in} = f^{-1}(g_{out}), \quad f \quad \text{non-decreasing function}$$

$$H(g_{in}) \Rightarrow H(g_{out}), \quad \text{namely}$$

$$H(g_{out}) = \frac{H[f^{-1}(g_{out})]}{f'[f^{-1}(g_{out})]}, \quad f' = \frac{\partial f}{\partial g}$$

More formally

- The histogram modification process can be considered to be a monotonic point transformation $g_d=T\{f_c\}$ for which the input amplitude variable $f_1 \leq f_c \leq f_C$ is mapped into an output variable $g_1 \leq g_d \leq g_D$ such that the output probability distribution $Pr\{g_d=b_d\}$ follows some desired form for a given input probability distribution $Pr\{f_c=a_c\}$ where a_c and b_d are reconstruction values of the c^{th} and d^{th} levels.
  - Clearly, the input and output probability distributions must each sum to unity.

$$\sum_{c=1}^{C} P_R\{f_c = a_c\} = 1$$

$$\sum_{d=1}^{D} P_R\{g_d = b_d\} = 1$$

Histogram equalization

– Furthermore, the cumulative distributions must equate for any input index *c*.

- the probability that pixels in the input image have an amplitude less than or equal to a_c must be equal to the probability that pixels in the output image have amplitude less than or equal to b_d, where $b_d=T\{a_c\}$ because the transformation is monotonic. Hence

$$\sum_{n=1}^{d} P_R\{g_n = b_n\} = \sum_{m=1}^{c} P_R\{f_m = a_m\} \qquad \text{(a)}$$

cumulative probability distribution of the output

cumulative probability distribution of the input

$$P_f(f) \approx \sum_{m=1}^{c} H_F(m)$$

histogram

- in the continuous domain (easier for calculations)

$$\int_{g_{\min}}^{g} p_g(g)\,dg = \int_{f_{\min}}^{f} pf(f)\,df$$

$p_f(f)$ and $p_g(g)$ are the probability densities of f and g

Histogram equalization

cumulative probability distribution of the output

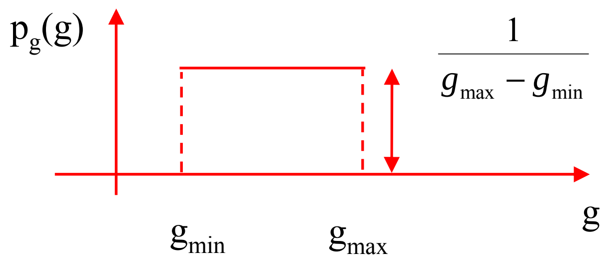
(a) $$\sum_{n=1}^{d} P_R\{g_n = b_n\} = \sum_{m=1}^{c} H_F(m)$$

cumulative histogram

$$\int_{g_{min}}^{g} p_g(g)\, dg = P_f(f)$$ (b)

When the output density is forced to be the uniform density

$$p_g(g) = \frac{1}{g_{max} - g_{min}} \quad \text{(area=1)}$$

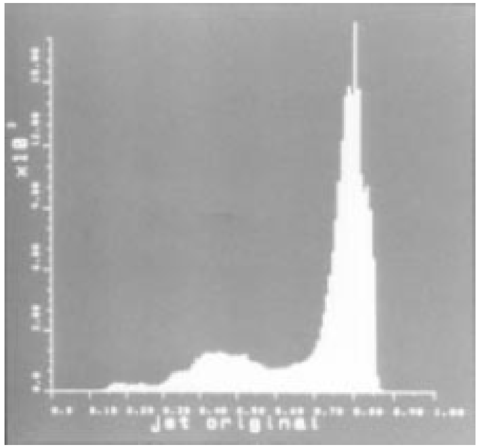


Solving (b) for g we get the histogram equalization transfer function:

