DISCRETE TIME AND SPACE MODELING OF ACOUSTIC WAVE REFLECTION PHENOMENA

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This seminar

Well-established principles of **acoustic reflection** and **absorption** are reformulated into a discrete wave-based framework.

- Fundamentals of acoustic reflection:
  1. wave-based approach
  2. frequency-dependent absorption
  3. some real figures.
- Discrete time and space wave scattering:
  1. Digital Waveguides
  2. isotropic, lossless wave propagation -> **scattering**
  3. frequency-dependent (lossy) scattering.
- Embedding wave reflection inside scattering:
  1. lossy **scattered reflection**
  2. lossy **scattered diffusion**.

Fundamentals of acoustic wave reflection

- Reflected **pressure** and **velocity** waves are affected by a change in amplitude and phase:

$$P^-(j\omega) = RP^+(j\omega) = |R|e^{j\omega} P^+(j\omega)$$

- Define the **impedance** as the ratio between pressure and *normal* velocity:

$$Z = \frac{P}{(V)_n} = \frac{P^+ + P^-}{(V^+ + V^-)_n}$$

(note: $Z = Z_0$ in the air).

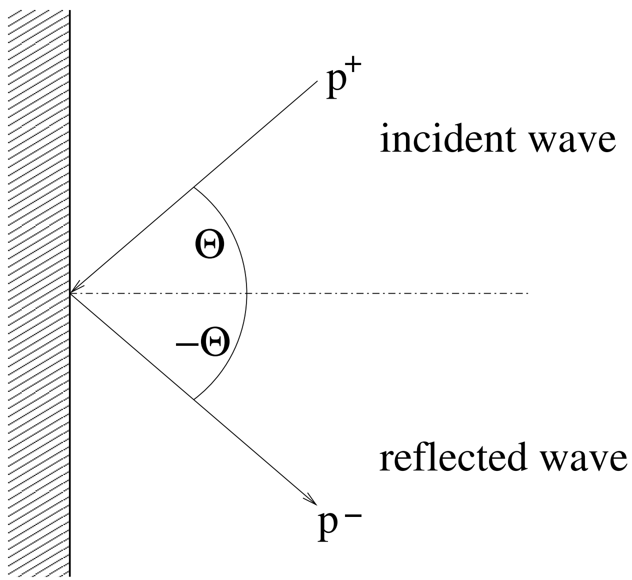
- It can be seen that, when $V = (V)_n$:

$$\frac{Z}{Z_0} = \frac{1+R}{1-R}$$

holds at the reflection point.

Non-orthogonal wave reflection

$$(V)_n = V\cos\Theta = V^+\cos\Theta + V^-\cos\Theta$$



An algebra similar to that seen in the orthogonal case leads to

$$\frac{Z}{Z_0} = \frac{1}{\cos\Theta}\frac{1+R}{1-R}$$

In practice

$$R(j\omega) = \frac{Z(j\omega)\cos\Theta - Z_0(j\omega)}{Z(j\omega)\cos\Theta + Z_0(j\omega)}$$

From Kuttruff:

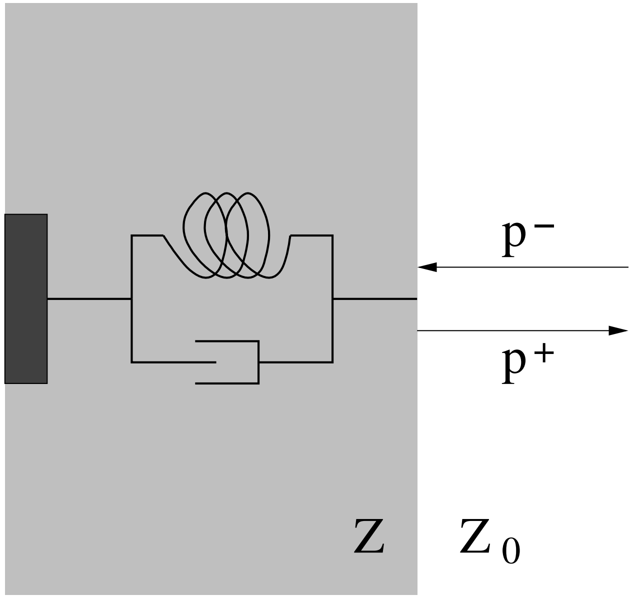
Material	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
Hard surface (bricks, plaster...)	0.02	0.02	0.03	0.03	0.04	0.05
Slightly vibrating wall	0.10	0.07	0.05	0.04	0.04	0.05
Strongly vibrating wall	0.40	0.20	0.12	0.07	0.05	0.05
Carpet	0.02	0.03	0.05	0.10	0.30	0.50
Plush curtain	0.15	0.45	0.90	0.92	0.95	0.50
Polyurethane foam	0.08	0.22	0.55	0.70	0.85	0.75
Acoustic plaster	0.08	0.15	0.30	0.50	0.60	0.70

Absorption coefficients $\alpha = 1 - R$ *at different frequencies for some materials.*

Orthogonal wave reflection in the digital domain

The **Digital Waveguide Filter** (DWF, from Van Duyne & Smith):

$$Z = k_m/j\omega + D$$



$$R(j\omega) = \frac{Z(j\omega) - Z_0}{Z(j\omega) + Z_0} = \frac{k_m/j\omega + D - Z_0}{k_m/j\omega + D + Z_0}$$

Orthogonal wave reflection in the digital domain (2)

Bilinear mapping to the discrete-time domain:

$$j\omega \longleftarrow 2F_s \frac{1-z^{-1}}{1+z^{-1}}$$

hence, the following DWF:

$$R(z) = \frac{\frac{k_m - 2F_s(Z_0 - D)}{k_m + 2F_s(Z_0 + D)} + \frac{k_m + 2F_s(Z_0 - D)}{k_m + 2F_s(Z_0 + D)} z^{-1}}{1 + \frac{k_m - 2F_s(Z_0 + D)}{k_m + 2F_s(Z_0 + D)} z^{-1}}$$

Material	k_m	D
Hard surface (bricks, plaster...)	30.25	69.40
Carpet	9.42	6.49
Acoustic plaster	0.56	3.78

Parameters of the spring/damper system for simulating some materials ($Z_0 = 414$ kgm^{-2}s^{-1}, $F_s = 8$ kHz).

Discrete time and space acoustic wave propagation

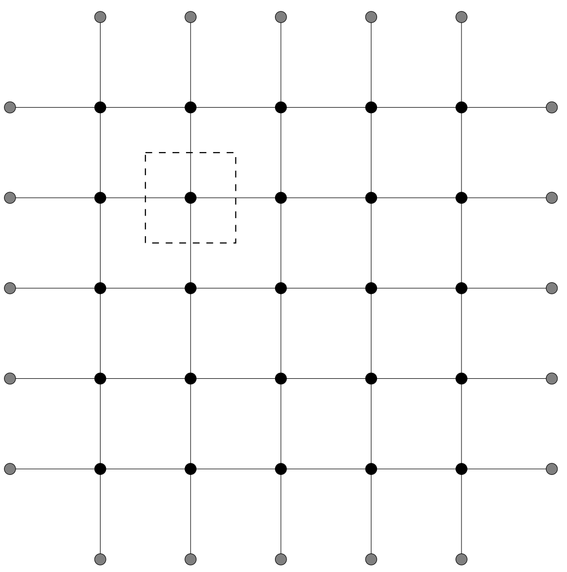
The **Waveguide Mesh**:

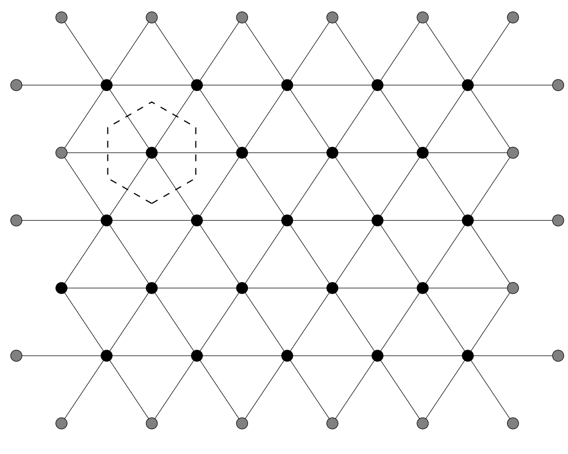
- makes use of *wave scattering*:

$$p_i^-(\boldsymbol{J}, nT) = \frac{2}{N} \sum_{k=1}^{N} p_k^+(\boldsymbol{J}, nT) - p_i^+(\boldsymbol{J}, nT) \quad , \quad i = 1, \ldots, N$$

- outgoing values are sent to their neighbor scattering elements: they will become new incoming samples at the *next* temporal step.