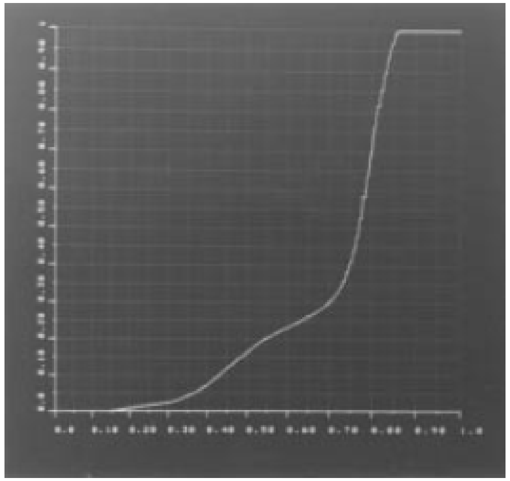
$$g = (g_{max} - g_{min}) P_f(f) + g_{min}$$

example

(*a*) Original



(*b*) Original histogram



(*b*) Transfer function

(*c*) Histogram equalized

Some mappings

TABLE 10.2-1. Histogram Modification Transfer Functions

Output Probability Density Model		Transfer Function[a]
Uniform	$p_g(g) = \frac{1}{g_{max} - g_{min}} \quad g_{min} \le g \le g_{max}$	$g = (g_{max} - g_{min})P_f(f) + g_{min}$
Exponential	$p_g(g) = \alpha \exp\{-\alpha(g - g_{min})\} \quad g \le g_{min}$	$g = g_{min} - \frac{1}{\alpha}\ln\{1 - P_f(f)\}$
Rayleigh	$p_g(g) = \frac{g - g_{min}}{\alpha^2}\exp\left\{-\frac{(g - g_{min})^2}{2\alpha^2}\right\} \quad g \ge g_{min}$	$g = g_{min} + \left[2\alpha^2 \ln\left\{\frac{1}{1 - P_f(f)}\right\}\right]^{1/2}$
Hyperbolic (Cube root)	$p_g(g) = \frac{1}{3}\frac{g^{-2/3}}{g_{max}^{1/3} - g_{min}^{1/3}}$	$g = \left[g_{max}^{1/3} - g_{min}^{1/3}[P_f(f)] + g_{max}^{1/3}\right]^3$
Hyperbolic (Logarithmic)	$p_g(g) = \frac{1}{g[\ln\{g_{max}\} - \ln\{g_{min}\}]}$	$g = g_{min}\left(\frac{g_{max}}{g_{min}}\right)^{P_f(f)}$

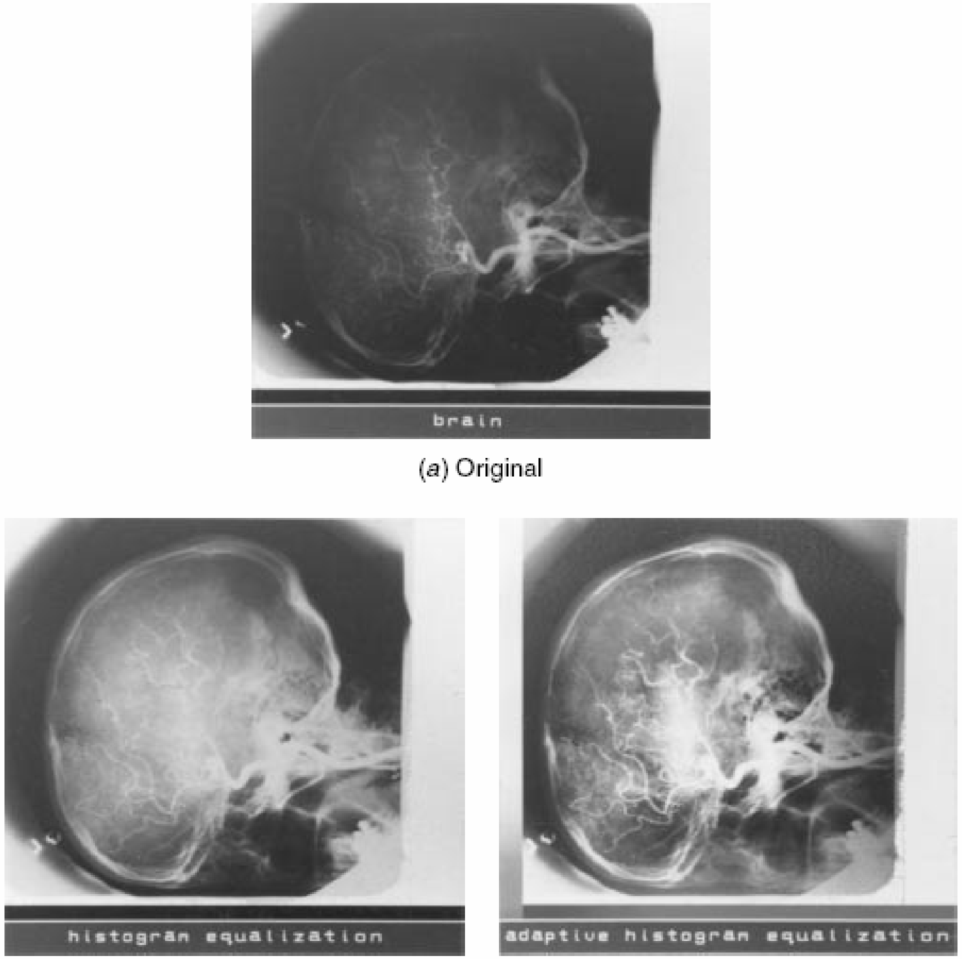
[a]The cumulative probability distribution $P_f(f)$, of the input image is approximated by its cumulative histogram:

$$p_f(f) \approx \sum_{m=0}^{j} H_F(m)$$

Adaptive hist. equalization

- The mapping function can be made spatially adaptive by applying histogram modification to each pixel based on the histogram of pixels *within a moving window neighborhood*.
  - This technique is obviously computationally intensive, as it requires histogram generation, mapping function computation, and mapping function application at each pixel.
  - Some interpolation-based solutions can be envisioned to improve computational efficiency

example

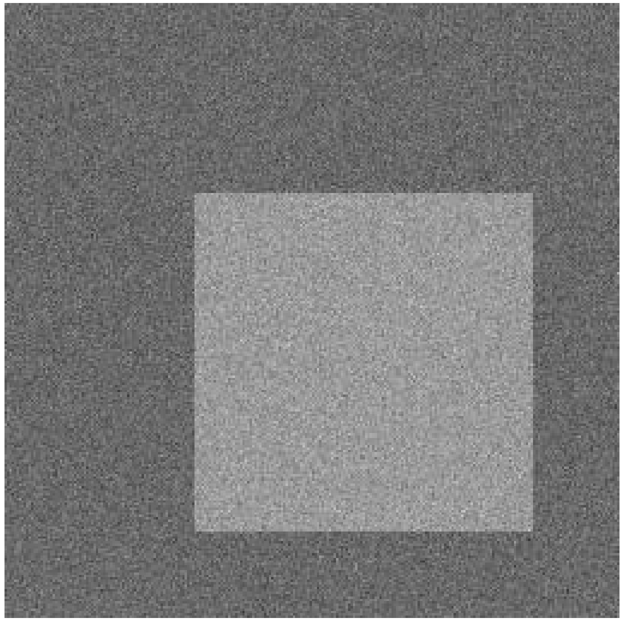


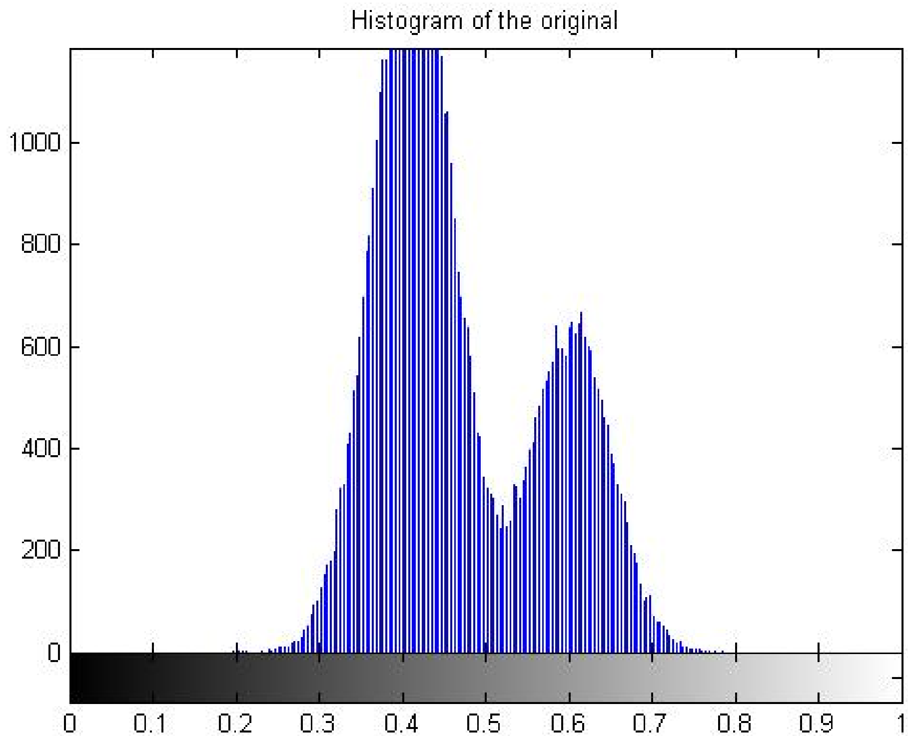