Example (2-D with $N = 4$ and $N = 6$):





Scattering boundaries

Wave scattering with varying waveguide impedances:

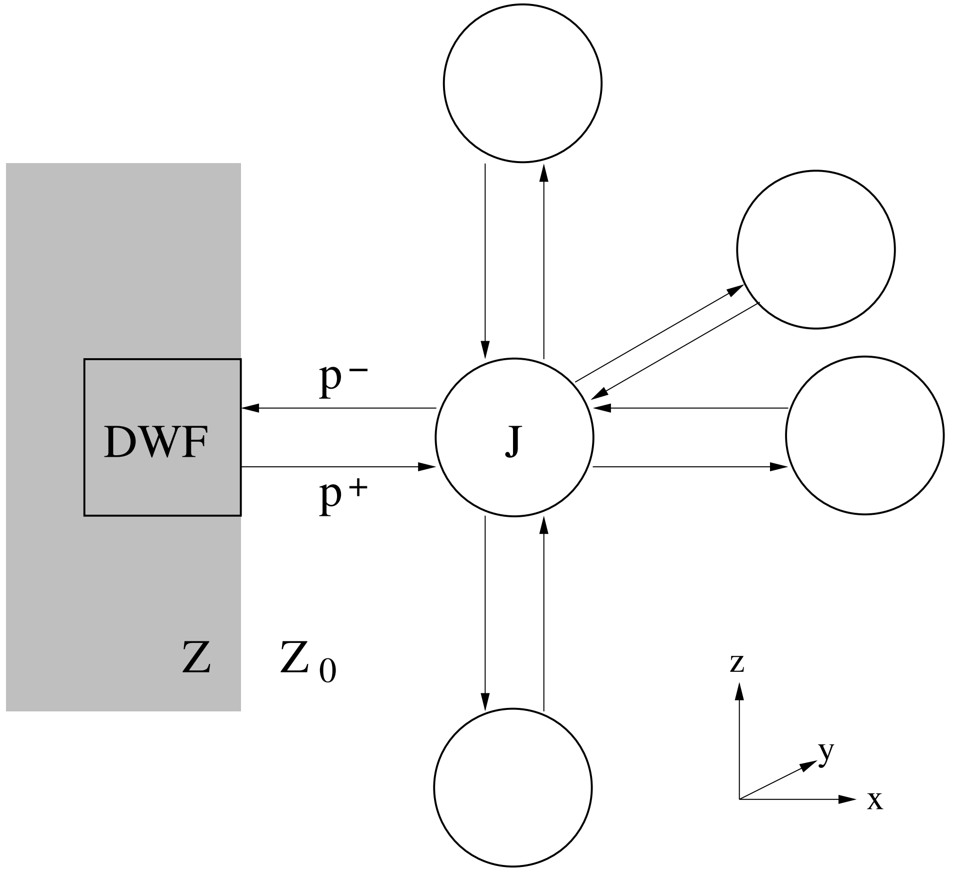
$$P_i^-(\boldsymbol{J}, j\omega) = \frac{2\sum_{k=1}^{N} P_k^+(\boldsymbol{J}, j\omega)/Z_k(j\omega)}{\sum_{k=1}^{N} 1/Z_k(j\omega)} - P_i^+(\boldsymbol{J}, j\omega) \quad , \quad i = 1, \ldots, N$$

This formula will be conveniently used to model the interaction between propagation medium and boundary in the case when $N-1$ waves incide onto the reflection point. In this case we assume that neither incoming nor outgoing waves travel along the waveguide lumped in the boundary, i.e. $P_N^- = P_N^+ = 0$:

$$P_i^-(\boldsymbol{J}, j\omega) = \frac{2\sum_{k=1}^{N-1} P_k^+(\boldsymbol{J}, j\omega)}{N - 1 + \frac{Z_0}{Z(j\omega)}} - P_i^+(\boldsymbol{J}, j\omega), \quad i = 1, \ldots, N-1$$

A simple case: 2-D with N = 4 (or 3-D with N = 6)

These mesh topologies inherently call for orthogonal reflection: all the above treatment can be used to model the boundary.