(a) Original

(b) Nonadaptive

(c) Adaptive

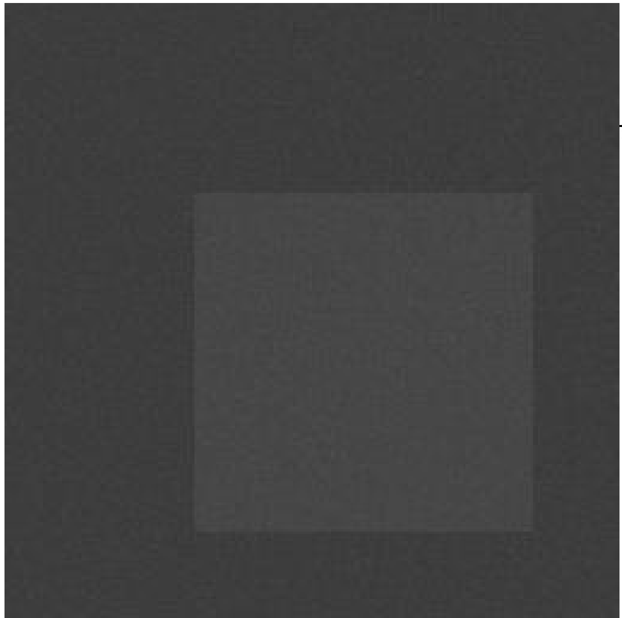
H. original

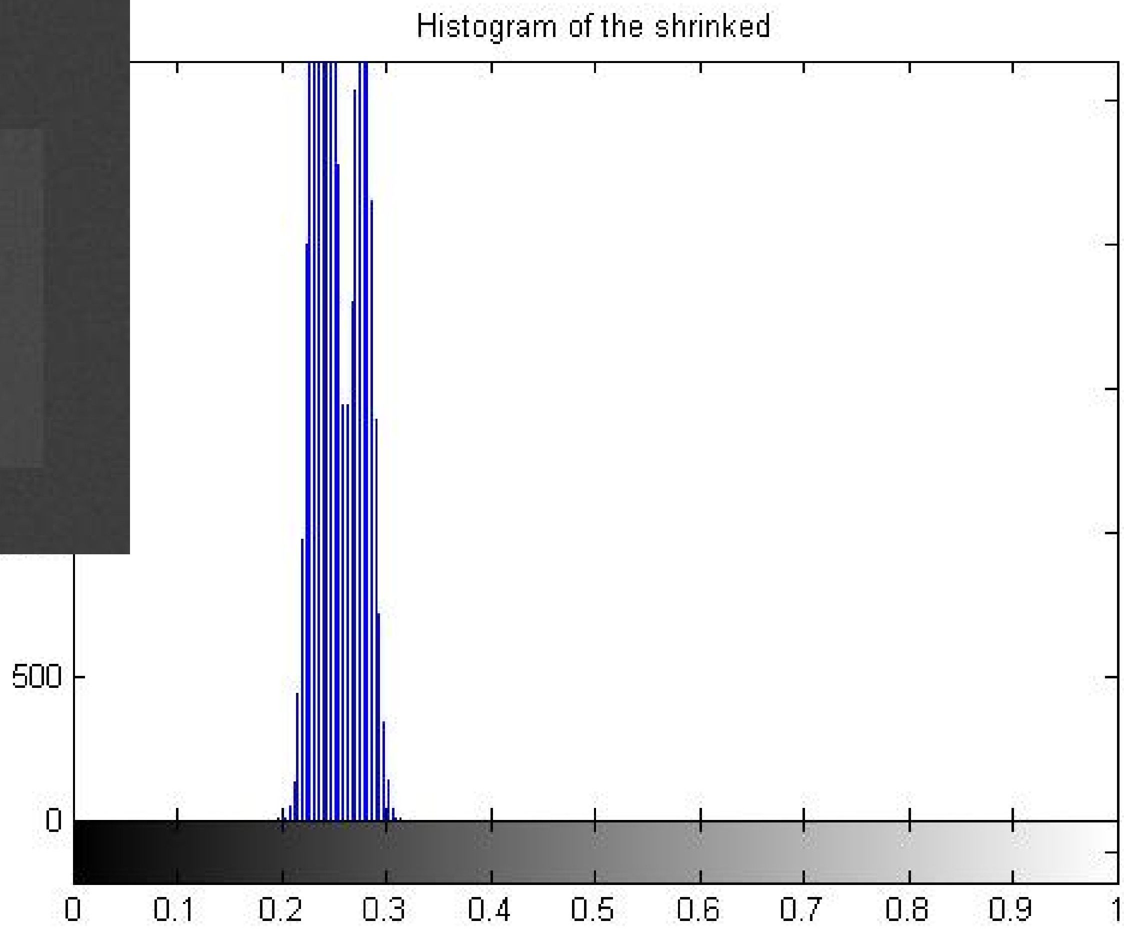




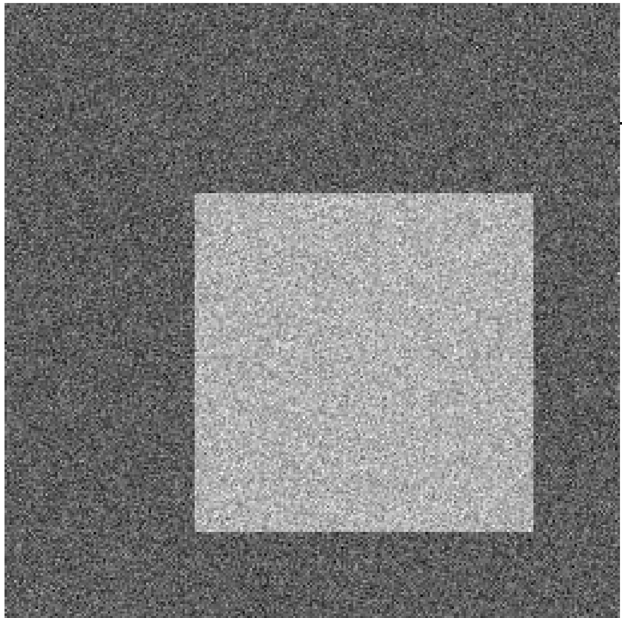
Histogram of the original
1000
800
600
400
200
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