Non-orthogonal boundary scattering

According with acoustic theory we postulate that

- the *parallel* component $p^{\|+}$ of every incoming wave is scattered out by a junction that is *not* influenced by the boundary:

$$P_i^{\|-}(\boldsymbol{J}, j\omega) = \frac{2}{N-1}\sum_{k=1}^{N-1} P_k^{\|+}(\boldsymbol{J}, j\omega) - P_i^{\|+}(\boldsymbol{J}, j\omega), \quad i = 1, \ldots, N-1$$

- the *perpendicular* component $p^{\perp+}$ of every incoming wave is scattered out by a junction that is *loaded* with the boundary impedance:

$$P_i^{\perp-}(\boldsymbol{J}, j\omega) = \frac{2\displaystyle\sum_{k=1}^{N-1} P_k^{\perp+}(\boldsymbol{J}, j\omega)}{N-1+\dfrac{Z_0}{Z}} - P_i^{\perp+}(\boldsymbol{J}, j\omega), \quad i = 1, \ldots, N-1$$

- moreover: $P_N^{\perp-}(\boldsymbol{J}, j\omega) = P_N^{\perp+}(\boldsymbol{J}, j\omega) = P_N^{\|-}(\boldsymbol{J}, j\omega) = P_N^{\|+}(\boldsymbol{J}, j\omega) = 0$.

Examples

- orthogonal reflection ($\Theta = 0$, $N = 2$) $\Rightarrow p_2 = 0\,,\; p_1^{\parallel +} = p_1^{\parallel -} = 0$. Hence:

$$P_1^-(j\omega) = P_1^{\perp -}(j\omega) = \frac{2P_1^{\perp +}(j\omega)}{1 + Z_0/Z} - P_1^{\perp +}(j\omega) = \frac{Z - Z_0}{Z + Z_0} P_1^{\perp +}(j\omega) = RP_1^+(j\omega)$$

- total absorption ($Z = 0$):

$$p_i^{\perp -}(\boldsymbol{J}, nT) = -p_i^{\perp +}(\boldsymbol{J}, nT) \quad , \quad i = 1, \ldots, N-1$$

(parallel components ruled by lossless scattering)

- total reflection ($Z = \infty$):

$$p_i^{\perp -}(\boldsymbol{J}, nT) = \frac{2}{N-1} \sum_{k=1}^{N-1} p_k^{\perp +}(\boldsymbol{J}, j\omega) - p_i^{\perp +}(\boldsymbol{J}, nT) \quad , \quad i = 1, \ldots, N-1$$

(parallel components ruled by lossless scattering).

Mirrored reflection ($\Theta_1 = -\Theta_2 = \Theta$, $N = 3$)

Suppose for simplicity $p_1^+ \neq 0$, $p_2^+ = 0$. Hence,

- total absorption yields:

$$
\begin{aligned}
p_1^{\|-} &= 0 \quad , \quad p_2^{\|-} = p_1^{\|+} \\
p_1^{\perp-} &= -p_1^{\perp+} \quad , \quad p_2^{\perp-} = 0
\end{aligned}
$$

thus

$$
\begin{aligned}
p_1 &= \sqrt{(p_1^{\|+} + p_1^{\|-})^2 + (p_1^{\perp+} + p_1^{\perp-})^2} = p_1^{\|+} \\
p_2 &= \sqrt{(p_2^{\|+} + p_2^{\|-})^2 + (p_2^{\perp+} + p_2^{\perp-})^2} = p_1^{\|+}
\end{aligned}
$$

Mirrored reflection (2)

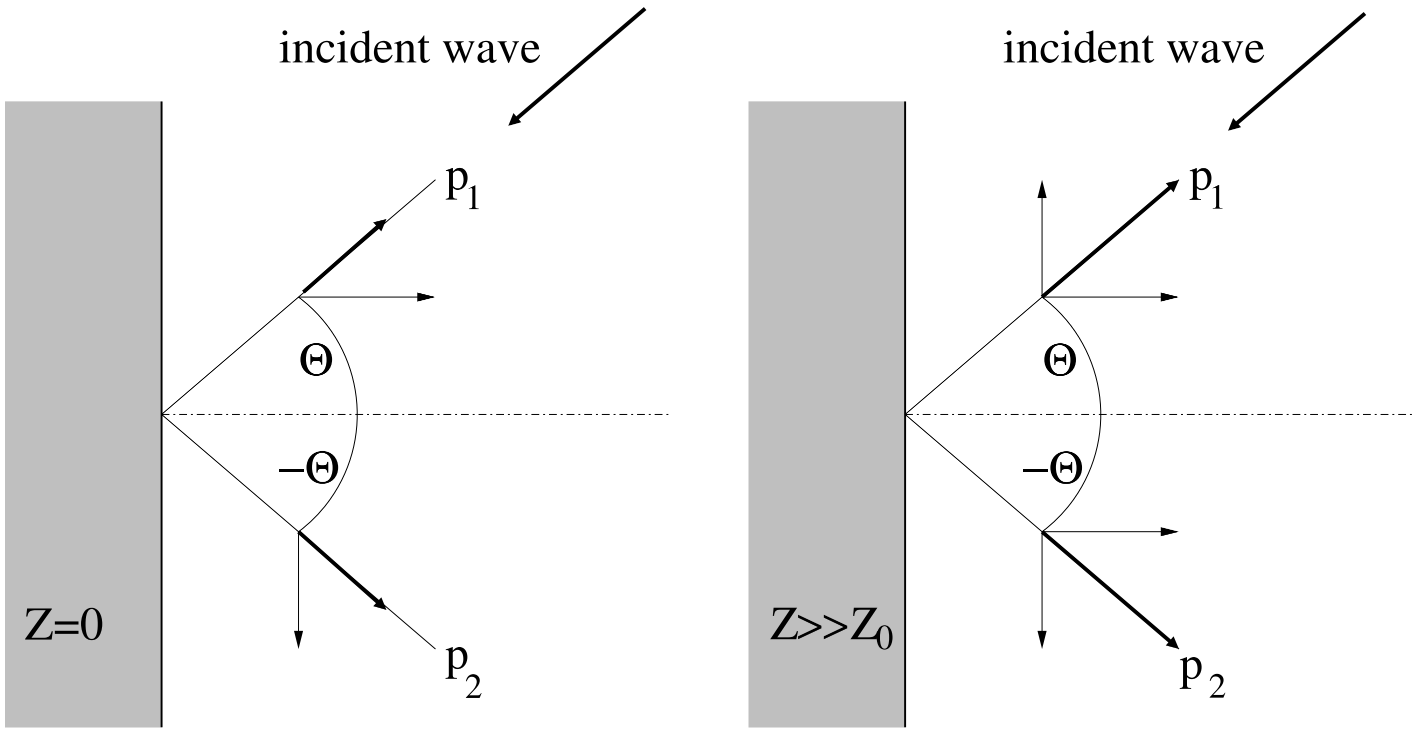
- total reflection yields:

$$
\begin{aligned}
p_1^{\|-} &= 0 \quad , \quad p_2^{\|-} = p_1^{\|+} \\
p_1^{\perp-} &= 0 \quad , \quad p_2^{\perp-} = p_1^{\perp+}
\end{aligned}
$$

thus

$$
\begin{aligned}
p_1 &= \sqrt{(p_1^{\|+} + p_1^{\|-})^2 + (p_1^{\perp+} + p_1^{\perp-})^2} = p_1^+ \\
p_2 &= \sqrt{(p_2^{\|+} + p_2^{\|-})^2 + (p_2^{\perp+} + p_2^{\perp-})^2} = p_1^+
\end{aligned}
$$

Mirrored reflection (3)



Mirrored reflection: the general case

$$
\begin{aligned}
P_1^{\|-} &= 0 \quad , \quad & P_2^{\|-} &= P_1^{\|+} \\
P_1^{\perp-} &= \tfrac{-Z_0}{Z_0+2Z} P_1^{\perp+} \quad , \quad & P_2^{\perp-} &= \tfrac{2Z}{Z_0+2Z} P_1^{\perp+}
\end{aligned}
$$