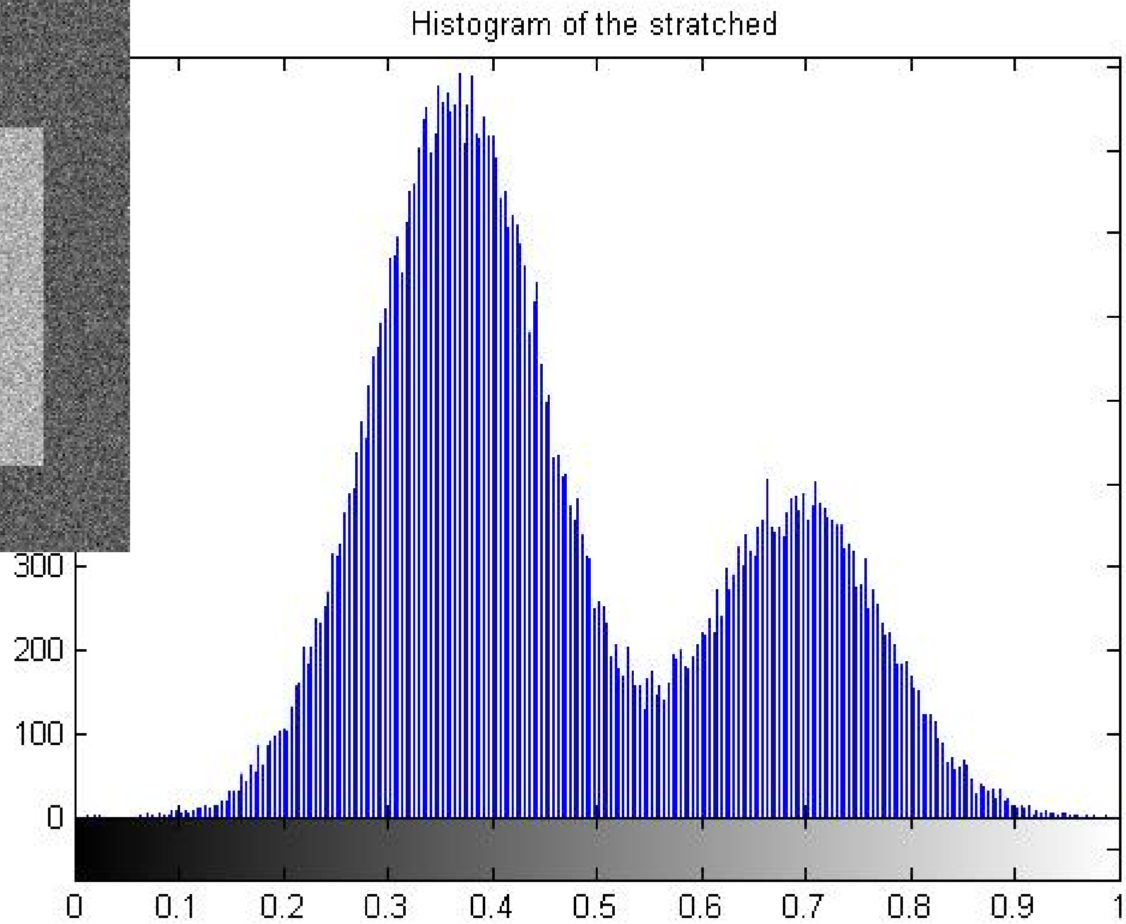
H. shrinked



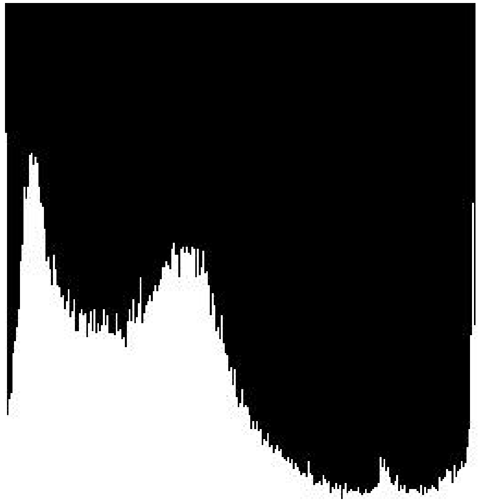


H. stratched



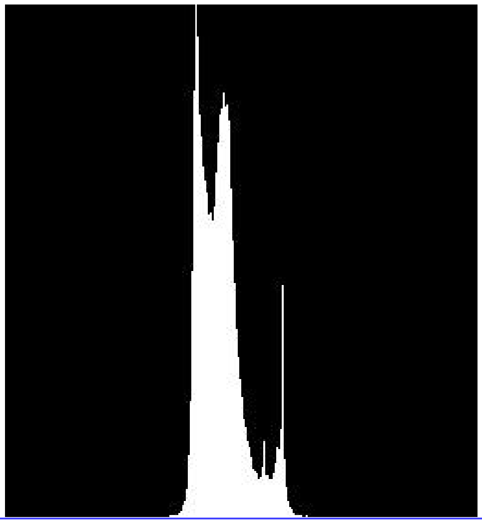

Histogram of the stratched
300
200
100
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