Define a *modified reflection factor*:

$$\rho^2(j\omega) = \frac{\sum_{i=1}^{N-1} |P_i^-(\boldsymbol{J}, j\omega)|^2}{\sum_{i=1}^{N-1} |P_i^+(\boldsymbol{J}, j\omega)|^2}$$

It descends (remind $P_i^{\|} = P_i \sin\Theta_i$ and $P_i^{\perp} = P_i \cos\Theta_i$):

$$
\begin{aligned}
\rho^2 &= \frac{|P_1^-|^2 + |P_2^-|^2}{|P_1^+|^2} = \frac{|P_1^{\|-}|^2 + |P_1^{\perp-}|^2 + |P_2^{\|-}|^2 + |P_2^{\perp-}|^2}{|P_1^{\|+}|^2 + |P_1^{\perp+}|^2} \\
&= \left|\frac{-Z_0}{Z_0+2Z}\right|^2 \cos^2\Theta + \sin^2\Theta + \left|\frac{2Z}{Z_0+2Z}\right|^2 \cos^2\Theta
\end{aligned}
$$

Passivity

Modified reflection factor:

$$\rho^2 = \frac{\sum_{i=1}^{N-1} |P_i^-|^2}{\sum_{i=1}^{N-1} |P_i^+|^2} = \frac{\sum_{i=1}^{N-1} |P_i^{\|-}|^2 + |P_i^{\perp -}|^2}{\sum_{i=1}^{N-1} |P_i^{\|+}|^2 + |P_i^{\perp +}|^2}$$

From the theory of Digital Waveguide Networks (see Bilbao):

$$\sum_{i=1}^{N-1} |P_i^-|^2/Z_i \leq \sum_{i=1}^{N} |P_i^-|^2/Z_i = \sum_{i=1}^{N} |P_i^+|^2/Z_i = \sum_{i=1}^{N-1} |P_i^+|^2/Z_i$$

and, since $Z_1 = \ldots = Z_{N-1} = Z_0$, then immediately $\rho^2 \leq 1$, hence *stability*.

Passivity - Further observations

From $\rho^2 \leq 1$ and

$$\sum_{i=1}^{N-1} |P_i^{\|-}|^2 = \sum_{i=1}^{N-1} |P_i^{\|+}|^2$$

then

$$\sum_{i=1}^{N-1} |P_i^{\perp-}|^2 \leq \sum_{i=1}^{N-1} |P_i^{\perp+}|^2$$

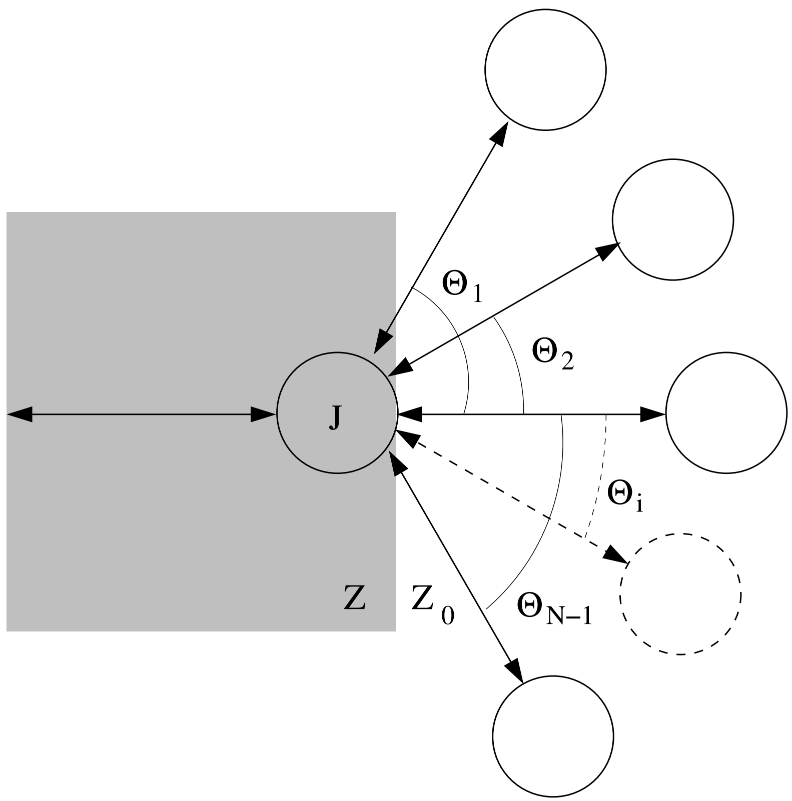
This means that passivity is only due to surface absorption, as expected.