H. stratching/shrinking

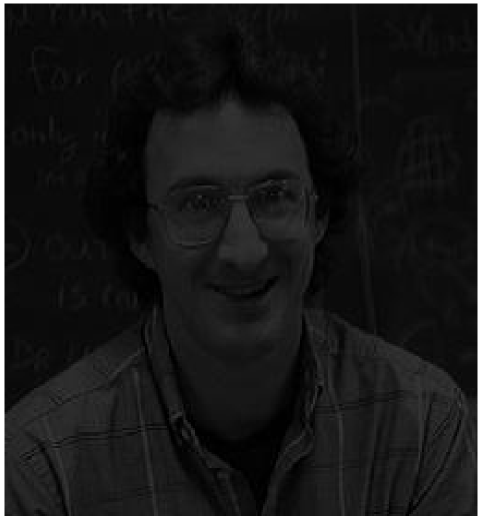


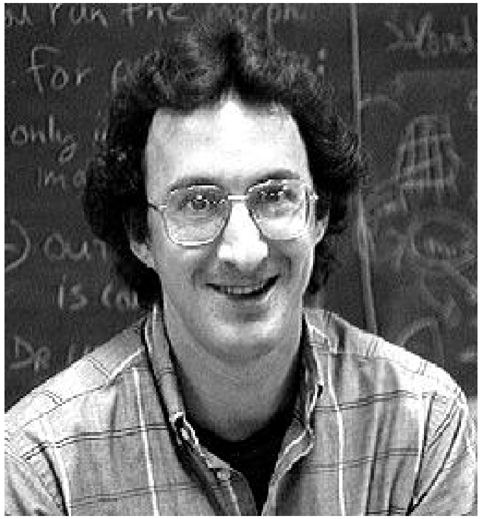
stratching

shrinking



H. stratching/shrinking





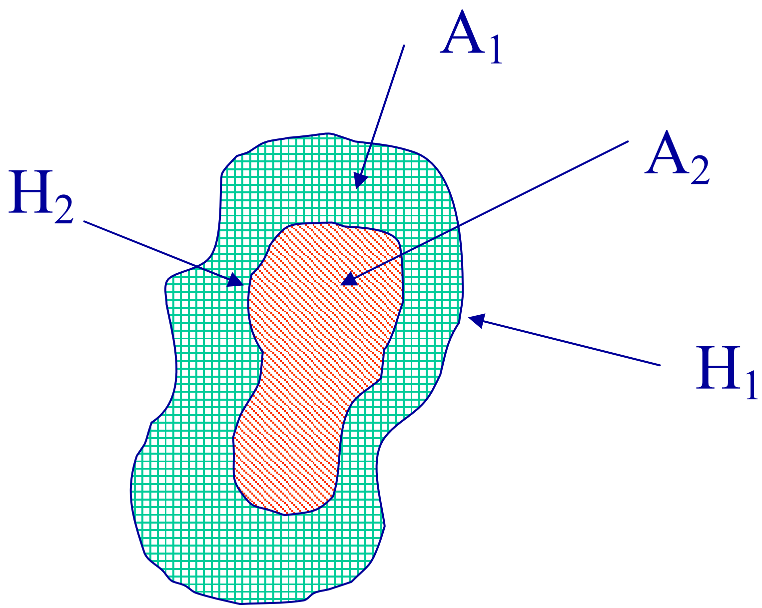






Example: region-based segmentation

- If the two regions have different graylevel distributions (histograms) then it is possible to split them by exploiting such an information



Example: region-based segmentation