Scattered boundaries - A bridge to diffusion (?)

Recent results in visual reflection (Ramamoorthi & Hanrahan):

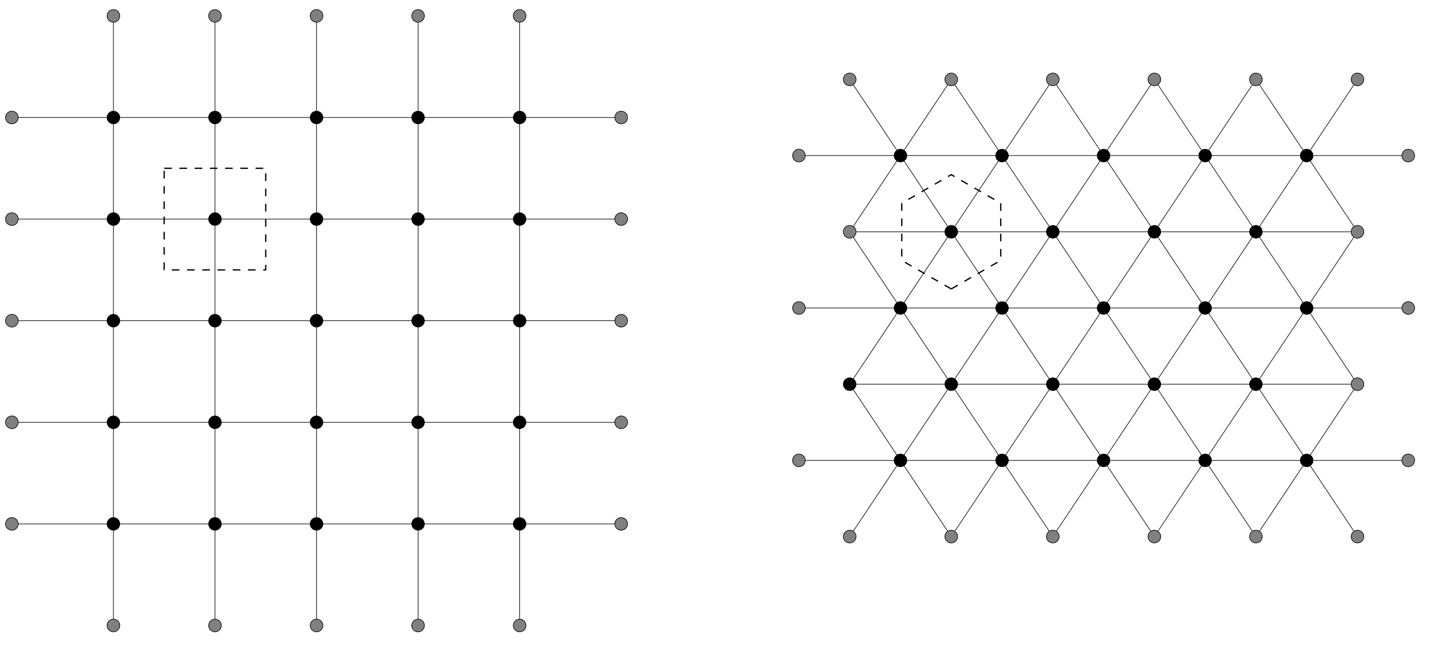
$$B(\boldsymbol{x}, \theta_0) = \int_{-\pi/2}^{\pi/2} L(\boldsymbol{x}, \theta_i)\rho(\theta_0, \theta_i)\cos\theta_i d\theta_i$$

Scattered reflection weights the outcoming energy among the waveguide branches. Is this a good model for discrete-space diffusion?



Simulations (still ongoing)

Application of scattered reflection to 2-D waveguide mesh geometries.



Any suggestion on possible figures to collect?

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